GEOMETRY

1. Line & Angle
2. Triangle
3. circle
4. Quadrilateral
5. Polygon
6. Co-ordinate geometry

* **Line & Angle :-**

Types of angle

1. Based on degree
2. Based on line

1. **Based on degree :-**

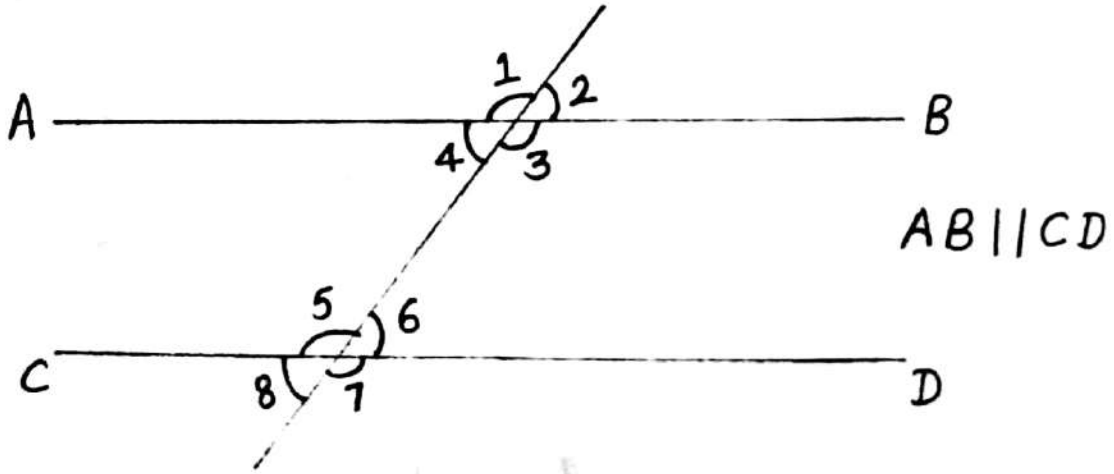
(a) Acute angle :- less than 90°.
(b) Right angle :- equal to 90°.
(c) obtuse angle :- $90° < \theta < 180°$
(d) straight line angle :- equal to 180°
(e) Reflex angle :- $180° < \theta < 360°$
(f) Complete / one round angle :- equal to 360°

Complementary angle :- Sum of two angles is 90°.

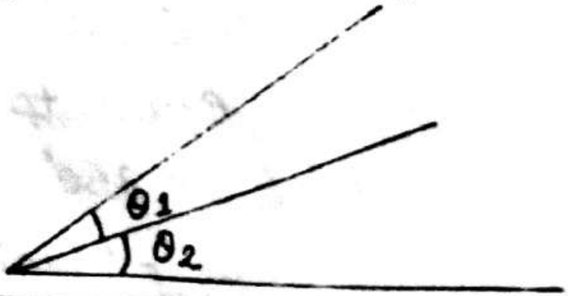
Supplementary angle :- Sum of two angles is 180°.
(Linear Pair)

2. Based on line :-

(a) Vertically opposite angle $\boxed{1 \Rightarrow 3}$
(b) Corresponding angle $\boxed{1 \Rightarrow 5}$ $\boxed{4 \Rightarrow 8}$
(c) Alternate angle (Interior + Exterior)
$\boxed{4 \Rightarrow 6}$ $\boxed{1 \Rightarrow 7}$
(d) Adjacent angle $\boxed{1 \Rightarrow 4}$ $\boxed{1 \Rightarrow 2}$



Adjacent angle :- Common point & line



$\angle 1 = \angle 3$ (Vert opposite) $\angle 1 + \angle 2 = 180°$ (Supp.)

$\angle 2 = \angle 6$ (Corresponding) $\angle 3 + \angle 6 = 180°$ (Supp.)

* <u>Triangle</u> :- A Closed figure, three lines or three sides is called triangle.

<u>Types of triangle</u> :-

1. Based on angle
2. Based on sides

1. <u>Based on angles</u> :-

(a) Acute angle triangle :- all angles are acute angle ($< 90°$)

(b) Right angle triangle :- one angle is 90°.

(c) obtuse angle triangle :- one angle is obtuse($> 90°$)

2. <u>Based on sides</u> :-

(a) Equilateral triangle :- All sides are equal & each angle is 60°.

(b) Isosceles triangle :- Two sides are equal.

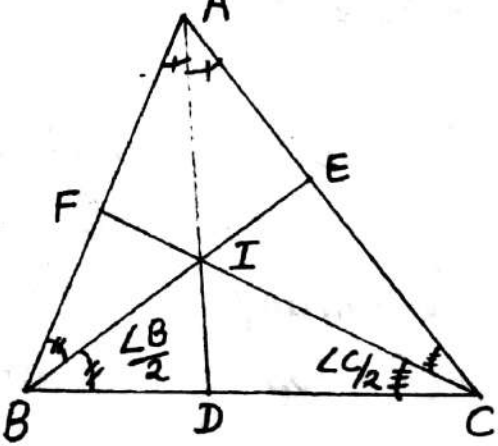
(c) Scalene triangle :- All sides are different.

Centre of triangle:-

1. Incentre
2. Circumcentre
3. Orthocentre
4. Centroid

1. Incentre (I):- Intersection point of angle bisectors is called incentre.

Angle property of Incentre:-



In ΔBIC -

$$\angle BIC = 180° - \frac{1}{2}(\angle B + \angle C)$$
$$= 180° - \frac{1}{2}(180° - \angle A)$$
$$= 180° - 90° + \frac{\angle A}{2}$$
$$\boxed{\angle BIC = 90° + \frac{\angle A}{2}}$$

Similarly,

$$\boxed{\angle AIC = 90^\circ + \frac{\angle B}{2}}, \quad \boxed{\angle AIB = 90^\circ + \frac{\angle C}{2}}$$

1. In a ΔABC, I is a incentre. If $\angle BAC = 80^\circ$, find angle $\angle BIC$.

Sol:- $\angle BIC = 90^\circ + \frac{80^\circ}{2}$

$\angle BIC = \boxed{130^\circ}$

2. In ΔABC, I is a incentre. If $\angle BIC = 135^\circ$, then $\angle ABC$ will be

(a) Acute angle triangle [✓ (b) Right angle triangle]

(c) obtuse angle triangle (d) None of these

Sol:- $135^\circ = 90^\circ + \frac{\angle A}{2}$

$\angle A = \boxed{90^\circ}$, Right angle triangle

3. In ΔABC, I is a incentre. If $\angle BIC = 145^\circ$, then ΔABC will be

Sol:- $145^\circ - 90^\circ = \angle A/2$

$\angle A = \boxed{110^\circ}$, obtuse angle triangle

4. In ΔABC, I is a incentre. If $\angle B = 60^\circ$, $\angle C = 80^\circ$ find $\angle BIC$.

Sol:- $\angle BIC = 180^\circ - \frac{1}{2}(\angle B + \angle C)$

$= 180^\circ - 70^\circ = \boxed{110^\circ}$

5. In a ΔABC, BO & CO are exterior angle bisector of $\angle B$ & $\angle C$, then $\angle BOC$ = ?

(a) $(\angle B + \angle C)$ [✓ (b) $\frac{1}{2}(\angle B + \angle C)$] (c) $\frac{1}{3}(\angle B + \angle C)$

(d) $\frac{1}{4}(\angle B + \angle C)$

Sol:- $\boxed{\angle BOC = \frac{1}{2}(\angle B + \angle C)}$

Or

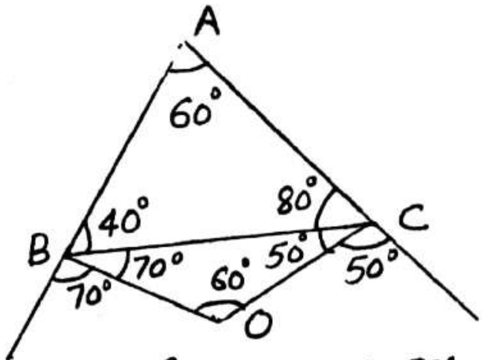
$\boxed{\angle BOC = 90^\circ - \frac{\angle A}{2}}$

6. In ΔABC, BO & CO are exterior angle bisector of $\angle B$ & $\angle C$, then $\angle BOC$ = ?

(a) 150° (b) 30° (c) 120° (d) 60°

Sol:- $\angle BOC = 90^\circ - \frac{60^\circ}{2}$

$= \boxed{60^\circ}$



7. In ΔABC, I is a incentre. BO & CO are exterior angle bisector of $\angle B$ & $\angle C$. Find the sum of $\angle BIC + \angle BOC = ?$

Sol:- $\angle BIC = 90^\circ + \frac{\angle A}{2}$

$\angle BOC = 90^\circ - \frac{\angle A}{2}$

$\therefore \boxed{\angle BIC + \angle BOC = 180^\circ}$

8. In ΔABC, I is a incentre. BO & CO are exterior angle bisector of $\angle B$ & $\angle C$. If $\angle BIC = 140^\circ$, $\angle BOC = ?$

Sol:- $\angle BOC = 180^\circ - 140^\circ$

$= \boxed{40^\circ}$

9. In ΔABC, I is a incentre of triangle. BO & CO are exterior angle bisector of $\angle B$ & $\angle C$. If $\angle BOC = 80^\circ$, $\angle BIC = ?$

SOL:- $\angle BIC = 180^\circ - 80^\circ$

$= \boxed{100^\circ}$

10. In ΔABC, BO & CO are exterior angle bisector of $\angle B$ & $\angle C$. If $\angle BAC = 60^\circ$, $\angle BOC = ?$

SOL:- $\angle BAC = 60^\circ$

$\angle BOC = 90^\circ - \frac{\angle A}{2}$

$= 90^\circ - 30^\circ$

$\angle BOC = \boxed{60^\circ}$

11. In ΔABC, BO & CO are exterior angle bisector of $\angle B$ & $\angle C$. If $\angle B = 40^\circ$, $\angle C = 60^\circ$, $\angle BOC = ?$

SOL:- $\angle BOC = \frac{1}{2}(\angle B + \angle C)$

$= \frac{1}{2} \times 100^\circ = \boxed{50^\circ}$

12. In ΔABC, BO & CO are exterior angle bisector of $\angle B$ & $\angle C$. If $\angle BOC = 45^\circ$, ΔABC will be

(a) Acute angle triangle (b) Right angle triangle

(c) obtuse angle triangle (d) None of these

Sol:- $\angle BAC = (90^\circ - \angle BOC) \times 2$

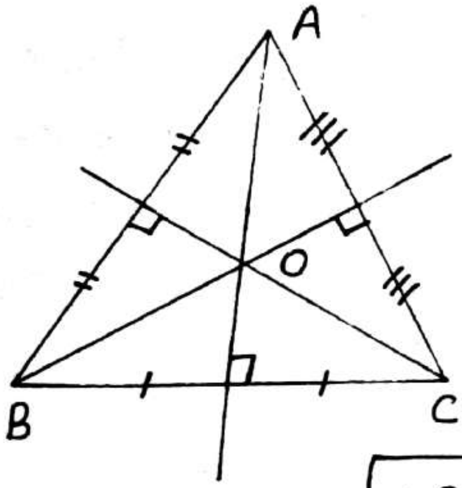
$= (90^\circ - 45^\circ) \times 2$

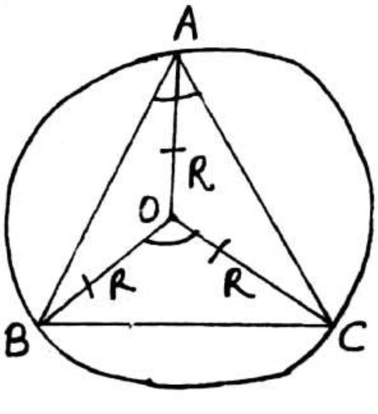
$= \boxed{90^\circ}$

$\therefore \boxed{B}$ Ans

2. <u>Circumcentre</u>:- Intersection point of sides perpendicular bisectors is called Circumcentre.

<u>Angle property of circumcentre</u>:-





$\boxed{\angle BOC = 2\angle A}$

$\boxed{\angle AOC = 2\angle B}$, $\boxed{\angle AOB = 2\angle C}$

$OA = OB = OC = R$

(Circumradius)

13. In ΔABC, O is a circumcentre. If $\angle BAC = 50^\circ$, find $\angle BOC = ?$

Sol:- $\angle BOC = 2\angle BAC$

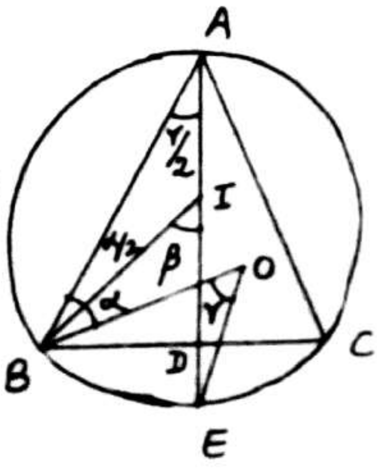
$= 2 \times 50^\circ = \boxed{100^\circ}$

14. In ΔABC O is a circumcentre. If $\angle BOC = 120^\circ$, $\angle BAC = ?$

Sol:- $\angle BAC = \frac{1}{2}\angle BOC$

$= \frac{1}{2} \times 120^\circ = \boxed{60^\circ}$

15. In ΔABC, points I & O are incentre & circumcentre. AD is a angle bisector of $\angle A$ intersect side BC at D and meet circumcircle of ΔABC at E. If $\angle ABC = \alpha^\circ$, $\angle BOE = \gamma^\circ$, $\angle BID = \beta^\circ$, $\frac{\alpha+\gamma}{\beta} = ?$



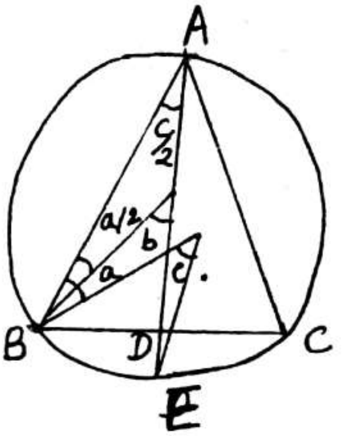
Sol:- In a ΔAIB,

Exterior angle = Sum of opposite interior angles

$$\beta = \frac{\gamma}{2} + \frac{\alpha}{2}$$

$$\frac{\alpha+\gamma}{\beta} = \boxed{2}$$

16. In ΔABC, points I & O are incentre & circum-centre. AD is a angle bisector of $\angle A$ intersect side BC at D and meet circumcircle of ΔABC at E. If $\angle ABC = a$, $\angle BOE = c$, $\angle BID = b$, $\frac{a+c}{b} = ?$



Sol:- $b = \frac{a}{2} + \frac{c}{2}$

$$b = \frac{a+c}{2}$$

$$\frac{a+c}{b} = \boxed{2}$$

17. A pole standing on centre of triangle if all elevation angle to top of the pole from all vertices are equal then centre of triangle will be

(a) Incentre ✓(b) Circumcentre (c) orthocentre
(d) Centroid

$\boxed{B}$ Ans

Sol:- $\boxed{\text{Circumcentre}}$

18. In a triangle, how many points which are equal distance from all vertices?

✓(a) 1 (b) 2 (c) 3 (d) 4

19. Which of the following centeres is equal distance from all vertices ?

(a) Incentre [✓ (b) Circumcentre] (c) Centroid

(d) orthocentre

20. In a ΔABC, points I & O are incentre & circumcentre. If $\angle BIC = 130^\circ$, $\angle AOC = 100^\circ$ then $\angle ACB = ?$

Sol:-

$$\angle BIC = 130^\circ$$
$$\angle BIC = 90^\circ + \frac{\angle A}{2}$$
$$130^\circ = 90^\circ + \frac{\angle A}{2}$$
$$\angle A = 80^\circ$$

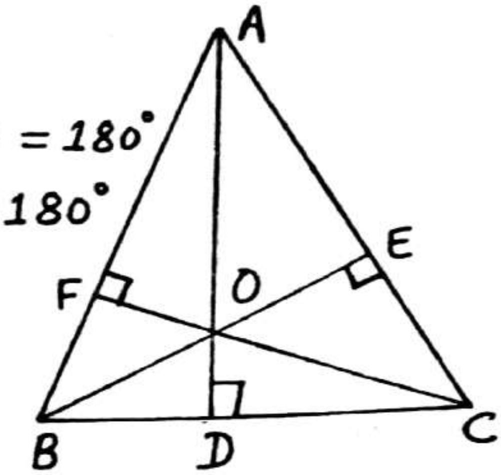
$$\angle AOC = 100^\circ$$
$$\angle AOC = 2\angle B$$
$$100^\circ = 2\angle B$$
$$\angle B = 50^\circ$$

$$\angle A + \angle B + \angle C = 180^\circ$$
$$\angle C = 180^\circ - 130^\circ$$
$$\angle C = \boxed{50^\circ}$$

3. Orthocentre :- Intersection point of heights or perpendicular is called orthocentre.

Angle property of orthocentre :-

In $\square AFOE$ -



$\angle AFO + \angle AEO = 180^\circ$

$\angle A + \angle EOF = 180^\circ$

$\angle EOF = \angle BOC$

(Vert. opposite)

$\angle A + \angle BOC = 180^\circ$

$\boxed{\angle BOC = 180^\circ - \angle A}$

$\boxed{\angle AOC = 180^\circ - \angle B}$

$\boxed{\angle AOB = 180^\circ - \angle C}$

21. In ΔABC, O is a orthocentre. If $\angle BAC = 46^\circ$, find $\angle BOC = ?$

Sol:- $\angle BOC = 180^\circ - \angle A$

$\angle BOC = 180^\circ - 46^\circ$

$= \boxed{134^\circ}$

22. In ΔABC, O is orthocentre. If $\angle BOC = 120^\circ$, $\angle BAC = ?$

Sol:- $\angle BAC = 180^\circ - \angle BOC$

$= 180^\circ - 120^\circ$

$= \boxed{60^\circ}$

23. O is orthocentre of ΔABC. If $\angle BOC = 70^\circ$, ΔABC will be

(a) Acute angle triangle (b) Right angle triangle
(c) obtuse angle triangle (d) None of these

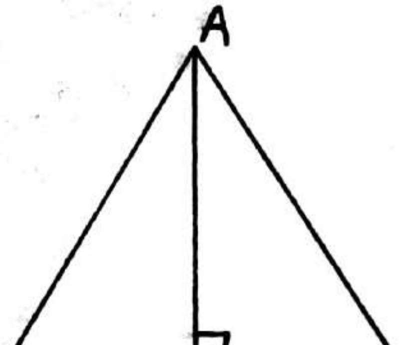
Sol:- $\angle BAC = 180^\circ - 70^\circ$

$= \boxed{110^\circ}$

24. In a triangle, ratio of sum of sides & sum of heights is-

(a) equal to 1 (b) less than 1
[✓ (c) greater than 1] (d) equal or greater than 1

Sol:



$AC > AD$

$S_1 > H_1$

$(S_1 + S_2 + S_3) > (H_1 + H_2 + H_3)$

$$\Rightarrow \frac{(S_1 + S_2 + S_3)}{(H_1 + H_2 + H_3)} > 1$$

25. In a triangle, ratio of sum of heights &

sum of sides is -

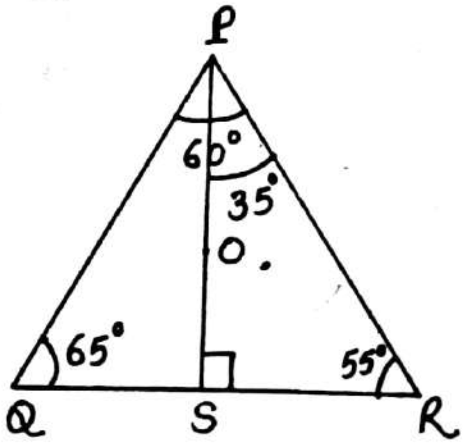
(a) equal to 1 [✓ (b) less than 1]

(C) greater than 1 (d) equal or greater than 1

SOL:- $\frac{H_1 + H_2 + H_3}{S_1 + S_2 + S_3} < 1$

Note:- Sum of sides is always greater than sum of heights.

26. In ΔPQR, points O & C are orthocentre & circumcentre. P[illegible] is extended meet QR at S. If $\angle PQR = 65°$, $\angle QCR = 120°$, find $\angle SPR = ?$



SOL:- $\angle QCR = 120°$

$\angle QCR = 2\angle P$

$120° = 2\angle P$

$\angle P = 60°$

$\angle R = 180° - 125°$

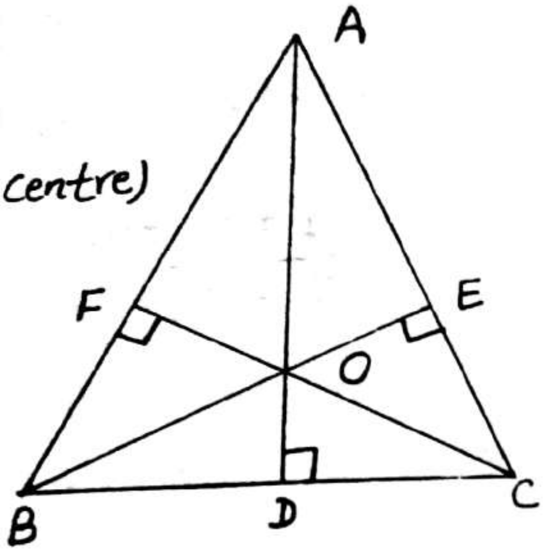
$= 55°$

$\angle SPR = 180° - 145° = \boxed{35°}$

27. In ΔABC, AD, BE & CF are heights & O is a orthocentre of ΔABC, then point A is orthocentre of triangle -

(a) ΔAOB [✓ (b) ΔBOC] (c) ΔAOC (d) ΔABC

Sol:- $\Delta BOC = A$
(ortho centre)



4 orthocentres
(O, A, B, C)

28. In ΔABC, AD, BE & CF are heights & O is a orthocentre of ΔABC, then point B is orthocentre of triangle -

(a) ΔAOB (b) ΔBOC [✓ (c) ΔAOC]
(d) ΔABC

Sol:- $\Delta AOC = B$
(orthocentre)

$\left\{ \begin{array}{l} \perp \text{ from A} \\ \text{AD, AE \& AF} \end{array} \right.$

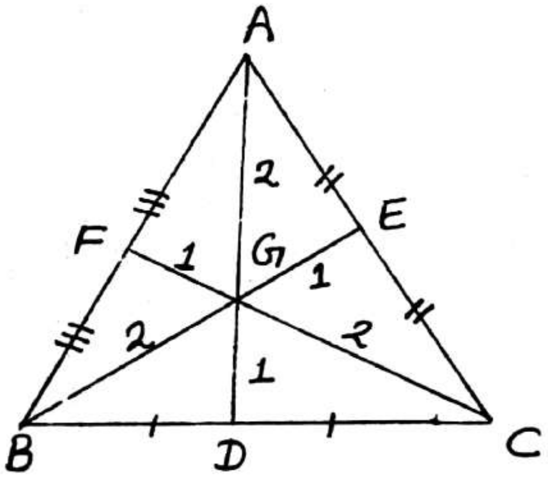
29. In ΔABC, AD, BE & CF are heights & O is orthocentre of ΔABC, then point C is orthocentre of triangle -

[✓ (a) ΔAOB] (b) ΔBOC (c) ΔAOC (d) ΔABC

SoL :- $\Delta AOB = C$ (orthocentre)

4. **Centroid (G)** :- Intersection point of meadians is called centroid.

Properties of centroid :-



$$\frac{AG}{GD} = \frac{BG}{GE} = \frac{CG}{GF} = \frac{2}{1}$$

$$\boxed{AG \neq BG \neq CG}$$

~~Ex~~ If $AD = BE = CF$

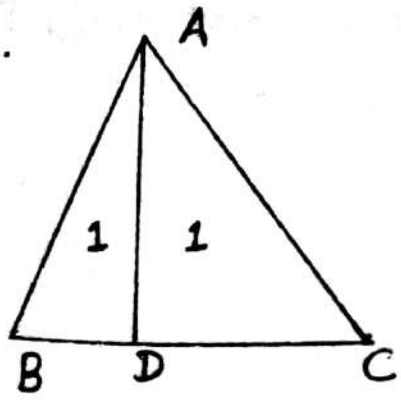
(Triangle is equilateral)

then,

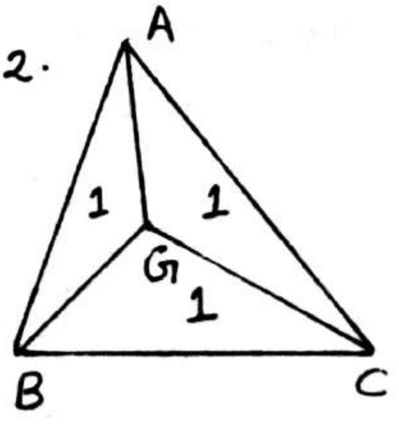
$$\boxed{AG = BG = CG \;\; \& \;\; GD = GE = GF}$$

Properties of Medians based on Area :-

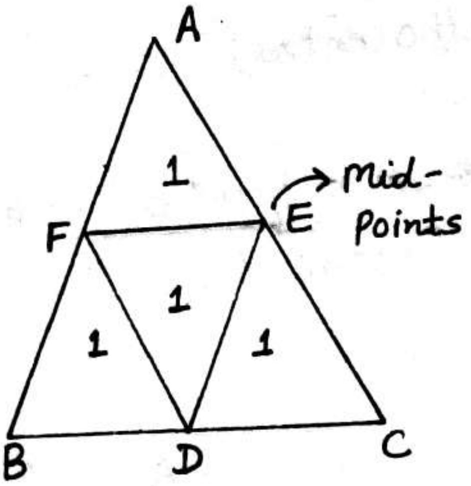
1.



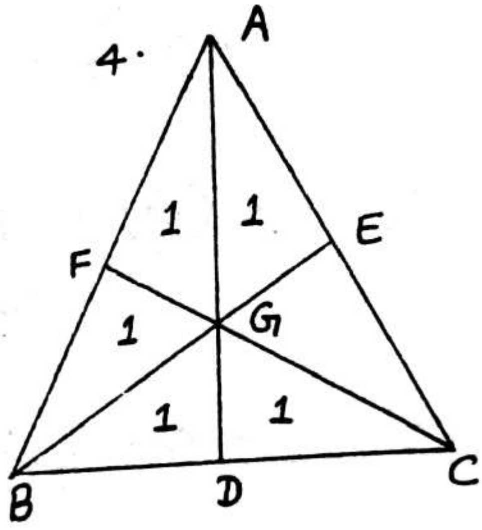
2.



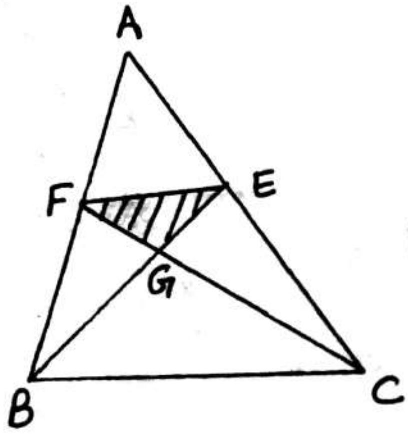
3.



4.



5.



$$\frac{\Delta ABC}{\Delta EGF} = \frac{12}{1}$$

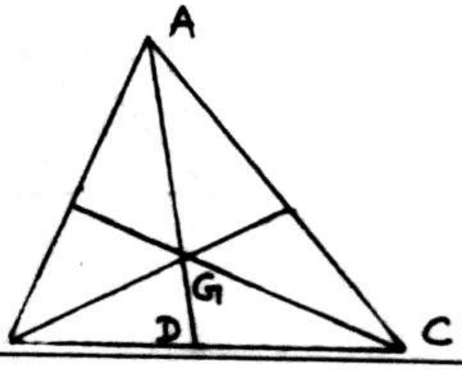
mid-Point Centroid

$$\Delta GEF = \square AFGE - \Delta AEF$$

$$= \frac{1}{3} - \frac{1}{4} = \frac{1}{12} \quad (\Rightarrow \text{Total} = 12)$$

30. In ΔABC, AD is meadian & G is centroid. If ar.$(\Delta ABC) = 300\ cm^2$, ar.$(\Delta BDG) = ?$

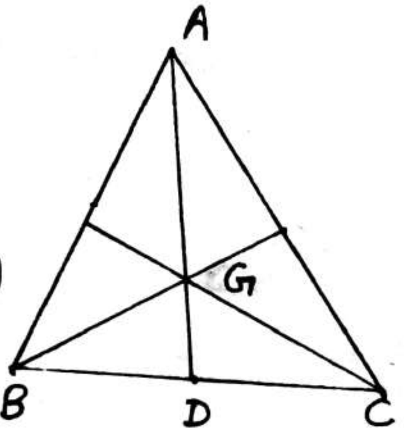
Sol:-



$$ar(\Delta BDG) = \frac{1}{6}\ ar(\Delta ABC)$$
$$= \frac{1}{6} \times 300 = \boxed{50\ cm^2}$$

31. In ΔABC, AD is meadian & G is centroid. If $ar(\Delta BDG) = 20\ cm^2$, $ar(\Delta ABC) = ?$

SOL:-

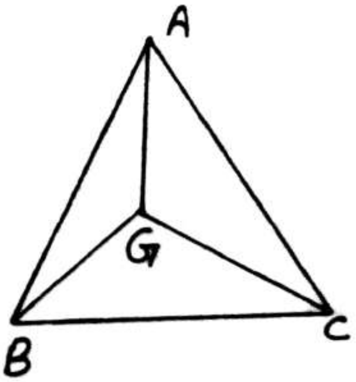


$\because ar(\Delta ABC) = 6\ ar(\Delta BDG)$

$\therefore ar(\Delta ABC) = 6 \times 20 = \boxed{120\ cm^2}$

32. In ΔABC, G is Centroid. Find ratio of area of ΔABC & ΔBGC.

Sol:- $$\frac{ar(\Delta ABC)}{ar(\Delta BGC)} = \frac{3}{1}$$

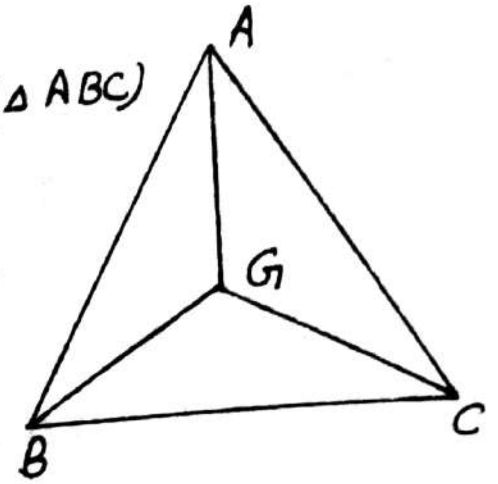


33. In ΔABC, G is Centroid. If $ar(\Delta ABC) = 120\ cm^2$, then find $ar(\Delta BGC) = ?$

Sol:- $ar(\Delta BGC) = \frac{1}{3} ar(\Delta ABC)$

$= \frac{1}{3} \times 120$

$= \boxed{40 \text{ cm}^2}$

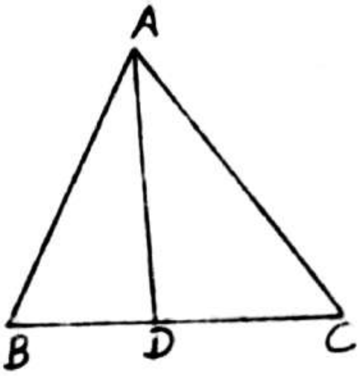


34. In ΔABC, AD is a meadian of $ar.(\Delta ABD) = 100 \text{ cm}^2$, then $ar.(\Delta ABC) = ?$

Sol:- $ar(\Delta ABC) = 2\, ar(\Delta ABD)$

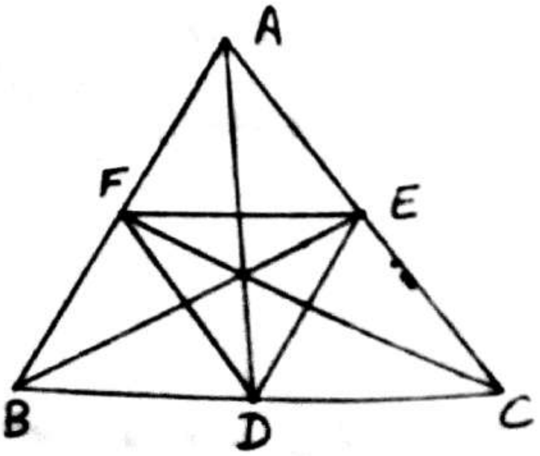
$= 2 \times 100$

$= \boxed{200 \text{ cm}^2}$



35. In ΔABC, points D, E & F are mid-points of sides BC, AC & AB. Find ratio of $ar(\Delta ABC)$ & $ar(\Delta DEF)$

Sol:- $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{4}{1}$

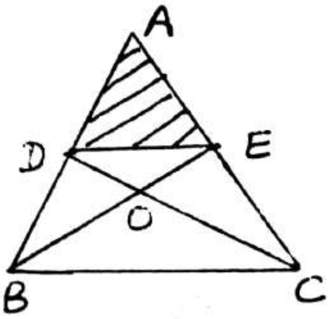


36. In ΔABC, points D & E are midpoints of sides AB & AC. Find ratio of ar (ΔABC) & ar(ΔADE)

Sol:- $\frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \frac{4}{1}$

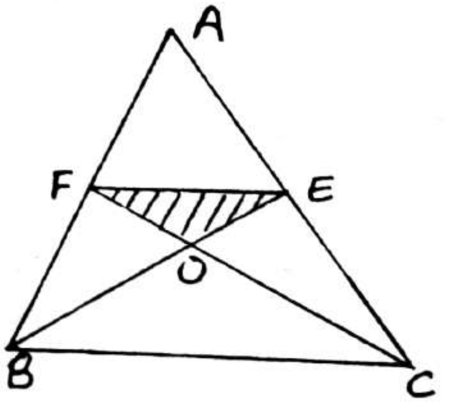
$\Delta ADE = \square ADOE - \Delta DOE$

$= \frac{1}{3} - \frac{1}{12} = \frac{1}{4}$



37. In ΔABC, BE & CF are median intersect at O. Find ratio of ar. (ΔABC) & ar(ΔEOF).

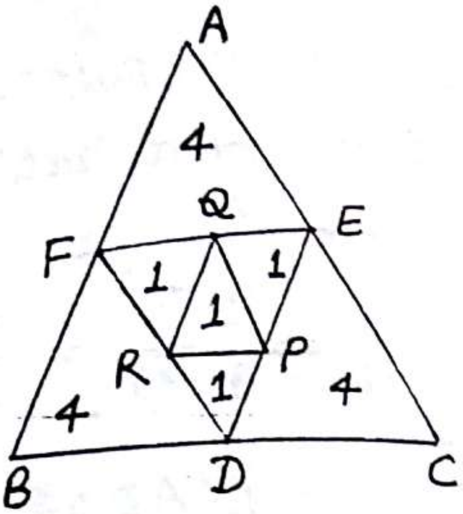
Sol:- $\frac{ar(\Delta ABC)}{ar(\Delta EOF)} = \frac{12}{1}$



38. In ΔABC, points D, E & F are mid-points of sides BC, AC & AB. Points P, Q & R mid-points of DE, EF & DF. Find ratio of ar(ΔABC) & ar (ΔPQR).

Sol:-

$$\frac{ar.(\Delta ABC)}{ar(\Delta PQR)} = \frac{16}{1}$$

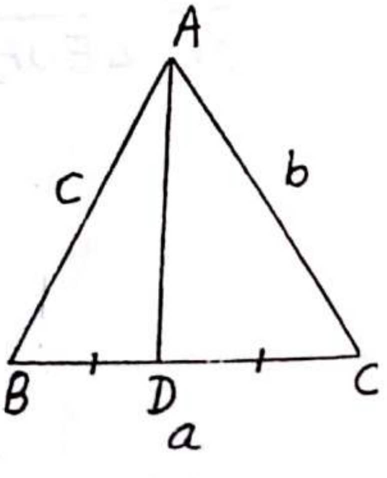


* Relation between sides & medians :-

1. $\boxed{\text{Sum of sides} > \text{Sum of medians}}$

2. $\boxed{3(\text{Sum of sides}) < 4(\text{Sum of medians})}$

3. In ΔABC, AD is a median.



$AB = c$

$AC = b$

$BC = a$

By Apollonius theorems-

$$\boxed{AB^2 + AC^2 = 2(AD^2 + BD^2)}$$

$$\frac{c^2+b^2}{2} = AD^2 + \frac{a^2}{4}$$

$$\boxed{AD^2 = \frac{2c^2+2b^2-a^2}{4}}$$

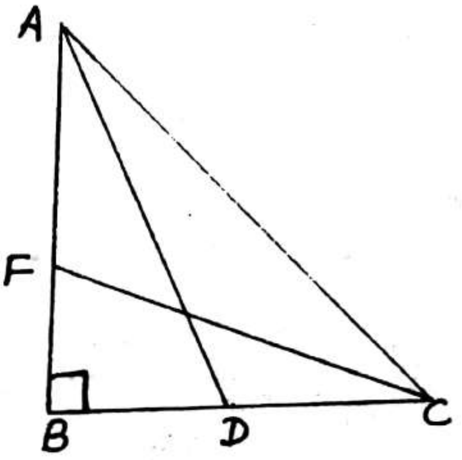
$$\boxed{BE^2 = \frac{2a^2+2c^2-b^2}{4}}, \quad \boxed{CF^2 = \frac{2a^2+2b^2-c^2}{4}}$$

(4) $AD^2 + BE^2 + CF^2 = \frac{3}{4}\,(a^2 + b^2 + c^2)$

$$\boxed{3\,(AB^2 + BC^2 + AC^2) = 4\,(AD^2 + BE^2 + CF^2)}$$

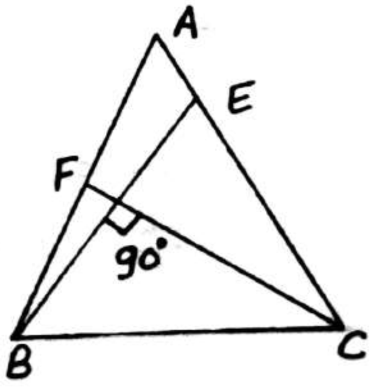
5. $$\boxed{3\,(\underset{\substack{\downarrow\\ \text{Area.}}}{\Delta ABC}) = 4\,(\Delta M_1 M_2 \underset{\substack{\downarrow\\ \text{Median}}}{M_3})}$$

6. If



$$\boxed{4\,(AD^2 + CF^2) = 5AC^2}$$

7. If $BE \perp CF$



$$\boxed{AB^2 + AC^2 = 5BC^2}$$

39 In ΔABC, AD, BE & CF are medians, then which of the following is true -

[✓ (a)] $(AB+BC+AC) > (AD+BE+CF)$

(b) $(AB+BC+AC) < (AD+BE+CF)$

(c) $(AB+BC+AC) \geq (AD+BE+CF)$

(d) $(AB+BC+AC) \leq (AD+BE+CF)$

40. In ΔABC, AD, BE & CF are medians, then which of the following is true -

(a) $3(AB+BC+AC) > 4(AD+BE+CF)$

[✓ (b)] $3(AB+BC+AC) < 4(AD+BE+CF)$

(c) $3(AB+BC+AC) \geq 4(AD+BE+CF)$

(d) $3(AB+BC+AC) \leq 4(AD+BE+CF)$

41. In ΔABC, AB = 2 cm, BC = 3 cm, AC = 4 cm. Find length of median AD.

42. The length of medians of ΔABC are 9 cm, 12 cm & 15 cm. Find the sum of square of all sides.

43. The length of sides of ΔABC are 12 cm, 16 cm & 20 cm. Find the sum of squares of medians.

44. Find the ratio of sum of square of sides & sum of square of medians in a triangle.

45. Find the ratio of sum of square of medians & sum of square of sides in a triangle.

46. In Δ ABC, BE & CF are medians & G is Centroid. AG & EF intersect at O. Find the ratio of -

(a) AG & OG
(b) AO & OG
(C) AO & AG

47. In Δ ABC. AD, BE & CF are medians & G is Centroid. AD & EF intersect at O. Find the ratio of -

(a) AO & AD
(b) AG & GD
(C) AD & AO
(d) AG & AO
(e) OG & GD

48. The length of medians of Δ ABC are 9 cm, 12 cm & 15 cm. Find the ar. (ΔABC).

49. The length of medians of ΔABC are 18 cm, 24 cm & 30 cm. Find the ar.(ΔABC)

50. In ΔABC, AD is median. E is the mid-point of AD. BE is extended meets side AC at F. Find the ratio of

(a) AF & FC

(b) AC & AF

(c) AC & FC

51. In ΔABC, AD is median and E is mid-point of AD. BE is extended meets side AC at F. If AB = 9 cm, CA = 15 cm, AF = ?

52. In ΔABC, AD is median and E is mid-point of AD. BE is extended meets side AC at F. Find $\dfrac{\Delta ABC}{\Delta AEF} = ?$

53. In ΔABC, AD is median and E is mid-point of AD. BE is extended meets side AC at F. Find $\dfrac{\Delta ABC}{\square CFED} = ?$

54. In ΔABC, BE & CF are medians intersect at right angle then which of the following is true-

(a) $(AB^2 + AC^2) = BC^2$

(b) $(AB^2+AC^2) = 2(BC^2)$

(c) $(AB^2+AC^2) = 4(BC^2)$

(d) $(AB^2+AC^2) = 5(BC^2)$

55. In ΔABC, medians $BE \perp$ medians CF. Find ratio of $\frac{AB^2+AC^2}{BC^2} = ?$

56. In ΔABC, $BE \perp CF$. If $AB = 19$ cm, $AC = 22$ cm, $BC = ?$

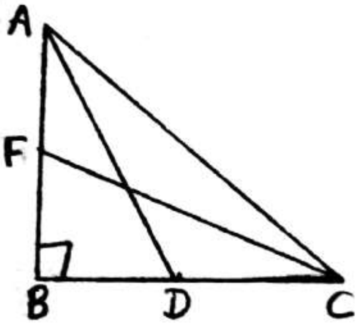
57. In a right angle ΔABC, AD & CF are medians then which of the following is true-

(a) $AD^2 + CF^2 = AC^2$

(b) $2(AD^2+CF^2) = 3AC^2$

(c) $3(AD^2+CF^2) = 4AC^2$

[✓ (d)] $4(AD^2+CF^2) = 5AC^2$



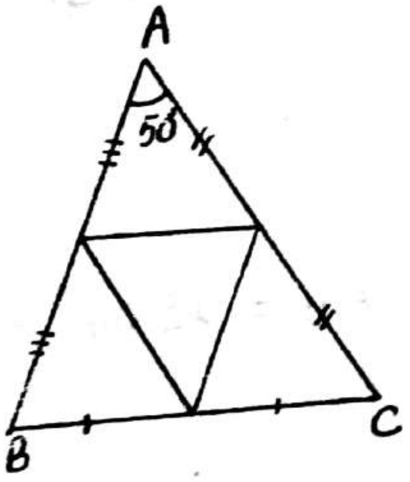
58. In a right angle ΔABC, AD & CF are medians then find ratio of $\frac{AD^2+CF^2}{AC^2} = ?$

59. In a right angle ΔABC, AD & CF are medians. If $AC = 5$ cm, $AD = 3$ cm, $CF = ?$

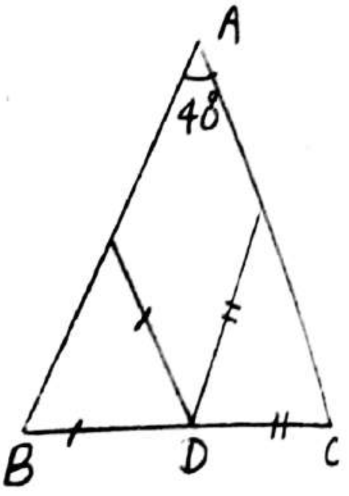
60 In ΔABC, D, E & F are the mid-points of BC, AC & AB. If $\angle A = 50^\circ$, then find $\angle EDF = ?$

61. In ΔABC, D, E & F are on sides BC, AC & AB. If $BD = DF$, $CD = DE$ and $\angle A = 40^\circ$, then find $\angle EDF$

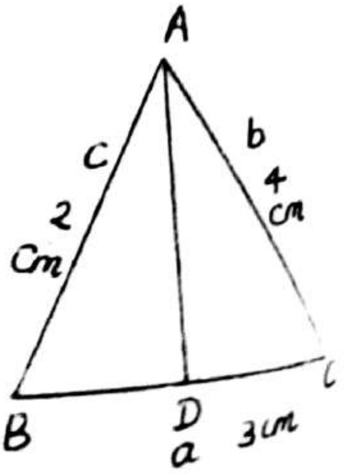
60.



61.



Solutions



41.

$$AD = \sqrt{\frac{2c^2+2b^2-a^2}{4}}$$

$$= \sqrt{\frac{2\times2^2+2\times4^2-3^2}{4}}$$

$$= \boxed{\frac{\sqrt{31}}{2}\ cm}$$

42. Sum of square of sides $= \frac{4}{3}(81+144+225)$

$$= \frac{4}{3} \times 450$$

$$= \boxed{600\ cm^2}$$

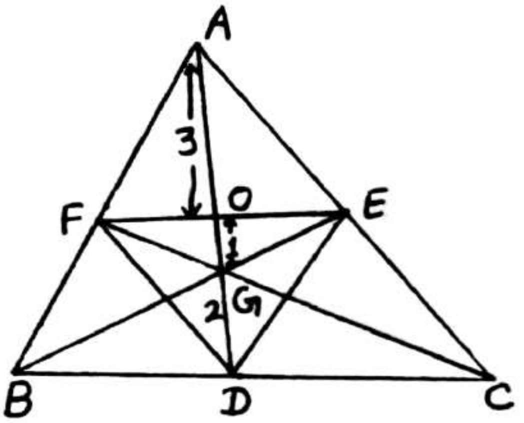
43. Sum of square of medians $= \frac{3}{4}(144+256+400)$

$= \frac{3}{4} \times 800 = \boxed{600\,cm^2}$

44. Required ratio $= \frac{4}{3}$

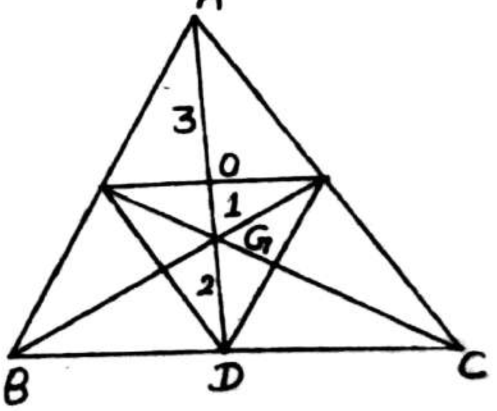
45. Required ratio $= \frac{3}{4}$

46.



(a) $\frac{AG}{OG} = \frac{4}{1}$ (b) $\frac{AO}{OG} = \frac{3}{1}$ (c) $\frac{AO}{AG} = \frac{3}{4}$

47.

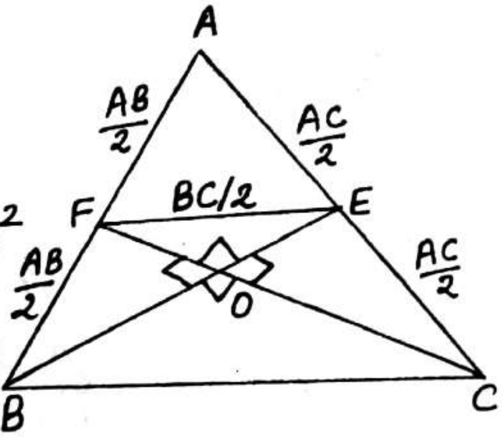


(a) $\frac{AO}{AD} = \frac{3}{6} = \frac{1}{2}$ (b) $\frac{AD}{AO} = \frac{6}{3} = \frac{2}{1}$ (c) $\frac{OG}{GD} = \frac{1}{2}$

(b) $\frac{AG}{GD} = \frac{4}{2} = \frac{2}{1}$

(d) $\frac{AG}{AO} = \frac{4}{3}$

54 & 55.



In ΔBOC -

$$BC^2 = BO^2 + CO^2$$

$$= \frac{AB^2}{4} - FO^2 + \frac{AC^2}{4} - EO^2$$

$$= \frac{AB^2}{4} + \frac{AC^2}{4} - (FO^2 + EO^2)$$

$$= \frac{AB^2}{4} + \frac{AC^2}{4} - \frac{BC^2}{4}$$

$$\frac{5BC^2}{4} = \frac{AB^2 + AC^2}{4}$$

$$\boxed{AB^2 + AC^2 = 5BC^2}$$

$$\frac{AB^2 + AC^2}{BC^2} = \boxed{5}$$

56. $AB^2 + AC^2 = 5BC^2$

$$361 + 484 = 5BC^2$$

$$BC^2 = \frac{845}{5}$$

$$BC = \sqrt{169} = 13 \text{ cm}$$

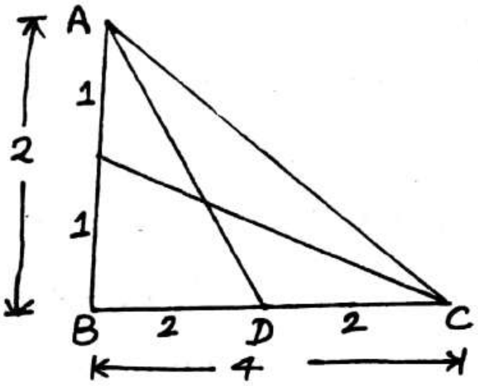
57 & 58.

$$\frac{AD^2+CF^2}{AC^2} = \frac{8+17}{20}$$

$$= \frac{25}{20} = \frac{5}{4}$$

$$\frac{AD^2+CF^2}{AC^2} = \frac{5}{4}$$

$$\boxed{4(AD^2+CF^2) = 5\ AC^2}$$



59.

$$4(AD^2+CF^2) = 5AC^2$$

$$4(9+CF^2) = 5(25)$$

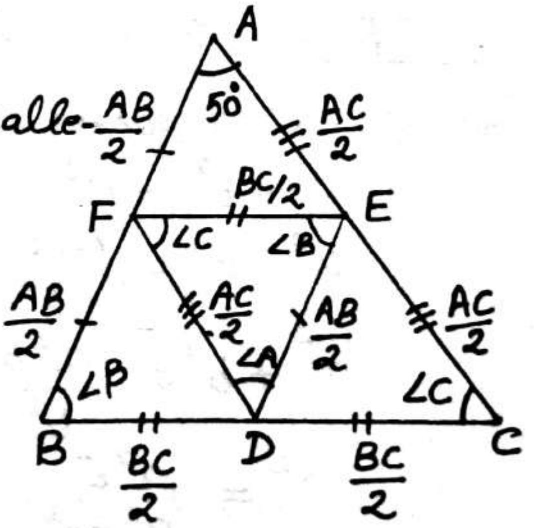
$$9+CF^2 = \frac{125}{4}$$

$$CF^2 = \frac{125}{4} - 9 = \frac{89}{4}$$

$$CF = \frac{\sqrt{89}}{2}\ cm$$

60. $\angle EDF = 50^\circ$

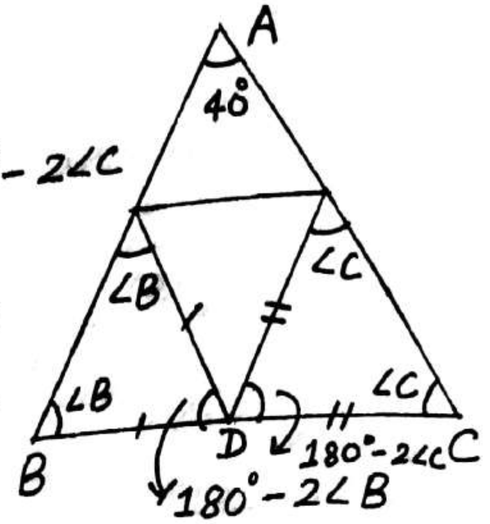
Opposite angles of paralle-ogram are equal.



□ AFDE
□ BDEF
□ CDFE

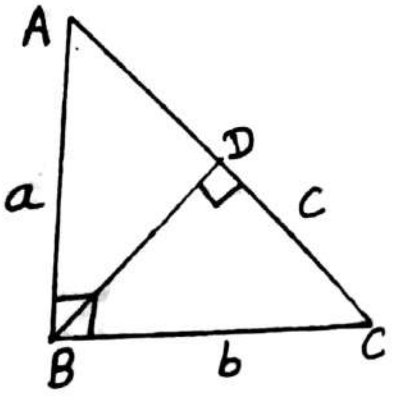
} Paralleogram

61.



$$180° - 2\angle B + \angle EDF + 180° - 2\angle C$$
$$= 180°$$
$$\angle EDF = 2(\angle B + \angle C) - 180°$$
$$= 2(180° - 40°) - 180°$$
$$= 280° - 180°$$
$$= \boxed{100}$$

Right angle triangle :- one angle is 90°.



In a right angle ΔABC.

$AB = a$, $BC = b$, $AC = c$

$BD \perp AC$

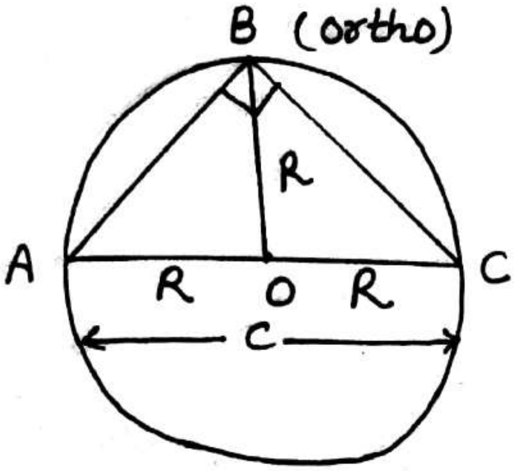
$$\text{Area} = \frac{1}{2}ab = \frac{1}{2}pc$$

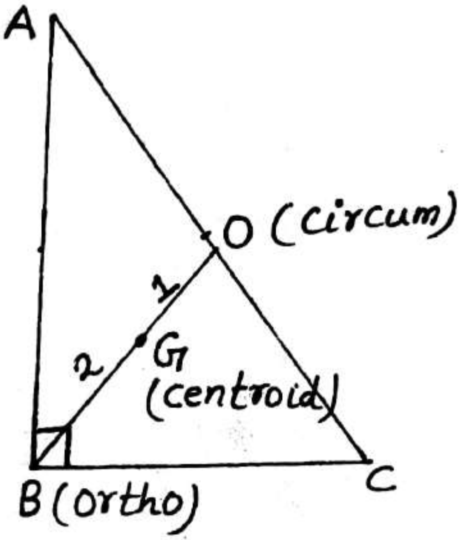
$$\boxed{ab = pc} \Rightarrow c = \frac{ab}{p}$$

By Pythagoras theorem -

$$a^2 + b^2 = c^2$$
$$a^2 + b^2 = \left(\frac{ab}{p}\right)^2$$
$$a^2 + b^2 = \frac{a^2b^2}{p^2}$$
$$p^2 = \frac{a^2b^2}{a^2+b^2}$$
$$\frac{1}{p^2} = \frac{a^2+b^2}{a^2b^2} \Rightarrow \boxed{\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}}$$

Concept of right angle triangle based on circle:-





1. $2R = C \rightarrow$ (max. side)
$\Rightarrow R = \frac{C}{2}$ (Circumradius)

2. Distance from orthocentre to circumcentre $= R = \frac{C}{2}$

3. $\overset{R}{\text{ortho} \xrightarrow{\quad 2 \quad} G \xrightarrow{\quad 1 \quad} \text{Circum}}$

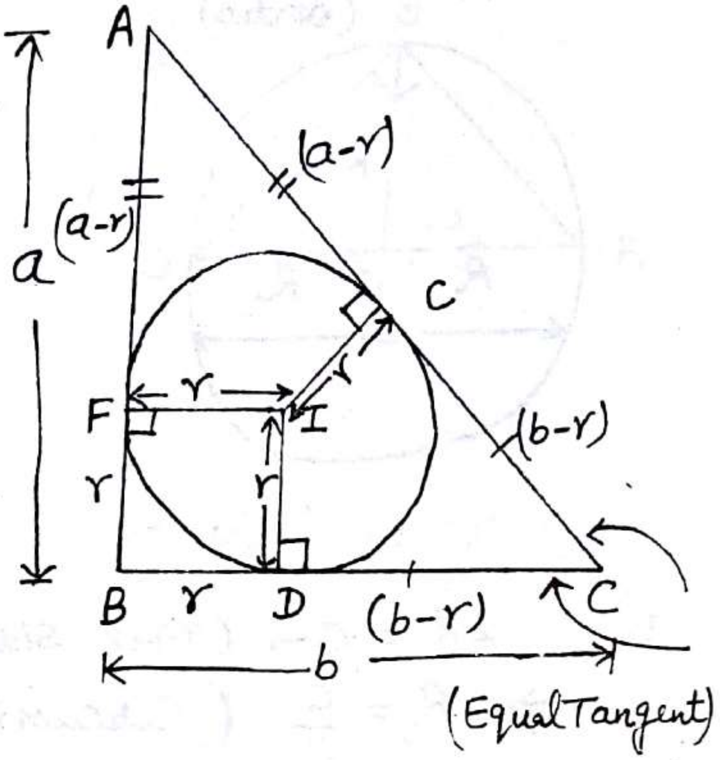
4. Distance between ortho & G $= \frac{2R}{3} = \frac{2}{3} \times \frac{C}{2} = \frac{C}{3}$

5. Distance between G & Circum $= \frac{1}{3} R = \frac{1}{3} \times \frac{C}{2} = \frac{C}{6}$

Note :- If the median of a triangle is half of its maximum side - Right angle triangle

Inradius of right angle triangle:-

In a right angle ΔABC, I is an incentre $AB=a$, $BC=b$ & $AC=c$

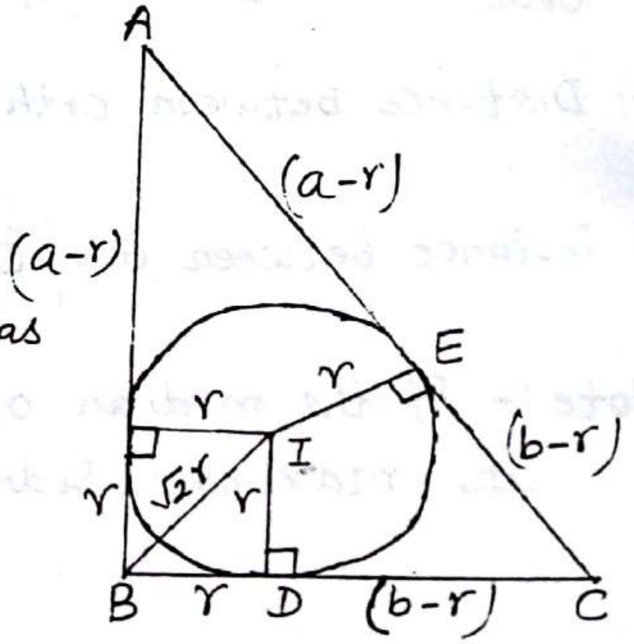


$$AE+EC = AC$$
$$(a-r)+(b-r) = c$$
$$a+b-2r = c$$
$$2r = a+b-c$$
$$\boxed{r = \frac{a+b-c}{2}}$$

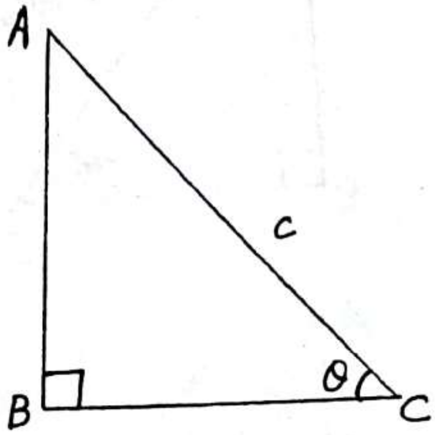
Distance between orthocentre & Incentre in right angle:-

Distance between ortho & Incentre $= \sqrt{2}r$

(In ΔBID, using Pythagoras Theorem we get
$BI = \sqrt{2}r$)



Area of right angle triangle in terms of maximum side (c) & base angle :-



$\cos\theta = \frac{BC}{c}$, $\sin\theta = \frac{AB}{c}$

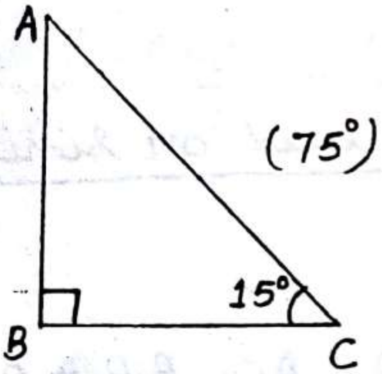
$BC = c\cos\theta$, $AB = c\sin\theta$

$\text{Area} = \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} c^2 \sin\theta \cos\theta$

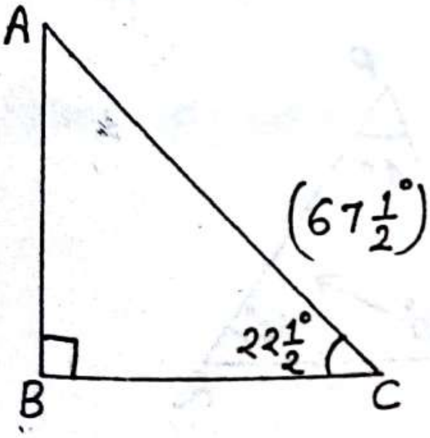
$$\boxed{\text{Area}(A) = \frac{c^2}{4}\sin 2\theta}$$

1.



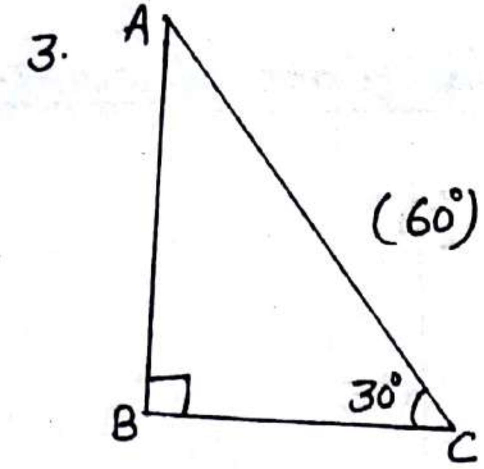
$A = \frac{c^2}{4}\sin 30° = \boxed{\frac{c^2}{8}}$

2.



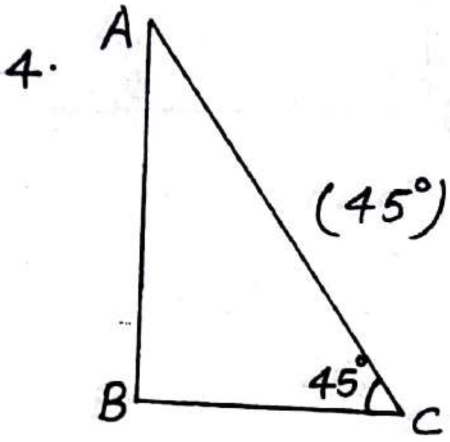
$A = \frac{c^2}{4}\sin 45° = \boxed{\frac{c^2}{4\sqrt{2}}}$

3.



$$A = \frac{c^2}{4} \sin 60^\circ = \boxed{\frac{\sqrt{3}c^2}{8}}$$

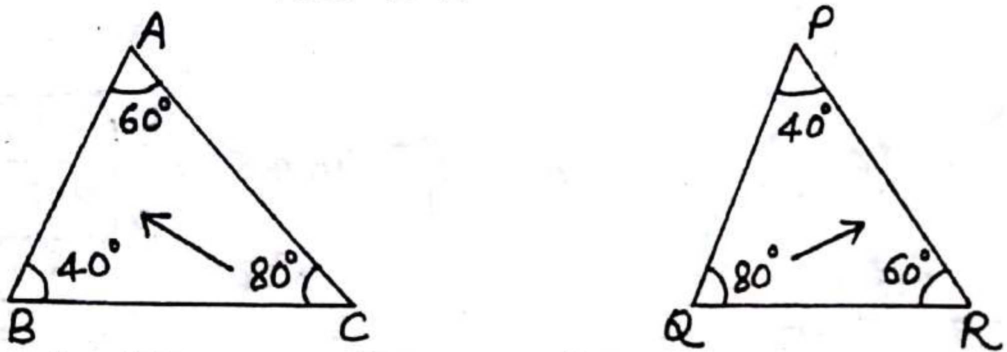
4.



$$A = \frac{c^2}{4} \sin 90^\circ = \boxed{\frac{c^2}{4}}$$

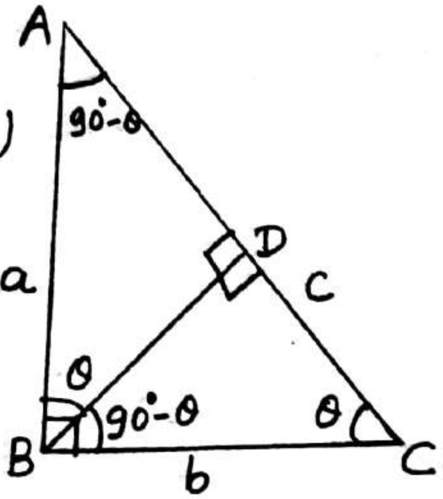
Concept of right angle triangle based on similar triangle :-

In a right angle ΔABC, $BD \perp AC$. $AB = a$, $BC = b$ & $AC = c$



$$\frac{AB}{PR} = \frac{AC}{QR} = \frac{BC}{PQ}$$

$$\Delta \overset{90^\circ-\theta}{A}\underset{90^\circ}{B}\overset{\theta}{C} \sim \Delta \overset{90^\circ-\theta}{A}\underset{90^\circ}{D}\overset{\theta}{B} \sim \Delta \overset{90^\circ-\theta}{B}\underset{90^\circ}{D}\overset{\theta}{C}\ (AAA)$$



$\Delta ABC \sim \Delta ADB$

$$\frac{AB}{AD} = \frac{AC}{AB}$$

$$\boxed{AB^2 = AC \cdot AD} \quad -(i)$$

$\Delta ABC \sim \Delta BDC$

$$\frac{BC}{CD} = \frac{AC}{BC}$$

$$\boxed{BC^2 = CD \cdot AC} \quad -(ii)$$

$\Delta ADB \sim \Delta BDC$

$$\frac{BD}{CD} = \frac{AD}{BD}$$

$$\boxed{BD^2 = CD \cdot AD}$$

$$\frac{ar(\Delta ABC)}{ar(\Delta ADB)} = \left(\frac{c}{a}\right)^2, \quad \frac{ar(\Delta ABC)}{ar(\Delta BDC)} = \left(\frac{c}{b}\right)^2$$

$$\frac{ar(\Delta ADB)}{ar(\Delta BDC)} = \left(\frac{a}{b}\right)^2$$

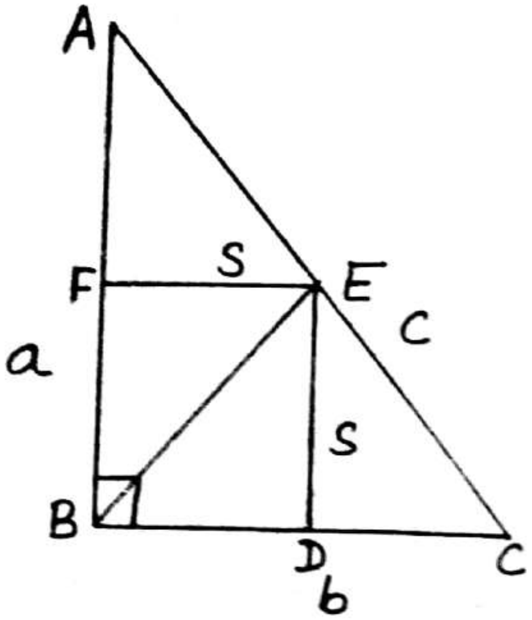
(Ratio of areas = square of ratio of correspondig sides)

From equation (i) & (ii)

$$\frac{AB^2}{BC^2} = \frac{AD}{CD}$$

$$\boxed{\frac{AB}{BC} = \sqrt{\frac{AD}{CD}}}$$

angle triangle:-



$$\Delta ABC = \Delta AEB + \Delta BEC$$

$$\frac{1}{2}ab = \frac{1}{2}as + \frac{1}{2}bs$$

$$ab = (b+a)s$$

$$\boxed{S = \frac{ab}{a+b}}$$

↓ Side of square

Questions based on right angle triangle:-

62. If the sides of a right angle triangle are 6 cm, 8 cm, 10 cm, then -

(a) Find Inradius

(b) Find Circumradius

(c) Distance between orthocentre & circumcentre

(d) Distance between orthocentre & centroid

(e) Distance between centroid & circumcentre

(f) Distance between incentre & orthocentre

(g) Area

63. In a right angle ΔABC, $BD \perp AC$. $AB = 15$ cm,

BC = 20 cm. Find -

(a) ratio of inradius of ΔABC & ΔADB.

(b) ratio of circumradius of ΔABC & ΔADB

(c) ratio of areas of ΔABC & ΔADB.

64. In a right angle ΔABC, $BD \perp AC$. $AB = 9$ cm, $BC = 12$ cm. Find the ratio of -

(a) Inradius of ΔABC & ΔBDC

(b) Circumradius of ΔABC & ΔBDC

(c) Area of ΔABC & ΔBDC

65. In a right angle ΔABC $BD \perp AC$. If $AB = 30$ cm $BC = 40$ cm. Find the ratio of -

(a) Inradius of ΔADB & ΔBDC

(b) Circumradius of ΔADB & ΔBDC

(c) Area of ΔADB & ΔBDC

66. In a right angle ΔABC, $BD \perp AC$. If $AB = 6$ cm, $BC = 8$ cm. Find the length of BD.

67. In a right angle ΔABC, $BD \perp AC$. $AB = 15$ cm, $BC = 20$ cm. Find the difference between BD & Circumradius.

68. In a right angle ΔABC, $BD \perp AC$. If $AB = 3BC$. Find $AD : CD$.

69. In right angle ΔABC, $AB = 2BC$ & $BD \perp AC$ then $CD = ?$

(a) $\frac{AC}{2}$ (b) $\frac{AC}{3}$ (c) $\frac{AC}{4}$ [✓ (d) $\frac{AC}{5}$]

70. In a right angle triangle ABC, the length of Circumradius is 12 cm. Find the distance between orthocentre & Centroid.

71. In a right angle ΔABC, the length of Circumradius is 8 cm. Find the distance between orthocentre & circumcentre.

72. In a right angle ΔABC, $BD \perp AC$. Area of ΔABC & ΔADB are $40\ cm^2$ & $10\ cm^2$. If $AB = 9\ cm$ $AC = ?$

73. In a right angle ΔABC, $BD \perp AC$. Area of ΔABC & ΔABD are $90\ cm^2$ & $10\ cm^2$. If $BC = 24\ cm$, $AC = ?$

74. In a right angle ΔABC, $BD \perp AC$. $AB = 6$ cm, $BC = 8$ cm. Find area of ΔADB.

75. In right angle ΔABC, $BD \perp AC$. $AB = 15$ cm, $BC = 20$ cm. Find the area of ΔBDC.

76. In a right angle ΔABC, $BD \perp AC$ & $AB = 2BC$. Find the ratio of area of ΔADB & ΔBDC.

77. In a right angle ΔABC, $BD \perp AC$. If $AD = 9$ cm, $CD = 4$ cm. Find the length of BD.

78. In a right angle ΔABC, $BD \perp AC$. $AD = 16$ cm, $CD = 9$ cm, find the length of BC.

79. In a right angle ΔABC, $BD \perp AC$. $AD = 1$ cm, $CD = 4$ cm. Find the length of AB[illegible] = ?

80. In a right angle ΔABC, $BD \perp AC$. $AD = 9$ cm, $CD = 4$ cm. Find the ratio of AB & BC.

81. In a right angle ΔABC, $\angle ACB = 15^\circ$, $AC = 1$ m. Find area of ΔABC (in cm^2).

82. In a right angle ΔABC, $\angle ACB = 22\frac{1}{2}^\circ$, $AC = 40$ cm. Find area of ΔABC.

83. In a right angle ΔABC, $\angle ACB = 30^\circ$, $AC = 30$ cm. Find area of ΔABC.

84. In a right angle ΔABC, $\angle ACB = 45^\circ$, $AC = 20$ cm. Find area of ΔABC.

85. In right angle ΔABC, $\angle ACB = 60^\circ$, $AC = 40$ cm. Find area of ΔABC.

86. In right angle ΔABC, $\angle ACB = 75^\circ$ & $AC = 24$ cm. Find area of ΔABC.

87. In a ΔABC, if length of one median is equal to half of its base, then ΔABC will be
(a) acute angle triangle [✓ (b) right angle triangle]
(c) obtuse angle triangle (d) None of these

88. In ΔABC, if orthocentre lies on one of its Vertex, then ΔABC will be
(a) acute angle triangle [✓ (b) right angle triangle]
(c) obtuse angle triangle (d) None of these

89. In a ΔABC, if circumcentre lie on its side then ΔABC will be

(a) acute angle triangle [✓ (b) right angle triangle]
(c) Obtuse angle triangle (d) None of these

90. Sides of ΔABC are $2a, (a^2-1)$ & (a^2+1), then ΔABC will be

(a) acute angle triangle [✓ (b) right angle triangle]
(c) Obtuse angle triangle (d) None of these

91. The ratio of sides of ΔABC are $1 : 2\frac{2}{5} : 2\frac{3}{5}$, then the ΔABC will be

(a) acute angle triangle (b) right angle triangle
(c) obtuse angle triangle (d) None of these

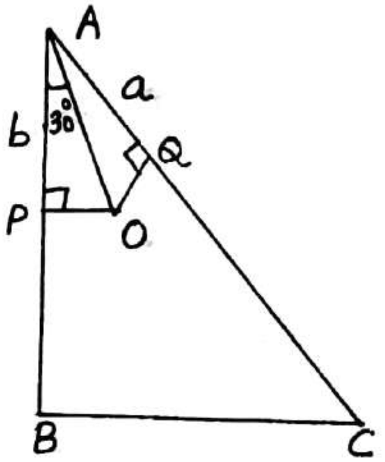
92. In a ΔABC if orthocentre lie on one its Vertex, then circumcentre will be lie on -

(a) inside the triangle (b) outside the triangle
(c) on another Vertex [✓ (d) on one its side]

93. In a ΔABC, if circumcentre lie on one its side, then orthocentre will be lie on -

(a) inside the triangle (b) outside the triangle
(c) On another side [✓ (d) on one its Vertex]

94. In a isosceles right angle ΔABC. O is any point OP & OQ are perpendicular to sides AB & AC. If $AQ = a$, $AP = b$ & $\angle PAO = 30^\circ$, $\sin 75^\circ = ?$ (In terms of a & b)



95. In a isosceles right angle ΔABC, O is any point. OP & OQ are perpendicular to sides AB & AC. If $AQ = a$, $AP = b$ & $\angle QAO = 30^\circ$, $\sin 75^\circ = ?$ (In terms of a & b)

96. The perimeter of isosceles right angle triangle is 2P. Find the area of ΔABC.

97. The sides of right angle triangle are 3 cm, 4 cm & 5 cm. Find side & area of maximum square in a triangle.

98. In a right angle ΔABC, $BD \perp AC$. $AB = 12$ cm

BC = 12 cm. Find the difference between BD & circumcentre.

99. In a right angle ΔABC, if distance between orthocentre & incentre is $10\sqrt{2}$ cm. Find in-radius of ΔABC.

100. In a ΔABC, G is a centroid if AG = BC. Find $\angle BGC = ?$

Solutions

48. $$3\ \Delta ABC = 4(\Delta M_1 M_2 M_3)$$
$$\Delta ABC = \frac{4}{3} \times \frac{1}{2} \times 12 \times 9$$
$$= \boxed{72\ cm^2}$$

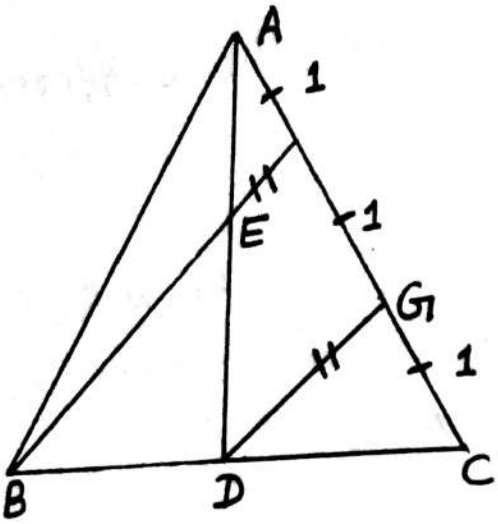
49. $$\Delta ABC = \frac{4}{3} \times \frac{1}{2} \times 18 \times 24$$
$$= 12 \times 24 = \boxed{288\ cm^2}$$

50 & 51.

DG || BF

ΔADG –

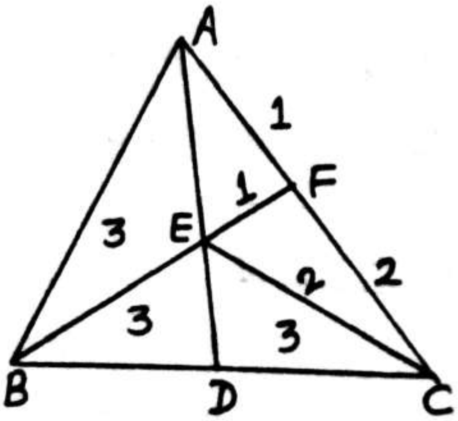
EF || DG
AF = FG
Δ BCF
BF = DG
FG = GC

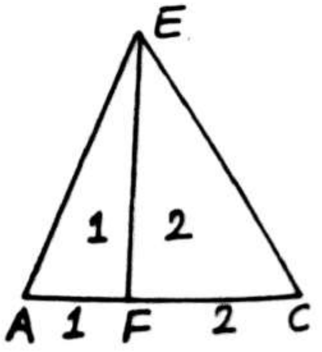


50. (a) $\frac{AF}{FC} = \frac{1}{2}$ (b) $\frac{AC}{AF} = \frac{3}{1}$ (c) $\frac{AC}{FC} = \frac{3}{2}$

51. AC = 15 cm
AF = $\boxed{5 cm}$

52/53.

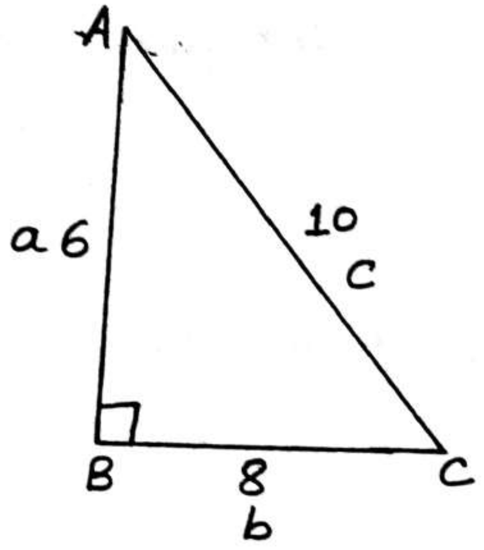




52. $\frac{\Delta ABC}{\Delta AEF} = \frac{12}{1}$

53. $\frac{\Delta ABC}{\square CDEF} = \frac{12}{5}$

62. (a) $r = \frac{a+b-c}{2} = \frac{6+8-10}{2}$
$= 2$ cm



(b) $R = \frac{c}{2} = \frac{10}{2} = 5\text{ cm}$

(c) $R = 5\text{ cm}$

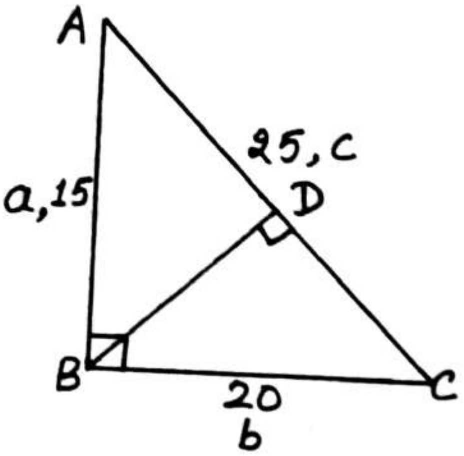
(d) $\frac{c}{3} = \frac{10}{3}\text{ cm}$

(e) $\frac{c}{6} = \frac{10}{6} = \frac{5}{3}\text{ cm}$

(f) $\sqrt{2}\,r = 2\sqrt{2}\text{ cm}$

(g) $A = \frac{1}{2} \times 6 \times 8 = 24\text{ cm}^2$

63. (a)



$$\frac{\text{Inradius of } \Delta ABC}{\text{Inradius of } \Delta ADB} = \frac{c}{a}$$

$$= \frac{25}{15} = \frac{5}{3}$$

(b) $$\frac{\text{Circumradius of } \Delta ABC}{\text{Circumradius of } \Delta ADB} = \frac{c}{a}$$

$$= \frac{25}{15} = \frac{5}{3}$$

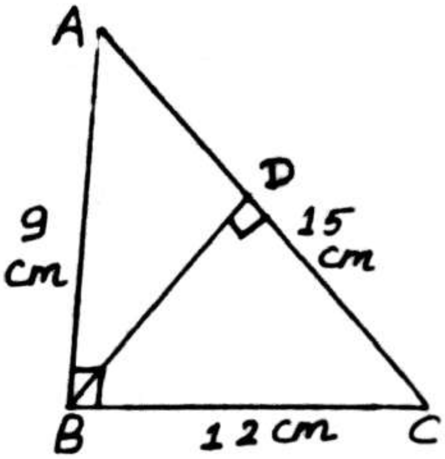
(c) $$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ADB} = \left(\frac{c}{a}\right)^2$$

$$= \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

Scanned by CamScanner

64. (a) $\frac{\text{Inradius of } \Delta ABC}{\text{Inradius of } \Delta BDC} = \frac{15}{12}$

$= \frac{5}{4}$

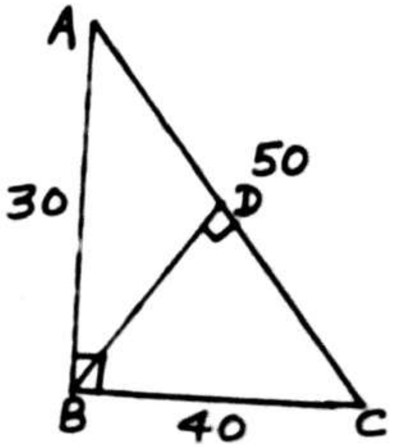


(b) $\frac{\text{Circumradius of } \Delta ABC}{\text{Circumradius of } \Delta BDC} = \frac{15}{12}$

$= \frac{5}{4}$

(c) $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BDC} = \left(\frac{5}{4}\right)^2 = \frac{25}{16}$

65.



(a) $\frac{\text{Inradius of } \Delta ADB}{\text{Inradius of } \Delta BDC} = \frac{30}{40}$

$= \frac{3}{4}$

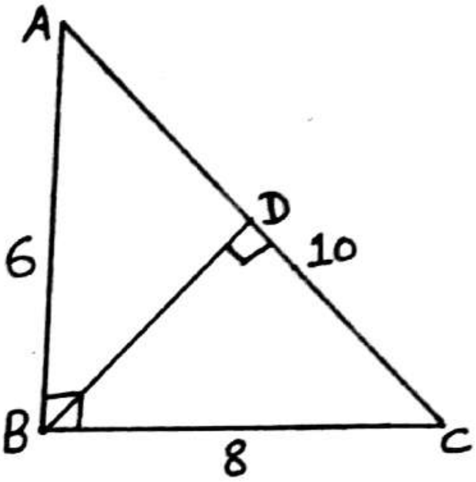
(b) $\frac{\text{Circumradius of } \Delta ADB}{\text{Circumradius of } \Delta BDC} = \frac{30}{40}$

$= \frac{3}{4}$

(c) $\frac{\text{Area of } \Delta ADB}{\text{Area of } \Delta BDC} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

66. $BD = \frac{AB \cdot BC}{AC}$

$$= \frac{6 \times 8}{10} = \frac{24}{5} \text{ cm}$$

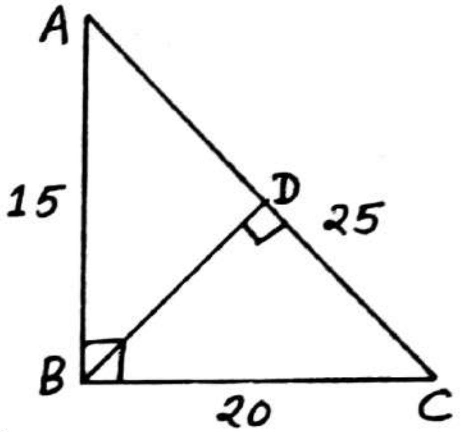


67.

$$BD = \frac{15 \times 20}{25}$$

$$= 12 \text{ cm}$$

$$R = \frac{25}{2} = 12.5 \text{ cm}$$



$$\text{Difference} = 12.5 - 12$$

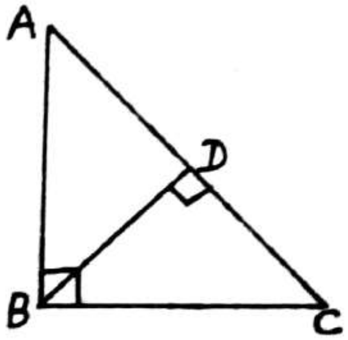
$$= 0.5 \text{ Cm}$$

68. $AB = 3BC$

$$\frac{AD}{CD} = \left(\frac{AB}{BC}\right)^2$$

$$= \left(\frac{3BC}{BC}\right)^2$$

$$= \frac{9}{1}$$

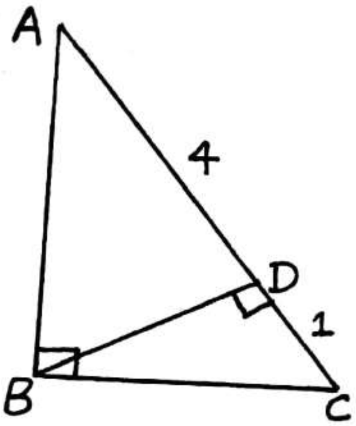


69. $AB = 2BC$

$$\frac{AD}{CD} = \left(\frac{AB}{BC}\right)^2 = \left(\frac{2BC}{BC}\right)^2$$

$$= \frac{4}{1}$$

$$CD = \frac{AC}{5}$$



70. $R = \frac{c}{2} = 12$ cm

Distance between orthocentre & centroid $= \frac{c}{3}$

$= \frac{24}{3}$

$= 8$ cm.

71. Distance between ortho & circumcentre = 18 cm.

72. $$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ADB} = \frac{40}{10} = \left(\frac{AC}{AB}\right)^2$$

$$\frac{AC}{AB} = \frac{2}{1}$$

$$AC = 2AB$$

$$= 2 \times 9 = 18 \text{ cm}$$

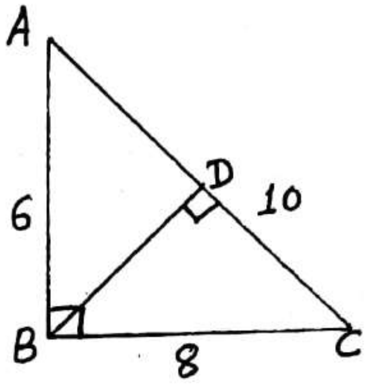
73. $$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BDC} = \frac{90}{10} = \left(\frac{AC}{BC}\right)^2$$

$$\frac{AC}{BC} = \frac{3}{1}$$

$$AC = 3\,BC$$

$$= 3 \times 24 = 72 \text{ cm}$$

74. $$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ADB} = \left(\frac{AC}{AB}\right)^2$$

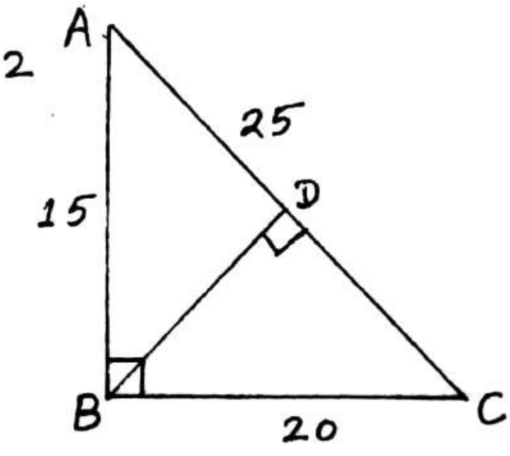


$$\frac{\frac{1}{2} \times 6 \times 8}{\text{Area of } \Delta ADB} = \frac{10 \times 10}{6 \times 6}$$

$$\text{Area of } \Delta ADB = \frac{3 \times 8 \times 9}{5 \times 5}$$

$$= \frac{216}{25} \text{ cm}^2$$

75. $$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BDC} = \left(\frac{AC}{BC}\right)^2$$



$$\frac{\frac{1}{2} \times 15 \times 20}{\text{Area of } \Delta BDC} = \frac{5 \times 5}{4 \times 4}$$

$$\text{Area of } \Delta BDC = 6 \times 16$$

$$= 96 \text{ cm}^2$$

76. $$\frac{\text{Area of } \Delta ADB}{\text{Area of } \Delta BDC} = \left(\frac{AB}{BC}\right)^2$$

$$= \left(\frac{2\,BC}{BC}\right)^2 = \frac{4}{1}$$

77. $BD^2 = AD \cdot CD$

$= 9 \times 4 = 36 \Rightarrow BD = 6$ cm

78. $BC^2 = CD \cdot AC = 9 \times 25$

$BC = 3 \times 5 = 15$ cm

79. $AB^2 = AC \cdot AD = 5 \times 1 \Rightarrow AB = \sqrt{5}$ cm

80. $AB = \sqrt{13 \times 9} = 3\sqrt{13}$

$BC = \sqrt{4 \times 13} = 2\sqrt{13}$

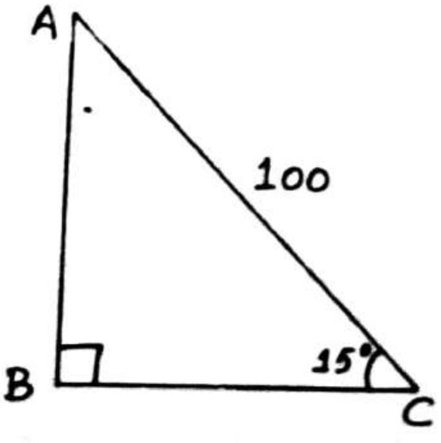
$\frac{AB}{BC} = \frac{3}{2}$ cm

81. Area of $\Delta ABC = \frac{c^2}{4} \sin 2\theta$

$= \frac{c^2}{8}$

$= \frac{100 \times 100}{8}$

$= 1250 \text{ cm}^2$



82. Area of $\Delta ABC = \frac{c^2}{4\sqrt{2}} = \frac{40 \times 40}{4\sqrt{2}}$

$= \frac{400}{\sqrt{2}} = 200\sqrt{2} \text{ cm}^2$

83. Area of $\Delta ABC = \frac{\sqrt{3}c^2}{8}$

$$= \frac{\sqrt{3} \times 30 \times 30}{8} = \frac{225\sqrt{3}}{2} \text{ cm}^2$$

84. Area of $\Delta ABC = \frac{c^2}{4} = \frac{400}{4} = 100 \text{ cm}^2$

85. Area of $\Delta ABC = \frac{\sqrt{3}c^2}{8} = \frac{\sqrt{3} \times 1600}{8} = 200\sqrt{3} \text{ cm}^2$

86. Area of $\Delta ABC = \frac{c^2}{8} = \frac{24 \times 24}{8} = 72 \text{ cm}^2$

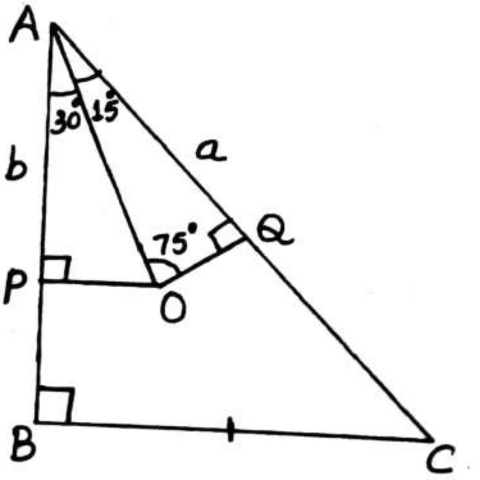
94. $\cos 30^\circ = \frac{b}{AO}$

$$\frac{\sqrt{3}}{2} = \frac{b}{AO}$$

$$AO = \frac{2b}{\sqrt{3}}$$

$$\sin 75^\circ = \frac{a}{AO} = \frac{a\sqrt{3}}{2b}$$

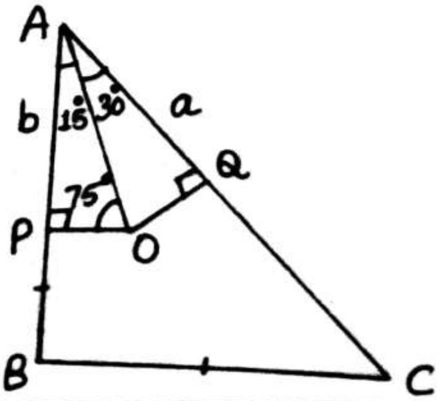
$$= \frac{\sqrt{3}}{2} \frac{a}{b}$$



95. $\cos 30^\circ = \frac{a}{AO}$

$$\frac{\sqrt{3}}{2} = \frac{a}{AO}$$

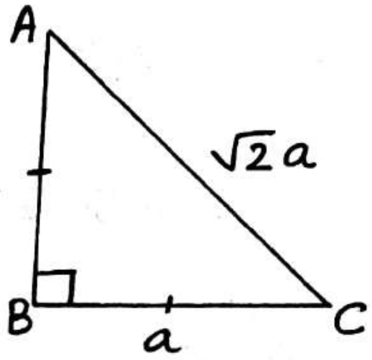
$$AO = \frac{2a}{\sqrt{3}}$$



$$\text{Sin } 75^\circ = \frac{b}{AO} = \frac{b}{2}\ \frac{\sqrt{3}}{a}$$
$$= \frac{\sqrt{3}}{2}\frac{b}{a}$$

96. $\text{Area} = \frac{1}{2} \times a \times a$

$$= \frac{1}{2} a^2$$

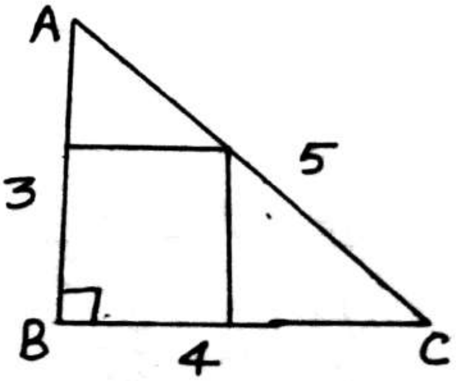


$$2a + \sqrt{2}\,a = 2P$$
$$a(2+\sqrt{2}) = 2P$$
$$a = \frac{2P}{(2+\sqrt{2})} \times \frac{(2-\sqrt{2})}{(2-\sqrt{2})}$$
$$a = (2-\sqrt{2})P$$

$$\text{Area} = \frac{1}{2}(2-\sqrt{2})^2 P^2$$
$$= (3-2\sqrt{2})P^2$$

97.

$$S = \frac{ab}{a+b}$$



$$S = \frac{12}{7}\ cm$$

$$\text{Area of} \quad \text{square} = \left(\frac{12}{7}\right)^2$$
$$= \frac{144}{49}\ cm^2$$

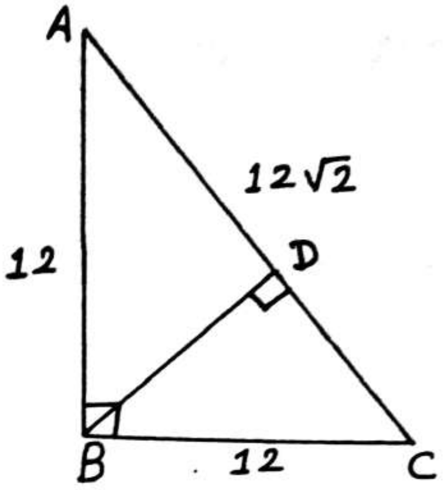
Scanned by CamScanner

98. $BD = \frac{12 \times 12}{12\sqrt{2}}$

$= 6\sqrt{2}$ cm

$R = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$ cm

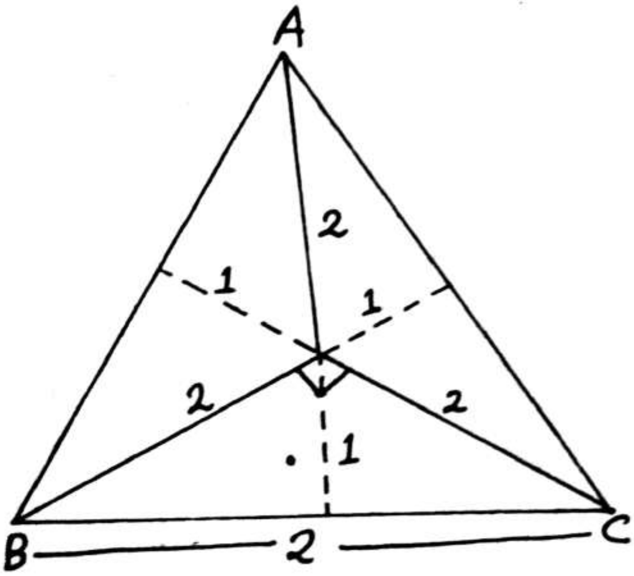
Difference = 0



99. $\sqrt{2}\,r = 10\sqrt{2}$

$r = 10$ cm.

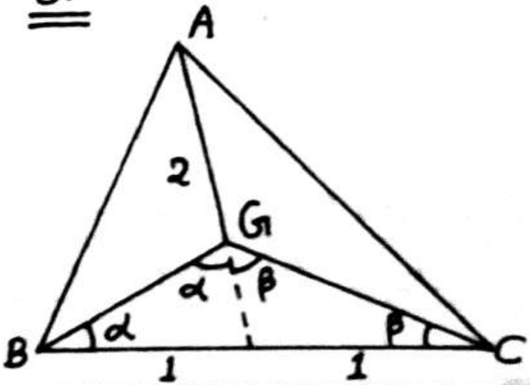
100.



$\Rightarrow \angle BGC = 90°$ or

$\alpha + \alpha + \beta + \beta = 180°$

$\alpha + \beta = 90°$

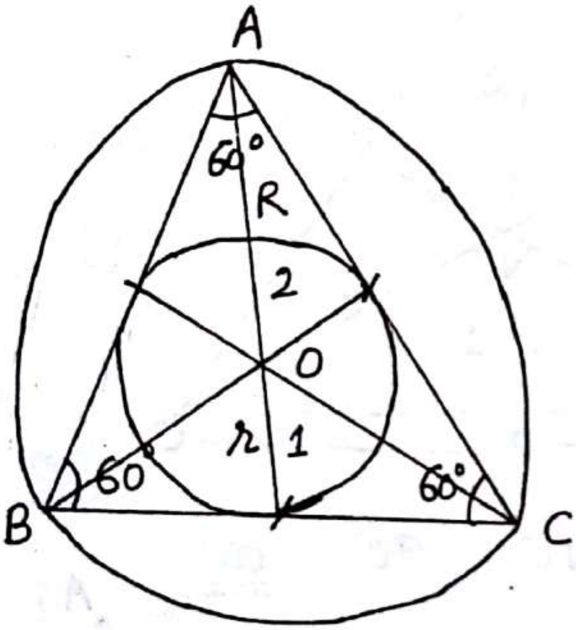


Equilateral Triangle:- All sides are equal & each angle is 60°.

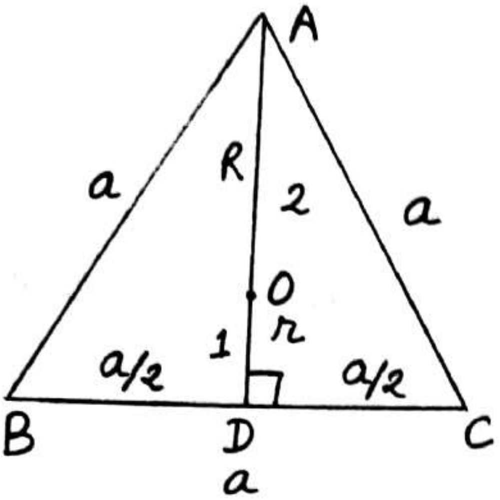
1. In equilateral triangle, all centres (incentre, circumcentre, orthocentre & centroid) are on same point.

2. In a equilateral triangle, the ratio of inradius & circumradius i.e. $\underset{(1-D)}{r : R} = 1 : 2$

3. In a equilateral triangle, the ratio of area of incircle & circumcircle i.e. $\underset{(2-D)}{A_r : A_R} = 1 : 4$



Inradius (r), Circumradius (R), Height or Median & Area of Equilateral triangle:-



In ΔADC -

$$AD^2 = a^2 - \frac{a^2}{4} = \frac{4a^2 - a^2}{4}$$

$$\Rightarrow \boxed{AD\,(H/M) = \frac{\sqrt{3}}{2}a}$$

r R

1 : 2

$$r = \frac{\sqrt{3}}{2}a \times \frac{1}{3} \quad , \quad \boxed{r = \frac{a}{2\sqrt{3}}}$$

$$R = \frac{\sqrt{3}}{2}a \times \frac{2}{3} \quad \text{or} \quad R = 2r$$

$$\boxed{R = \frac{a}{\sqrt{3}}} \qquad \boxed{R = \frac{a}{\sqrt{3}}}$$

$$\text{Area}(A) = \frac{1}{2} \times BC \times AD$$

$$= \frac{1}{2} \times a \times \frac{\sqrt{3}}{2}a$$

$$\boxed{A = \frac{\sqrt{3}}{4}a^2}$$

101. Find the ratio of inradius & circumradius of an equilateral triangle.

102. Find the ratio of area of incircle & circumcircle of equilateral triangle.

103. In a ΔABC, if all centres are on same point, then ΔABC will be
(a) Right angle triangle (b) Equilateral triangle
(C) Obtuse angle triangle (d) None of these

104. The side of equilateral triangle is 12 cm. Find inradius.

105. The side of equilateral triangle is 6 cm. Find circumradius.

106. Inradius of equilateral triangle is $8\sqrt{3}$ cm. Find area of triangle.

107. Circumradius of equilateral is $6\sqrt{3}$ cm. Find perimeter of triangle.

108. In a equilateral ΔABC, points D, E &

F are midpoints of Sides BC, AC & AB. Find $\angle EDF$.

109. In a equilateral ΔABC, AD is a angle bisector of $\angle A$ intersect side BC at D & circum circle of ΔABC at E. Find the ratio of AD & DE.

110. In a equilateral ΔABC. AD is a angle bisector of $\angle A$ intersect BC at D & circumcircle of ΔABC at E. Find $AB \cdot AC + AE \cdot DE = ?$

(a) AB^2 (b) AD^2 ✓(C) AE^2 (d) DE^2

111. In a equilateral ΔABC, D is a point on side BC such that the ratio of BD & CD is 1 : 4. Find $AB : AD = ?$

112. In a equilateral ΔABC, D is a point on side BC such that the ratio of BD & CD is 1 : 3. Find the ratio of AB & AD.

113. In a equilateral ΔABC, D is point on side BC such that the ratio of BD & CD is 1 : 2. Find $AB : AD = ?$

114. In a equilateral ΔABC, D is a point on side

BC such that the ratio of BD & CD is 1:1. Find $AB : AD = ?$

115. In a equilateral ΔABC, D is a point on side BC such that the ratio of BD & CD is 1:4, then Which of the following is true -

(a) $3AB^2 = 4AD^2$ (b) $7AB^2 = 9AD^2$

(c) $13AB^2 = 16AD^2$ [✓ (d) $21AB^2 = 25AD^2$]

116. In a equilateral ΔABC, D is a point on side BC such that the ratio of BD & CD is 1:3, then Which is true -

(a) $3AB^2 = 4AD^2$ (b) $7AB^2 = 9AD^2$

[✓ (c) $13AB^2 = 16AD^2$] (d) $21AB^2 = 25AD^2$

117. In equilateral ΔABC, D is a point on BC such that the ratio of BD & CD is 1:2, then which is true

(a) $3AB^2 = 4AD^2$ (b) $7AB^2 = 9AD^2$

(c) $13AB^2 = 16AD^2$ (d) $21AB^2 = 25AD^2$

118. In equilateral ΔABC, D is a point on BC such that the ratio of BD & CD is 1:1, then which is true

[✓ (a) $3AB^2 = 4AD^2$]   (b) $7AB^2 = 9AD^2$

(c) $13AB^2 = 16AD^2$ (d) $21AB^2 = 25AD^2$

119. In an equilateral ΔABC, $DE \parallel BC$, $DE = 8$ cm. Find the area of ΔADE.

120. In an equilateral ΔABC, D, E & F are mid-points of sides BC, AC & AB. Find the ratio of –

(a) Inradius of ΔABC & ΔDEF.

(b) Circumradius of ΔABC & ΔDEF.

(c) Area of ΔABC & ΔDEF

121. In an equilateral ΔABC, AD, BE & CF are medians & G is a centroid. AG & EF intersect at O. Find the ratio of AB & OG.

122. In an equilateral ΔABC, AD, BE & CF are medians & G is a Centroid. AG & EF intersect at O. If side of the triangle is $12\sqrt{3}$ cm, find the length OG.

123. In an equilateral ΔABC, I is a incentre. Find the ratio of area of ΔAIB & ΔBIC.

124. In an equilateral ΔABC, O is a circumcentre. Find the ratio of area of ΔAOB, ΔBOC & ΔAOC.

125. In an equilateral ΔABC, D, E & F are mid-points of sides BC, AC & AB. Find the ratio of perimeters of ΔABC & DEF.

126. In an equilateral ΔABC, O is any point. The perpendicular distance between O & sides are P_1, P_2 & P_3. Find -

(a) Side of triangle
(b) Area of triangle
(c) Perimeter of triangle
(d) Height of triangle

127. In an equilateral ΔABC, O is any point. The perpendicular distances between O & side are $\sqrt{3}$, $2\sqrt{3}$ & $5\sqrt{3}$. Find -

(a) Side of triangle
(b) Area of triangle
(c) Perimeter of triangle
(d) Height of triangle

128. In a equilateral ΔABC, O is any point. The

perpendicular distances between O & sides are 6, 7 and 8 cm. Find -

(a) Side of triangle
(b) Area of triangle
(c) Perimeter of triangle
(d) Height of triangle

Solutions

101. $\frac{r}{R} = \frac{1}{2}$

102. $\frac{A_r}{A_R} = \frac{1}{4}$

104. $r = \frac{a}{2\sqrt{3}} = \frac{12}{2\sqrt{3}} = 2\sqrt{3}$ cm

105. $R = \frac{a}{\sqrt{3}} = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ cm

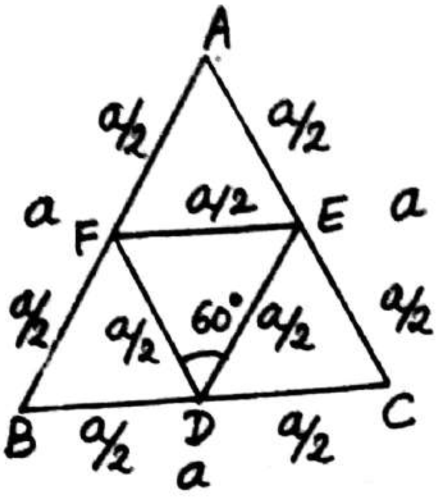
106. $\frac{a}{2\sqrt{3}} = 8\sqrt{3} \Rightarrow a = 48$ cm

$$\text{Area} = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times 48 \times 48$$
$$= 576\sqrt{3} \text{ cm}^2$$

107. $\frac{a}{\sqrt{3}} = 6\sqrt{3} \Rightarrow a = 18$ cm

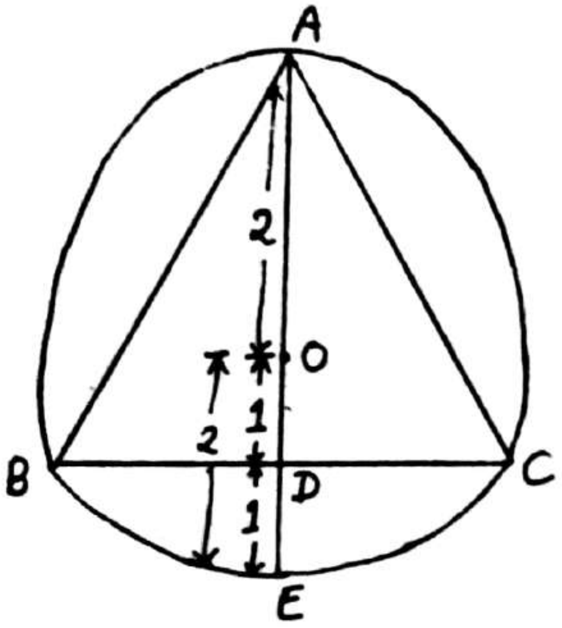
Perimetre $= 3a = 3 \times 18 = 54$ cm.

108.



$\Rightarrow \angle EDF = 60°$

109/110.



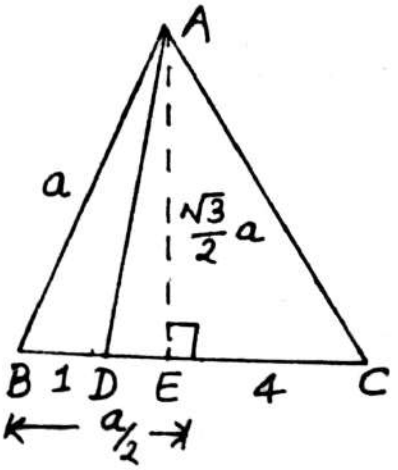
109. $\frac{AD}{DE} = \frac{3}{1}$

110. $AB \cdot AC + AE \cdot DE = a \cdot a + \frac{2a}{\sqrt{3}} \times \frac{a}{2\sqrt{3}}$

$$= a^2 + \frac{a^2}{3}$$

$$= \frac{4a^2}{3} = AE^2$$

111. $DE = \frac{a}{2} - \frac{a}{5} = \frac{3a}{10}$



$$AD^2 = \frac{3}{4}a^2 + \frac{9a^2}{100}$$

$$AD^2 = \frac{75a^2 + 9a^2}{100}$$

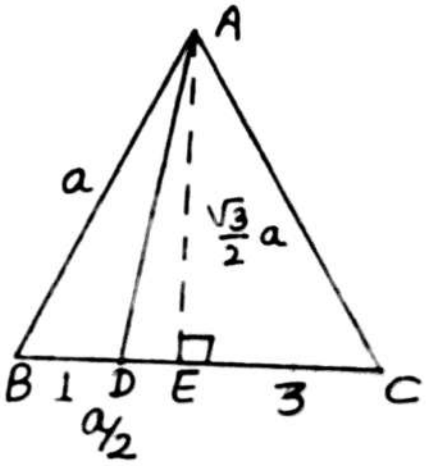
$$AD^2 = \frac{84a^2}{100}$$

$$\boxed{25\,AD^2 = 21\,AB^2}$$

$$\frac{AB}{AD} = \frac{5}{\sqrt{21}}$$

112. $DE = \frac{a}{2} - \frac{a}{4}$

$$= \frac{a}{4}$$



$$AD^2 = \frac{3}{4}a^2 + \frac{a^2}{16}$$

$$AD^2 = \frac{13a^2}{16}$$

$$\boxed{16AD^2 = 13AB^2}$$

113. $DE = \frac{a}{2} - \frac{a}{3}$

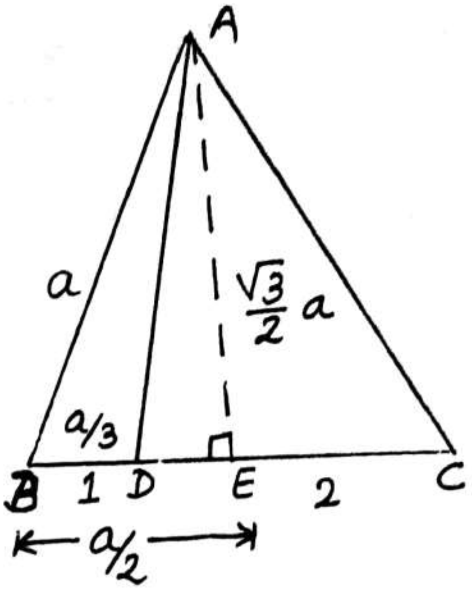
$$= \frac{a}{6}$$

$$AD^2 = \frac{3}{4}a^2 + \frac{a^2}{36}$$

$$AD^2 = \frac{27a^2 + a^2}{36}$$

$$\boxed{9AD^2 = 7AB^2}$$

$$\frac{AB}{AD} = \frac{3}{\sqrt{7}}$$



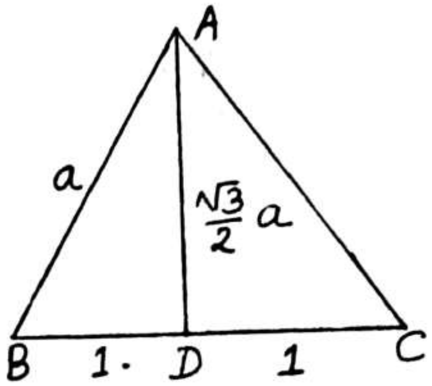
114.

$$\frac{AB}{AD} = \frac{a}{\frac{\sqrt{3}}{2}a}$$

$$\frac{AB}{AD} = \frac{2}{\sqrt{3}}$$

$$\frac{AB^2}{AD^2} = \frac{4}{3}$$

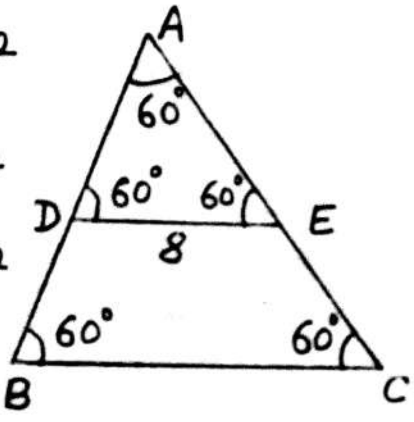
$$\boxed{4AD^2 = 3AB^2}$$



119. Area of $\Delta ADE = \frac{\sqrt{3}}{4}a^2$

$$= \frac{\sqrt{3}}{4} \times 8^2$$

$$= 16\sqrt{3}\ cm^2$$

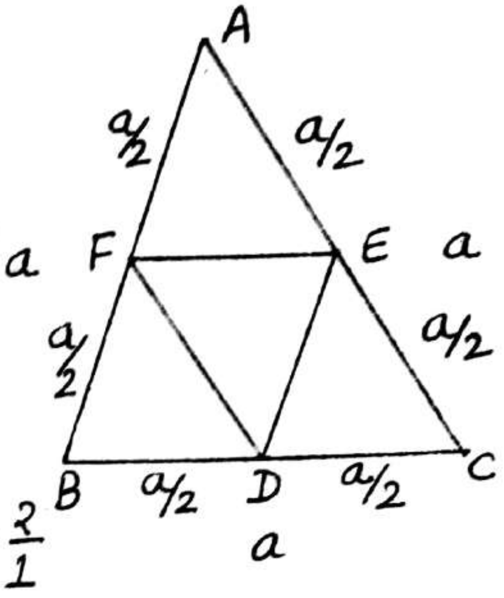


Scanned by CamScanner

120.

(a) $$\frac{\text{Inradius of } \Delta ABC}{\text{Inradius of } \Delta DEF} = \frac{a}{a/2}$$

$$= \frac{2}{1}$$

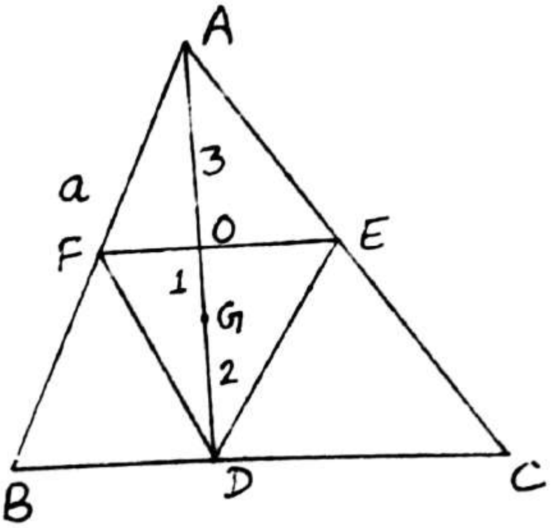


(b) $$\frac{\text{Circumradius of } \Delta ABC}{\text{Circumradius of } \Delta DEF} = \frac{2}{1}$$

(c) $$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \left(\frac{2}{1}\right)^2$$

$$= \frac{4}{1}$$

121/122.



121. $$OG = \frac{1}{6} AD$$

$$\frac{AB}{OG} = \frac{a}{\frac{1}{6} AD}$$

$$= \frac{a}{\frac{1}{6} \times \frac{\sqrt{3}}{2} \times a}$$

$$= \frac{12}{\sqrt{3}} = 4\sqrt{3}$$

$$AB : OG = 4\sqrt{3} : 1$$

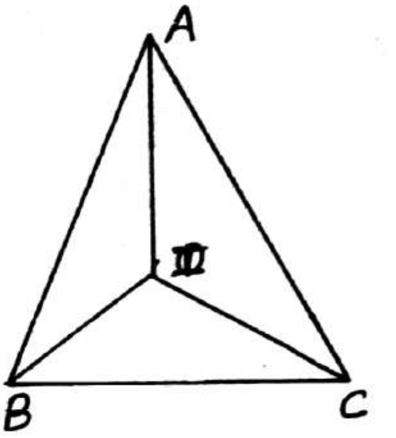
122.

$$AB : OG = 4\sqrt{3} : 1$$

$$\times 3 \downarrow \qquad \downarrow \times 3$$

$$12\sqrt{3} \qquad 3 \text{ cm}$$

123.

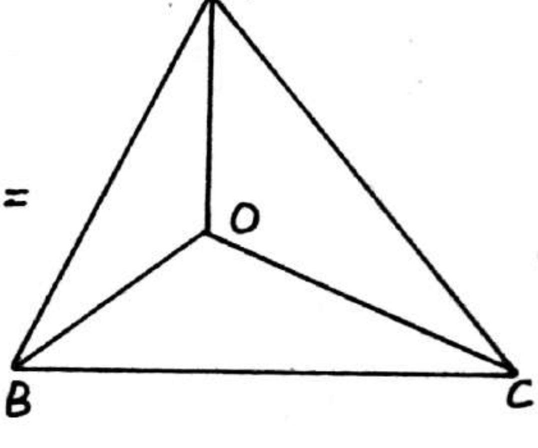


$$\Delta AIB : \Delta BIC = 1 : 1$$

124.

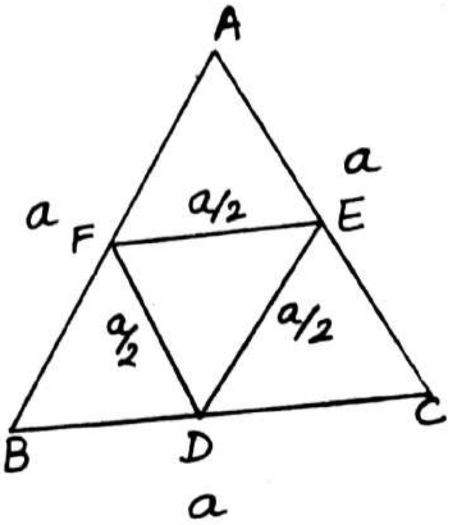
$$\Delta AOB : \Delta BOC : \Delta AOC =$$

$$1 : 1 : 1$$

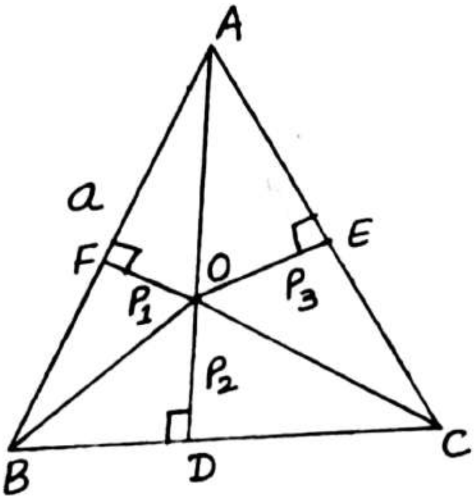


125. $$\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \frac{3a}{\frac{3a}{2}}$$

$$= \frac{2}{1}$$



126.



$$\Delta ABC = \Delta AOB + \Delta BOC + \Delta AOC$$

$$\frac{\sqrt{3}}{4} a^2 = \frac{1}{2} a P_1 + \frac{1}{2} a P_2 + \frac{1}{2} a P_3$$

$$\frac{\sqrt{3}}{2} a = P_1 + P_2 + P_3$$

$$\boxed{H = P_1 + P_2 + P_3}$$

(a) Side $(a) = \frac{2}{\sqrt{3}} (P_1 + P_2 + P_3)$

(b) Area $= 4/3 \, (P_1 + P_2 + P_3)^2 \cdot \frac{\sqrt{3}}{4}$

(C) Perimeter $= 2\sqrt{3}\,(P_1 + P_2 + P_3)$

(d) H $= P_1 + P_2 + P_3$

127. $P_1 = \sqrt{3}$, $P_2 = 2\sqrt{3}$, $P_3 = 5\sqrt{3}$

(a) Side $= \frac{2}{\sqrt{3}}\left(\sqrt{3} + 2\sqrt{3} + 5\sqrt{3}\right)$

$= \frac{2}{\sqrt{3}} \times 8\sqrt{3} \quad = 16$ cm

(b) Area $= (16)^2 \cdot \frac{\sqrt{3}}{4} = 256\ cm^2 \times \frac{\sqrt{3}}{4}$

(C) Perimeter $= 3 \times 16 = 48$ cm

(d) $H = 8\sqrt{3}$ cm

128. $P_1 = 6$, $P_2 = 7$, $P_3 = 8$

(a) Side $= \frac{2}{\sqrt{3}}\,(6 + 7 + 8)$

$= \frac{2}{\sqrt{3}} \times 21 \quad = 14\sqrt{3}$ cm

(b) Area $= \frac{\sqrt{3}}{4}(14\sqrt{3})^2 = 588\ cm^2 \times \frac{\sqrt{3}}{4}$

(C) Perimeter $= 3 \times 14\sqrt{3} \quad = 42\sqrt{3}$ cm

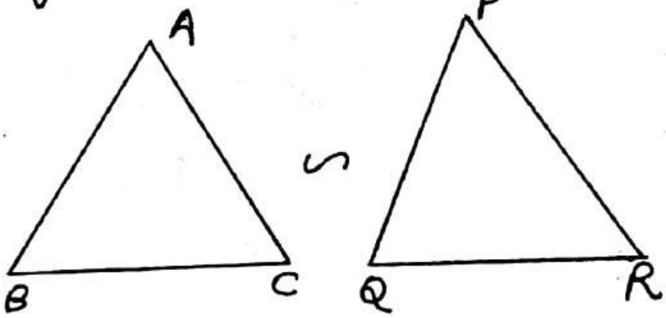
(d) Height = 21 Cm

Similar & Congurent triangle:-

1. Similar triangle :- Ratio of Corresponding sides of two triangles is equal.

Properties of Similar triangle:-

1. Ratio of 1-D is equal.



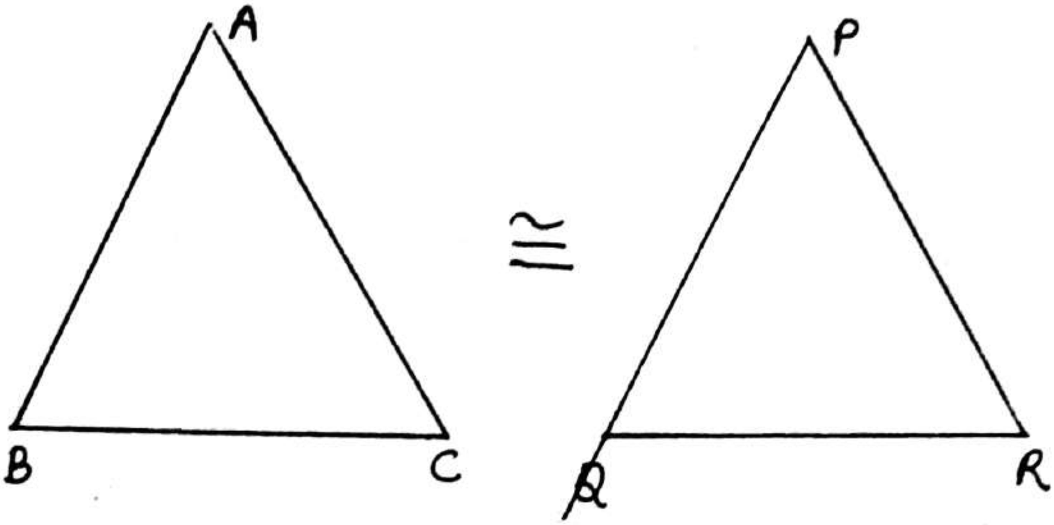
$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{r_1}{r_2} = \frac{R_1}{R_2} = \frac{H_1}{H_2} = \frac{M_1}{M_2} = \frac{AD}{PS} = \frac{P_1}{P_2}$$

2. Ratio of 2-D is equal

$$\frac{\Delta ABC}{\Delta PQR} = \left(\frac{AB}{PQ}\right)^2$$

2. Congurent triangles :- Ratio of corresponding sides of two triangles is 1.

Properties of Congurent triangle:-



1. Ratio of 1 - D is 1

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{r_1}{r_2} = \frac{R_1}{R_2} = \frac{H_1}{H_2} = \frac{M_1}{M_2} = \frac{P_1}{P_2} = \frac{AD}{PS} =$$

2. Ratio of 2 - D is 1.

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \left(\frac{AB}{PQ}\right)^2 = 1$$

Note :- 1. Every Congruent triangle is Similar but Vice - Versa is not true.

2. Ratio of Congruent triangle is as Simila triangle but Vice - Versa is not true.

Properties	$\sim$	$\cong$
AAA	✓	✗
SSS	✓	✓

SAS	✓	✓
ASA	✓	✓

129. In $\triangle ABC$, $DE \parallel BC$. The ratio of AD & BD is 3:7 Find $\triangle ABC : \triangle ADE = ?$

130. In $\triangle ABC$, $DE \parallel BC$ & $AD : BD = 2:3$. Find $\triangle ABC : \square BCED$.

131. In $\triangle ABC$, $DE \parallel BC$ & $AD : BD = 3:5$. Find $\frac{\triangle ADE}{\square BCED}$.

132. In $\triangle ABC$, $DE \parallel BC$ & $AD : BD = 3:4$. If $\triangle ABC = 980\ cm^2$, then $\triangle ADE = ?$

133. In $\triangle ABC$, $DE \parallel BC$ & $AD : BD = 2:3$. If $\triangle ADE = 100\ cm^2$, then $\triangle ABC = ?$

134. In $\triangle ABC$, $DE \parallel BC$ & $AD : BD = 3:5$. If $\triangle ABC = 16\ cm^2$, then $\triangle ADE = ?$

135. In $\triangle ABC$, $DE \parallel BC$ & $AD : BD = 3:7$. If $\triangle ABC = 500\ cm^2$, then $\triangle BCE = ?$

136. In ΔABC, $DE \parallel BC$ & $AD : BD = 3:5$ If $\Delta ABC =$ 16 cm^2, then $\Delta BDE = ?$

137. In ΔABC, $DE \parallel BC$ & $AD : BD = 4:5$. Find $\Delta ADE :$ $\Delta BDE = ?$

138. In ΔABC, $DE \parallel BC$ & $AD : BD = 3:5$. Find ΔBDE $: \Delta BCE = ?$

139. In ΔABC, $DE \parallel BC$ & $AD : BD = 1:2$. Find ΔADE $\Delta BDE : \Delta BCE = ?$

140. In ΔABC, $DE \parallel BC$ & $AD : BD = 3:5$. Find $\Delta ADE :$ $\Delta BDE : \Delta BCE = ?$

141. In ΔABC, $DE \parallel BC$ and $AD : BD = 4:5$. If ΔABC $= 810$ cm^2. Find $\Delta BCE = ?$

142. The ratio of corresponding sides of two triangles is 3:4. Find the ratio of their areas.

143. The ratio of areas of two similar triangles is 4:9. Find the ratio of corresponding sides

144. The ratio of corresponding sides of two similar triangles is 4 : 5. If perimeter of first triangle is 120 cm. Find the perimeter of second triangle.

145. In ΔABC, line $DE \parallel BC$ & divides ΔABC into two equal parts. Find $AD : BD = ?$

146. In a ΔABC, $DE \parallel BC$ & divides ΔABC into two equal parts. Find $AB : AD = ?$

147. In ΔABC, $DE \parallel BC$ & divides ΔABC into two equal parts. Find $AB : BD = ?$

148. In ΔABC, D & E are mid-points of sides AB & AC. Find the ratio of $\Delta ADE : \square BCED = ?$

149. In ΔABC, D & E are mid-points of AB & AC. P is a point on DE such that the ratio $DP : PE = 3 : 4$. If $DP = 12$ cm, find $BC = ?$

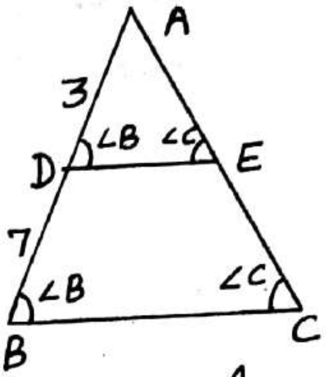
150. ΔABC & ΔPQR are congruent triangles. If $\Delta ABC = 200 \text{ cm}^2$. Find $\Delta PQR = ?$

Solutions

129.

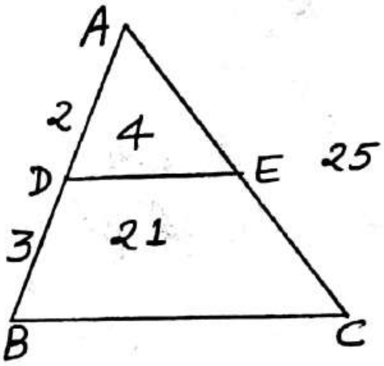
$$\frac{\Delta ABC}{\Delta ADE} = \left(\frac{10}{3}\right)^2$$

$$= \frac{100}{9}$$



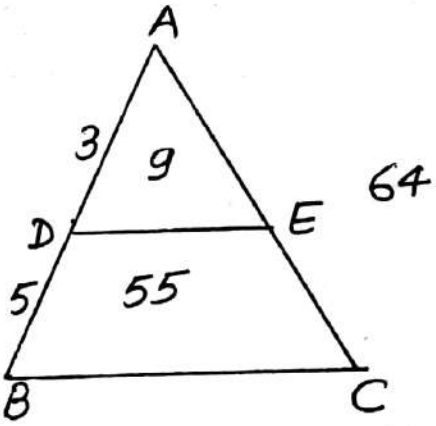
130. $\frac{\Delta ABC}{\Delta ADE} = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$

$$\frac{\Delta ABC}{\square BCED} = \frac{25}{21}$$



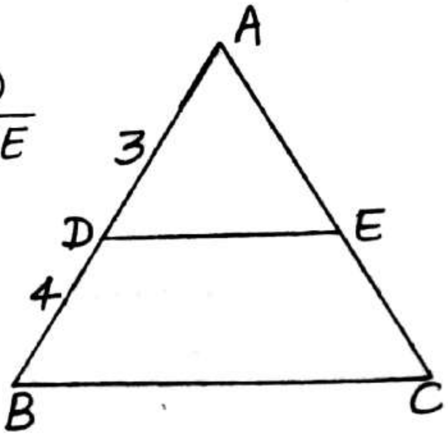
131. $\frac{\Delta ABC}{\Delta ADE} = \left(\frac{8}{3}\right)^2 = \frac{64}{9}$

$$\frac{\Delta ADE}{\square BCED} = \frac{9}{55}$$



132. $\frac{\Delta ABC}{\Delta ADE} = \left(\frac{7}{3}\right)^2 = \frac{980}{\Delta ADE}$

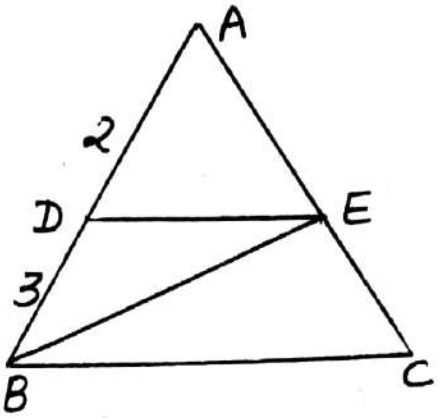
$$\Delta ADE = 180 \text{ cm}^2$$



133. $\frac{\Delta ABC}{\Delta ADE} = \frac{25}{4}$

$\frac{\Delta ABC}{100} = \frac{25}{4}$

$\Delta ABC = 625 \text{ cm}^2$

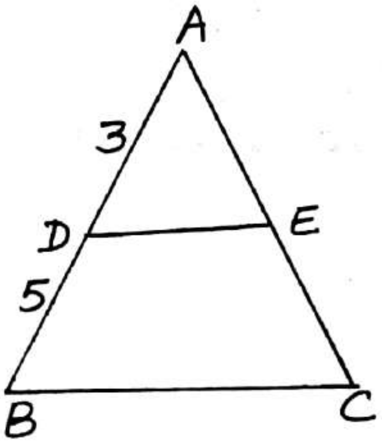


134. $\frac{\Delta ABC}{\Delta ADE} = \frac{64}{9}$

$\frac{16}{\Delta ADE} = \frac{64}{9}$

$\Delta ADE = \frac{9}{4}$

$= 2.25 \text{ cm}^2$

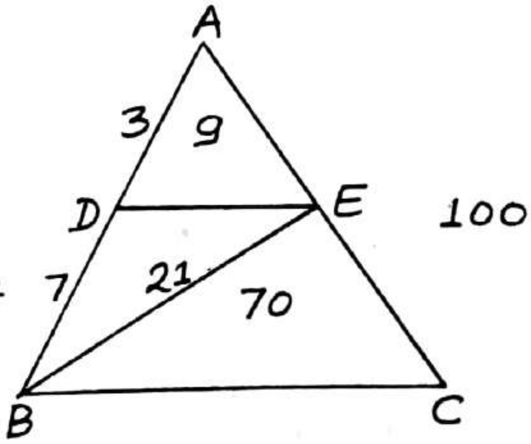


135. ΔABC

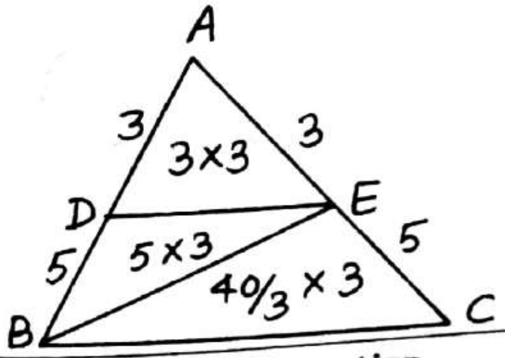
$100 \rightarrow 500$

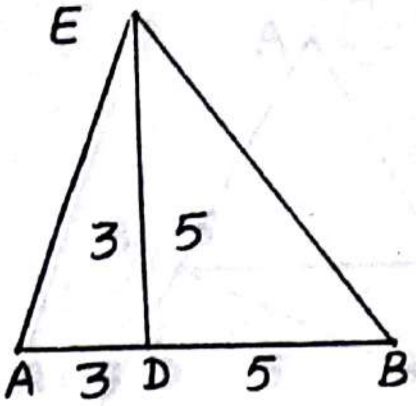
$1 \rightarrow 5$

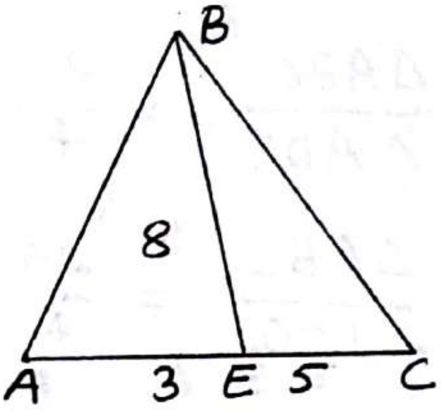
$\Delta BCE = 70 \times 5 = 350 \text{cm}^2$



136.







$3 \rightarrow 8$

$5 \rightarrow 40/3$

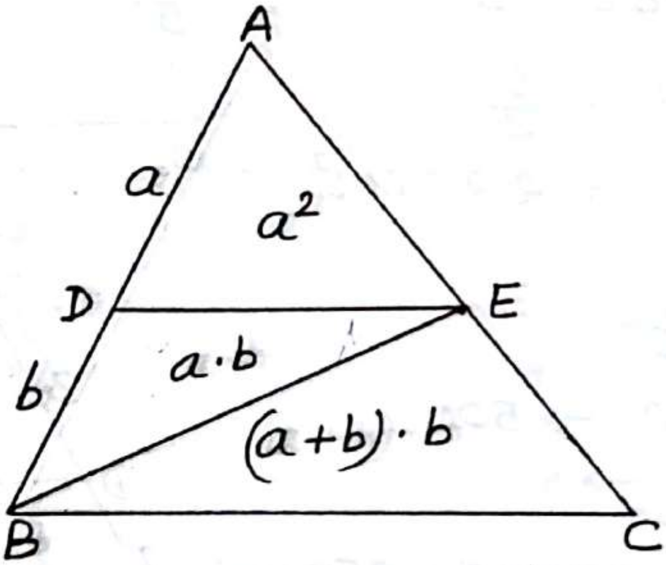
$\Delta ABC = 16\ cm^2$

$64 \rightarrow 16$

$1 \rightarrow \frac{1}{4}$

$\Delta BDE = 15 \times \frac{1}{4} = 3{\cdot}75\ cm^2$

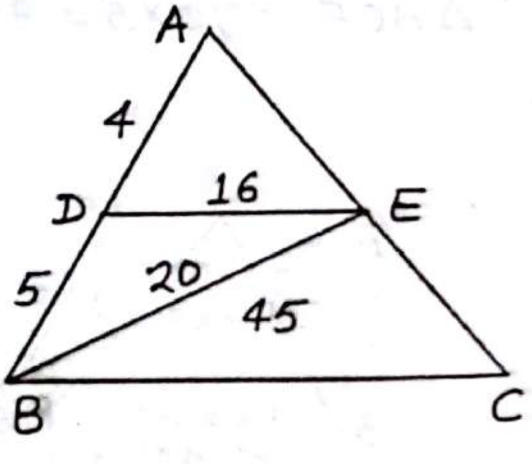
Concept :-



137.

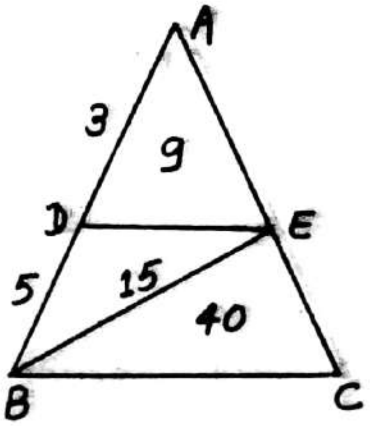
$$\frac{\Delta ADE}{\Delta BDE} = \frac{16}{20}$$

$$= \frac{4}{5}$$



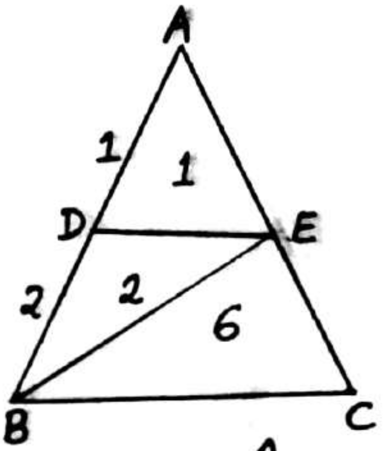
138. $\frac{\Delta BDE}{\Delta BCE} = \frac{15}{40}$

$= \frac{3}{8}$



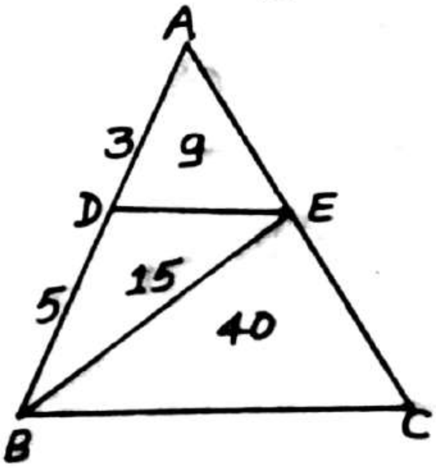
139. $\Delta ADE : \Delta BDE : \Delta BCE$

$1 : 2 : 6$



140. $\Delta ADE : \Delta BDE : \Delta BCE$

$9 : 15 : 40$

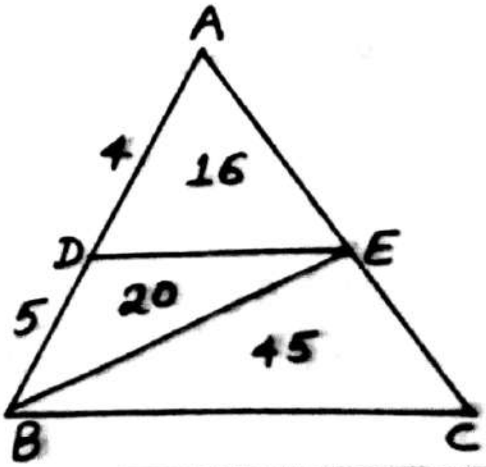


141. $\Delta ABC = 810\ cm^2$

$81 \rightarrow 810$

$1 \rightarrow 10$

$\Delta BCE = 45 \times 10 = 450\ cm^2$



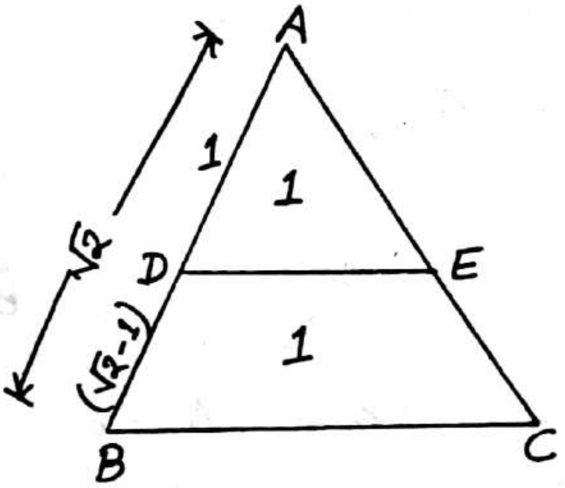
142. Ratio of areas $= \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

143. $\frac{S_1}{S_2} = \sqrt{\frac{4}{9}} = \frac{2}{3}$

144. Ratio of perimetres $= \frac{4}{5}$

$$\frac{4}{5} \begin{matrix} \xrightarrow{30} 120 \\ = \\ \xrightarrow[30]{} \boxed{150} \end{matrix}$$

145/146/147.



$$\frac{\Delta ABC}{\Delta ADE} = \frac{2}{1}$$

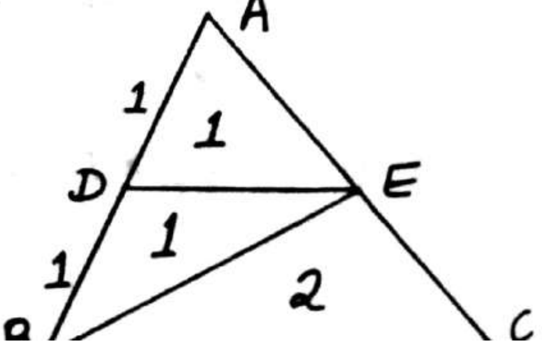
$$\frac{AB}{AD} = \frac{\sqrt{2}}{1}$$

145. $AD : BD = \frac{1}{(\sqrt{2}-1)} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = (\sqrt{2}+1)$

146. $AB : AD = \frac{\sqrt{2}}{1}$

147. $AB : BD = \frac{\sqrt{2}}{(\sqrt{2}-1)} \times \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)} = 2+\sqrt{2}$

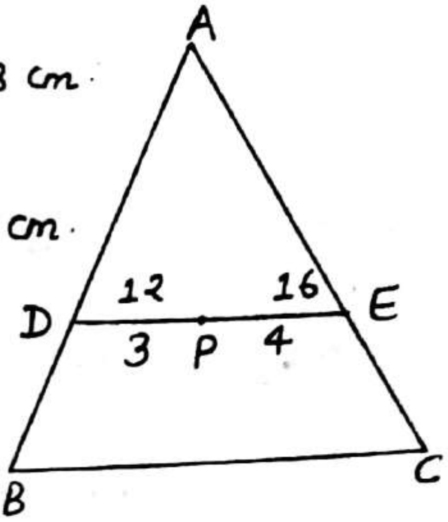
148. $\frac{\Delta ADE}{\square BCED} = \frac{1}{3}$



149. 3 → 12 $DE = 28\ cm$.

1 → 4

4 → 16 $BC = 56\ cm$.



150. $\Delta ABC \cong \Delta PQR$

If $\Delta ABC = 200\ cm^2$

$\Rightarrow \Delta PQR = 200\ cm^2$

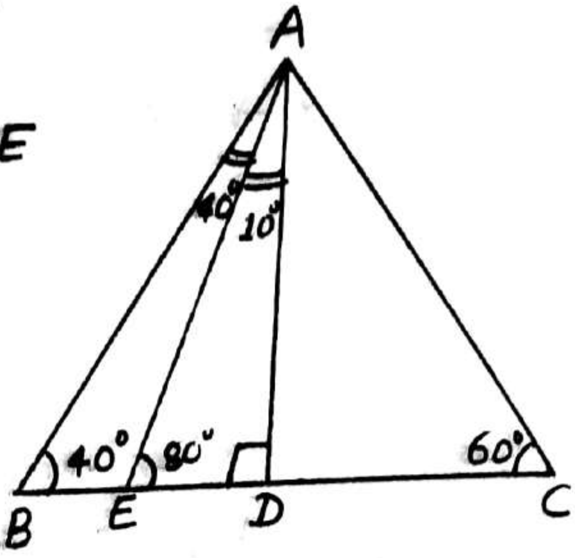
151. In ΔABC, $AD \perp BC$ & AE is an angle bisector of $\angle A$. If $\angle B = 40°$, $\angle C = 60°$, $\angle DAE = ?$

152. In ΔABC, $AD \perp BC$ & AE is an angle bisector of $\angle A$. If $\angle B = 80°$, $\angle C = 60°$, then $\angle DAE = ?$

153. In ΔABC, BD is an angle bisector of $\angle B$ & CD is an exterior angle bisector of $\angle C$. If $\angle BAC = 80°$, $\angle BDC = ?$

154. In ΔABC, BD is an angle bisector of $\angle B$ & CD is an exterior angle bisector of $\angle C$. Find the ratio of $\angle BAC$ & $\angle BDC$

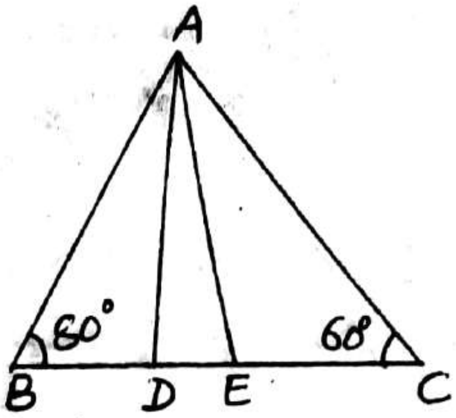
Solutions



151. $\angle AED = \angle ABE + \angle BAE$

$$\angle DAE = \frac{\angle B \sim \angle C}{2}$$

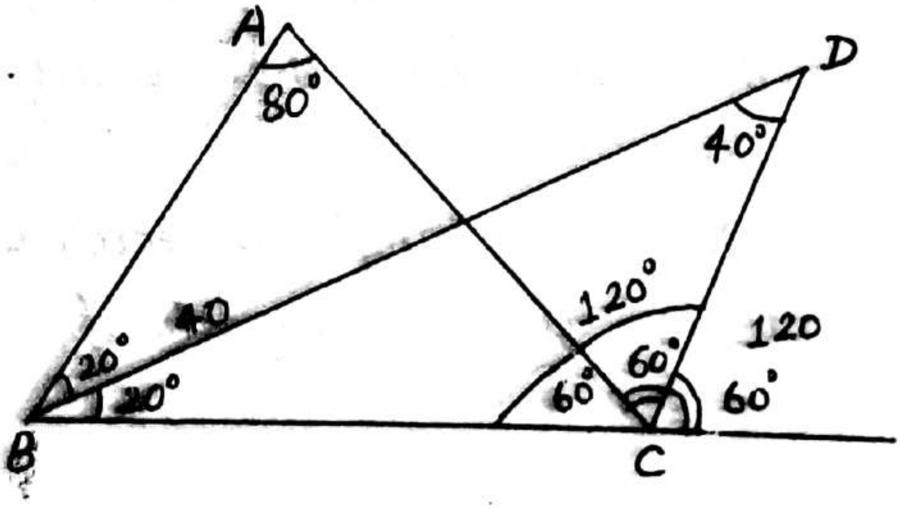
$$= \frac{60° - 40°}{2} = 10°$$



152. $\angle DAE = \frac{80-60}{2}$

$$= \frac{20}{2} = 10°$$

153/154.



153. $\angle BDC = 40°$

154. $\frac{\angle BAC}{\angle BDC} = \frac{80}{40} = \frac{2}{1}$

Questions based on sides of triangle:-

1. Sum of two sides is always greater than third side.

2. Difference of two sides is less than third side.

3. a, b and c are sides of ΔABC such that c is the maximum side of ΔABC, then

(i) $a^2+b^2 = c^2$

$\angle C$ is right angle. (opposite to c)

(ii) $a^2+b^2 < c^2$

$\angle C$ is obtuse angle.

$\Rightarrow$ ΔABC is obtuse angle triangle.

(iii) $a^2+b^2 > c^2$

ΔABC is acute angle triangle.

155. The ratio of sides of ΔABC is $1 : 2\frac{2}{5} : 2\frac{3}{5}$, then ΔABC will be -

(a) Acute angle triangle [✓ (b) Right angle triangle]
(c) obtuse angle triangle (d) None of these

156. The ratio of sides of ΔABC is $2:3:4$, then ΔABC will be -

(a) Acute angle triangle (b) Right angle triangle
[✓ (c) obtuse angle triangle] (d) None of these

157. The Ratio of sides of ΔABC is 5 : 6 : 7, then ΔABC will be -
[✓ (a) Acute angle triangle] (b) Right angle triangle
(c) Obtuse angle triangle (d) None of these

158. The length of four line segments are 2 cm, 3 cm, 4 cm & 5 cm. How many triangles Can be formed by these line segments.

Solutions

155. $1 : \frac{12}{5} : \frac{13}{5}$

$5 : 12 : 13$

$5^2 + 12^2 = 25 + 144 = 169$

$13^2 = 169$

156. $2 : 3 : 4$

$2^2 + 3^2 = 4 + 9 = 13$

$4^2 = 16$

157. $5 : 6 : 7$

$5^2 + 6^2 = 25 + 36 = 61$

$7^2 = 49$

158. Three triangles

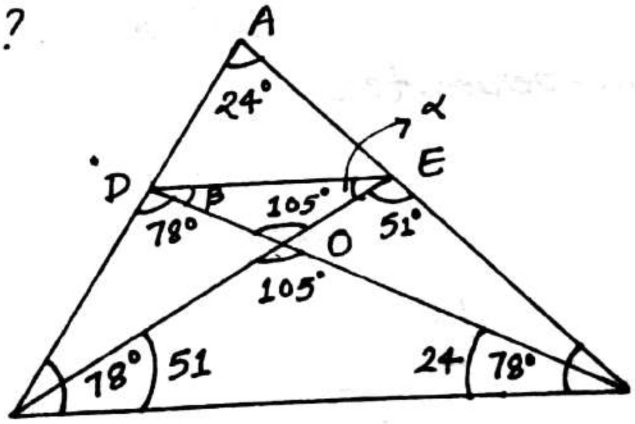
2, 3, 4 → ✓

3, 4, 5 → ✓

2, 3, 5 → ×

2, 4, 5 → ✓

159. In ΔABC, points D & E are any points on sides AB & AC such that $\angle B = \angle C = 78^\circ$, $\angle BCD = 24^\circ$ & $\angle CBE = 51^\circ$, $\angle ADE = ?$, $\angle AED = ?$, $\angle BED = ?$ & $\angle CDE = ?$



$\alpha + \beta = 75^\circ \Rightarrow \alpha + 51^\circ = \beta$

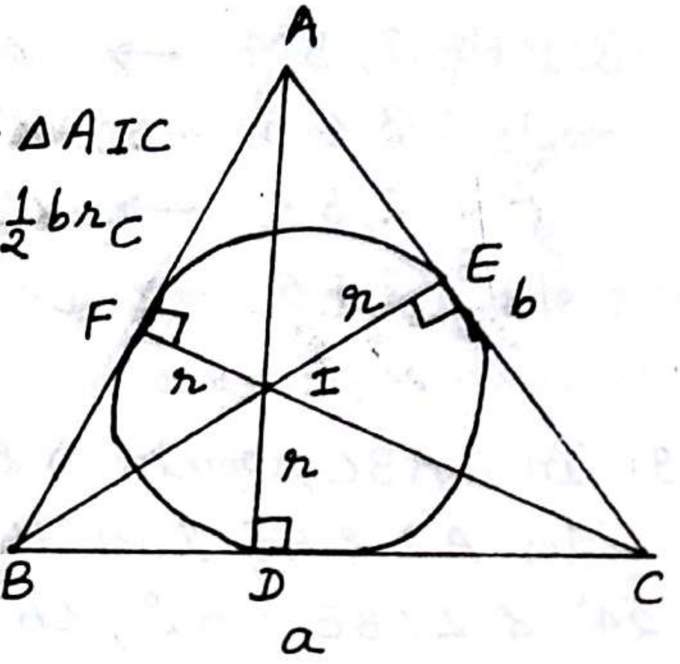
$\alpha + \alpha + 51^\circ = 75 \Rightarrow \alpha = 12^\circ$

$\beta = 75^\circ - 12^\circ = 63^\circ$

$\angle AED = 117^\circ$, $\angle ADE = 39^\circ$

Inradius of triangle :-

$$\text{Inradius of triangle} = \frac{\text{Area of triangle}}{\text{Semi-perimeter of triangle}}$$



$\Delta ABC = \Delta AIB + \Delta BIC + \Delta AIC$

$\Delta = \frac{1}{2}cr + \frac{1}{2}ar + \frac{1}{2}br$

$\Delta = \frac{1}{2}(a+b+c) \cdot r$

$\Delta = s \cdot r$

$$\boxed{r = \frac{\Delta}{s}}$$

$s \rightarrow$ Semi-perimeter

For equilateral triangle

$$r = \frac{\frac{\sqrt{3}}{4}a^2}{\frac{3}{2}a} = \frac{\sqrt{3}}{4}a \times \frac{2}{3}$$

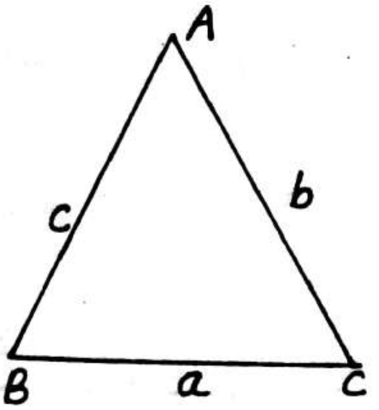
$$= \frac{a}{2\sqrt{3}}$$

Sin rule of triangle :-

1. Area $= \frac{1}{2} ab \sin C$

or $= \frac{1}{2} ac \sin B$

or $= \frac{1}{2} bc \sin A$

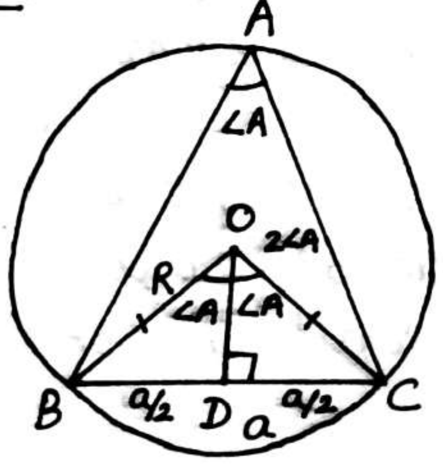


2. $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$a : b : c = \sin A : \sin B : \sin C$

$\sin A : \sin B : \sin C = a : b : c$

Circumradius of triangle:-



In ΔBCD

$\sin A = \frac{a}{2R}$

$R = \frac{a}{2\sin A}$

or $= \frac{b}{2\sin B}$

or $= \frac{c}{2\sin C}$

$\Delta = \frac{1}{2} bc \sin A$

$\sin A = \frac{2\Delta}{bC}$

$$\boxed{R = \frac{abC}{4\Delta}}$$

Ratio of Inradius & Circumradius:-

$$\frac{r}{R} = \frac{\Delta/s}{abc/4\Delta}$$

$$= \frac{4\Delta^2}{s(abc)}$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\boxed{\frac{r}{R} = \frac{4(s-a)(s-b)(s-c)}{abc}}$$

160. In ΔABC, D is a point on BC such that the ratio of BD & CD is 1:3. $\angle B = \frac{\pi}{4}$, $\angle C = \frac{\pi}{3}$ Find $\sin\angle BAD : \sin\angle CAD = ?$

161. In ΔABC, D is a point on BC such that the ratio BD:CD is 1:2. $\angle B = \frac{\pi}{4}$, $\angle C = \frac{\pi}{3}$. Find the ratio $\sin\angle BAD : \sin\angle CAD = ?$

162. In ΔABC, D is a point on BC such that the ratio BD:CD is 1:3. $\angle B = \pi/3$, $\angle C = \pi/4$. Find the ratio $\sin\angle BAD : \sin\angle CAD = ?$

163. In ΔABC, D is a point on BC such that the ratio BD:CD is 1:2. $\angle B = \pi/3$ & $< C = \pi/4$. Find the ratio $\sin\angle BAD : \sin\angle CAD = ?$

164. In ΔABC, $a = b = 5$, $c = 8$. Find $R = ?$

165. In ΔABC, $a = 4$, $b = 6$, $c = 8$. Find the ratio $r : R = ?$

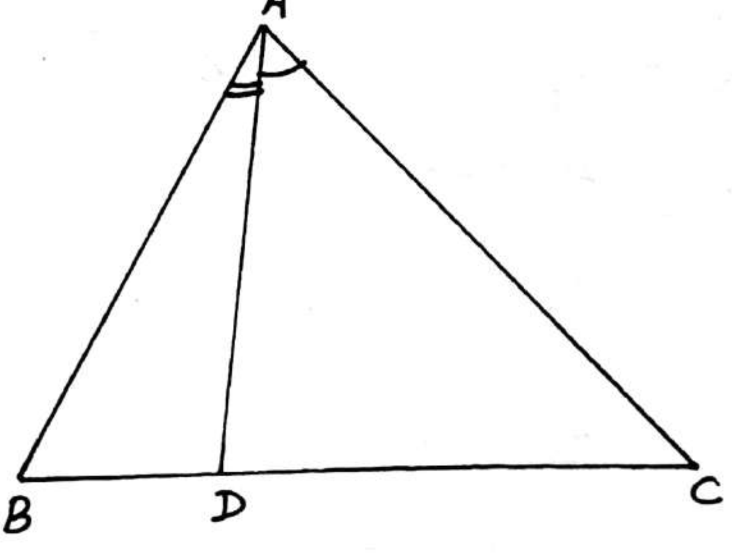
166. In ΔABC, $\angle A = 60^\circ$, $\angle B = 30^\circ$. Find the ratio of $AB : BC : AC = ?$

167. In ΔABC, $AB = 10$ cm, $BC = 20$ cm, $\angle B = 30^\circ$. Find the area of ΔABC.

168. In ΔABC, $BC = 10$ cm. If $\angle BAC = 30^\circ$, find circumradius of ΔABC.

Solutions

Result :-



$$\frac{\Delta ABD}{\Delta ACD} = \frac{\frac{1}{2} \cdot AB \cdot AD \cdot \sin \angle BAD}{\frac{1}{2} \cdot AC \cdot AD \cdot \sin \angle CAD} = \frac{\frac{1}{2} AB \cdot BD \cdot \sin B}{\frac{1}{2} \cdot CD \cdot AC \cdot \sin C}$$

$$\frac{\sin\angle BAD}{\sin\angle CAD} = \left(\frac{BD}{CD}\right)\cdot\frac{\sin B}{\sin C}$$

160. $\frac{\sin\angle BAD}{\sin\angle CAD} = \frac{1}{3}\ \frac{\sin \pi/4}{\sin \pi/3}$

$= \frac{1}{3} \times \frac{1/\sqrt{2}}{\sqrt{3}/2} = \frac{\sqrt{2}}{3\sqrt{3}}$

161. $\frac{\sin\angle BAD}{\sin\angle CAD} = \frac{1}{2}\ \frac{\sin \pi/4}{\sin \pi/3} = \frac{1}{2}\cdot\frac{\sqrt{2}}{\sqrt{3}} = \frac{1}{\sqrt{6}}$

162. $\frac{\sin\angle BAD}{\sin\angle CAD} = \frac{1}{3}\ \frac{\sin \pi/3}{\sin \pi/4} = \frac{1}{3}\times\frac{\sqrt{3}}{2}\times\sqrt{2}$

$= \frac{1}{\sqrt{6}}$

163. $\frac{\sin\angle BAD}{\sin\angle CAD} = \frac{1}{2}\ \frac{\sin \pi/3}{\sin \pi/4} = \frac{1}{2}\times\frac{\sqrt{3}}{2}\times\sqrt{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}}$

164. $R = \frac{abc}{4\Delta}$ $\quad\quad \Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$R = \frac{5\times5\times8}{4\times12}$ $\quad\quad = \sqrt{9\times4\times4}$

$= \frac{25}{6}$ $\quad\quad = 12$

165. $\frac{r}{R} = \frac{4(s-a)(s-b)(s-c)}{abc}$

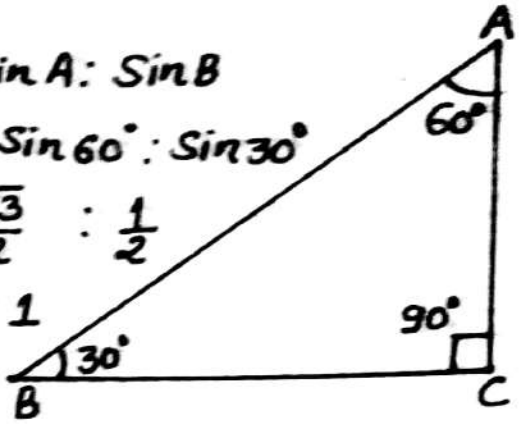
$= \frac{4\times5\times3\times1}{4\times6\times8} = \frac{5}{16}$

166. $AB : BC : AC = \sin C : \sin A : \sin B$

$= \sin 90^\circ : \sin 60^\circ : \sin 30^\circ$

$= 1 : \frac{\sqrt{3}}{2} : \frac{1}{2}$

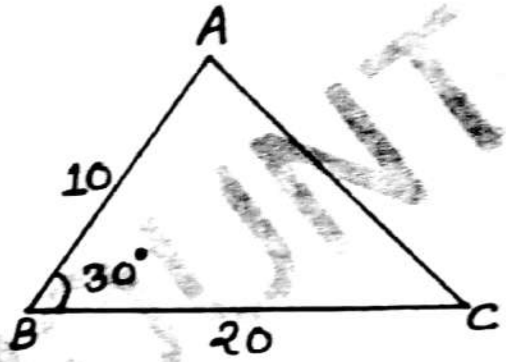
$= 2 : \sqrt{3} : 1$



167. Area of ΔABC

$= \frac{1}{2} \times 10 \times 20 \times \sin 30^\circ$

$= 50 \text{ cm}^2$



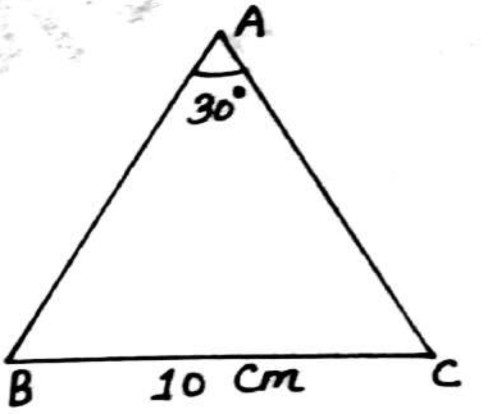
168. $R = \frac{abc}{4\Delta}$

'or'

$R = \frac{a}{2\sin A}$

$= \frac{10}{2 \sin 30^\circ}$

$= \frac{5}{1/2} = 10 \text{ cm}$



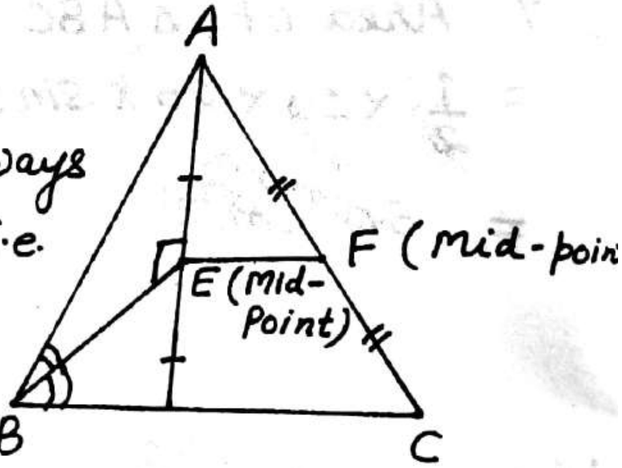
169. In ΔABC, E is a point such that BE is an angle bisector of $\angle B$ & $AE \perp BE$. F is a point on AC such that $EF \parallel BC$. If $AC = 12$ cm, find the length of AF.

170. In ΔABC, E is a point such that BE is a angle bisector of $\angle B$ & $AE \perp BE$. F is a point on AC such that $EF \parallel BC$. Find the ratio of-

(a) AF : FC

(b) AC : AF

(c) AC : FC



169/170.

A perpendicular bisector always cut line into equal parts i.e. it is a mid-point.

169. AF = 6 cm.

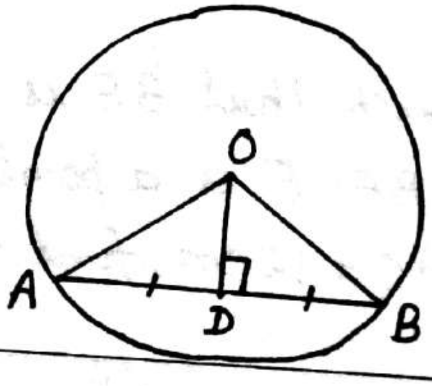
(In Isoc. & equli. Δ bise ctor is always $\perp$ & hence mid-point)

170. (a) 1 : 1

(b) 2 : 1

(c) 2 : 1

Questions based on Circle :-

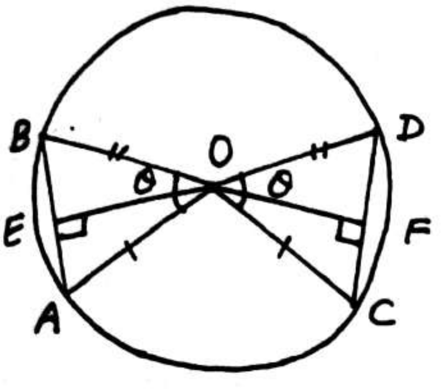


If $OD \perp AB$

then $AD = BD$

If $AD = BD$

then $OD \perp AB$

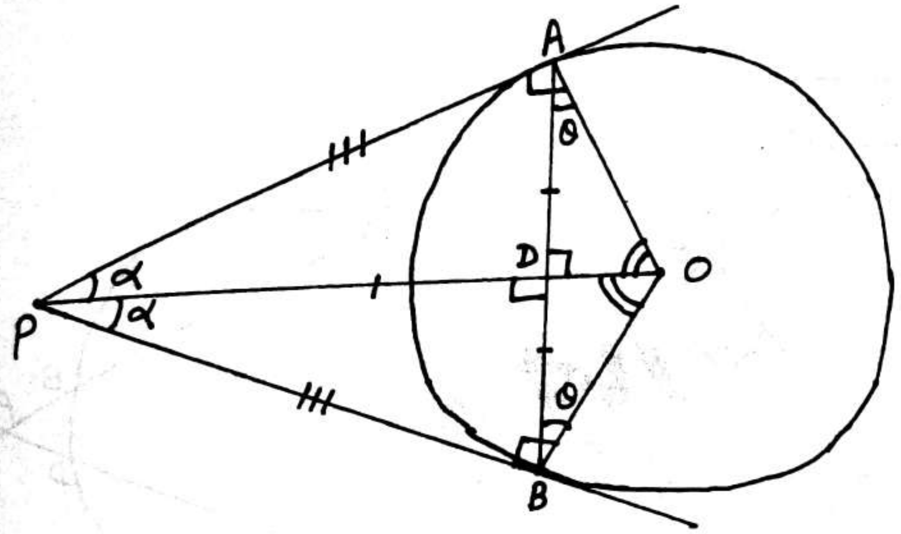


If $AB = CD$

$OE = OF$

$\angle AOB = \angle COD$

$\overset{\frown}{AB} = \overset{\frown}{CD}$



$$\Delta AOP \cong \Delta BOP$$

1. $AP = BP$
2. $\angle APO = \angle BPO$
3. $\angle AOP = \angle BOP$
4. $\angle APB + \angle AOB = 180^\circ$

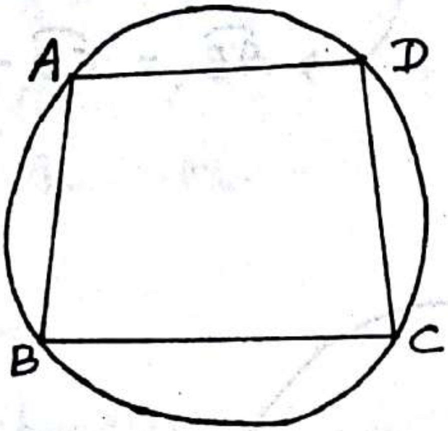
$$2\alpha + 180^\circ - 2\theta = 180^\circ$$

$$\alpha = \theta$$

5. $\angle ABO = \frac{1}{2} \angle APB$

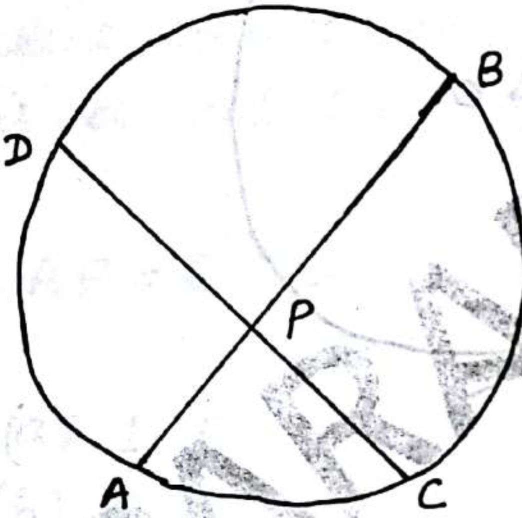
$$\triangle APO = \frac{1}{2}\,\overset{radius}{AO}\cdot\overset{tangent}{AP}$$

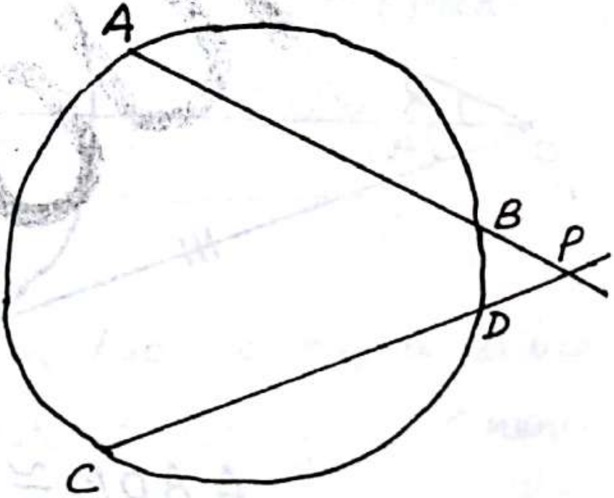
$$\boxed{\square APBO = AO\cdot AP}$$



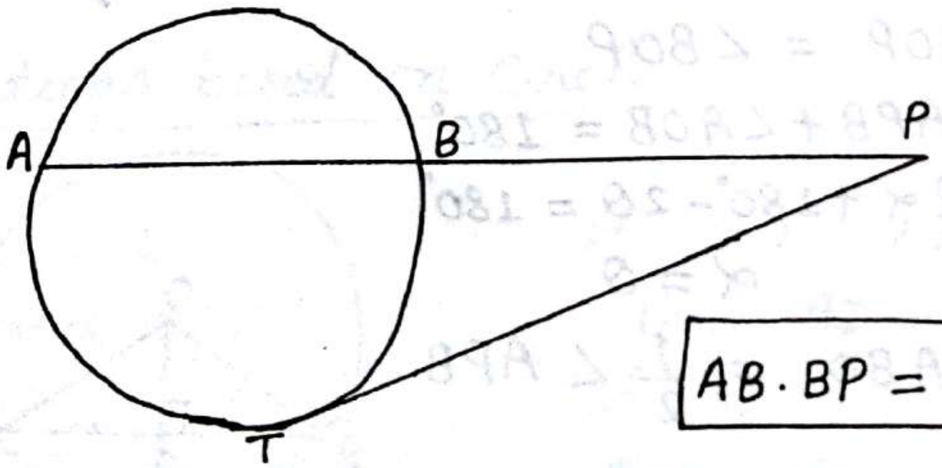
$\angle A + \angle C = 180°$

$\angle B + \angle D = 180°$





$AP\cdot BP = CP\cdot DP$

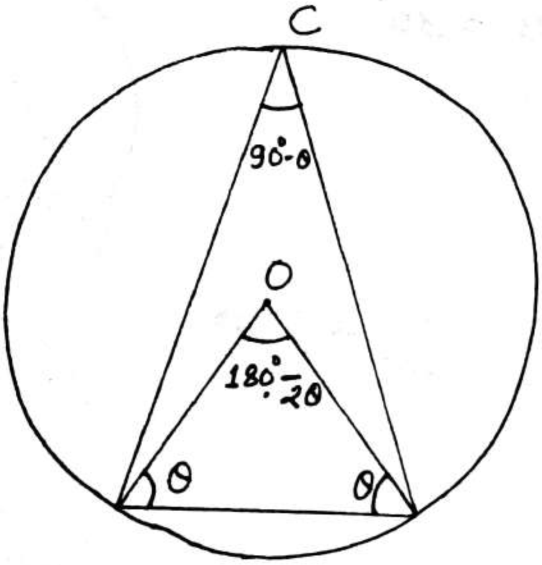


$$\boxed{AB\cdot BP = PT^2}$$

171. AB is a chord & O is the centre of circle. C is any point on major arc of AB. Find the sum of $\angle OAB$ & $\angle ACB$.

172. AB is a chord & O is the centre of circle. C is any point on minor arc of AB. Find $\angle ACB - \angle OAB = ?$

171.



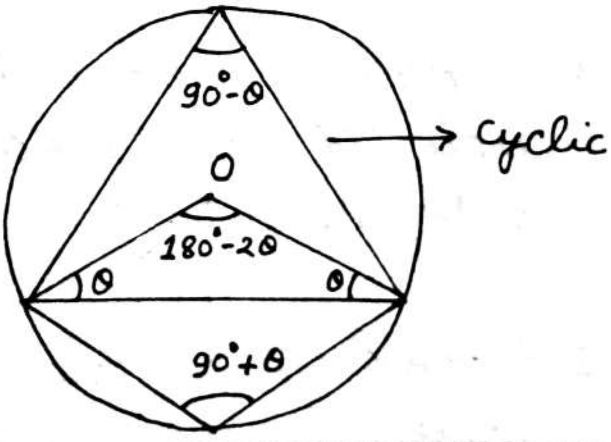
$$\angle OAB + \angle ACB = \theta + 90^\circ - \theta = 90^\circ$$

172.

$\angle ACB - \angle OAB$

$= 90^\circ + \theta - \theta$

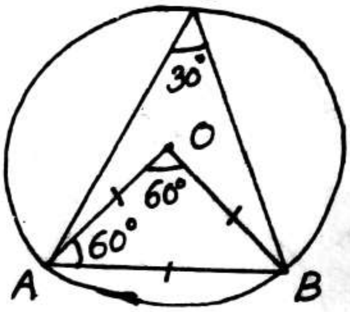
$= 90^\circ$



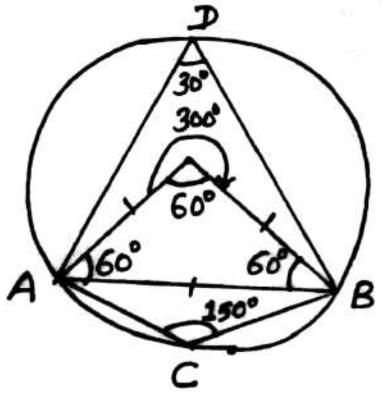
173. AB is a chord half of diameter. C is a point on major arc of AB. Find $\angle ACB = ?$

174. AB is a chord half of diameter. C is a point on minor arc of AB. Find $\angle ACB = ?$

173. $\angle ACB = 30^\circ$



174. $\angle ACB = 150^\circ$



175. AB & CD are two chords & O is the centre of circle. If $\angle AOB = 60^\circ$ & $\angle COD = 90^\circ$, find the ratio AB : CD = ?

176. AB & CD are two chords & O is the centre of circle. If $\angle AOB = 60^\circ$ & $\angle COD = 120^\circ$, find AB : CD.

177. AB & CD are two chords & O is the Centre of the circle. If $\angle AOB = 60^\circ$ & $\angle COD = 90^\circ$, $AB = a$ and $CD = b$ then which is true -

(a) $a^2 = 2b^2$ [✓ (b) $2a^2 = b^2$]

(c) $a^2 = 3b^2$ (d) $3a^2 = b^2$

178. AB & CD are two chords & O is the Centre of circle. If $\angle AOB = 60^\circ$ & $\angle COD = 120^\circ$, $AB = a$ & $CD = b$ then which is true -

(a) $a^2 = 2b^2$ (b) $2a^2 = b^2$

(c) $a^2 = 3b^2$ [✓ (d) $3a^2 = b^2$]

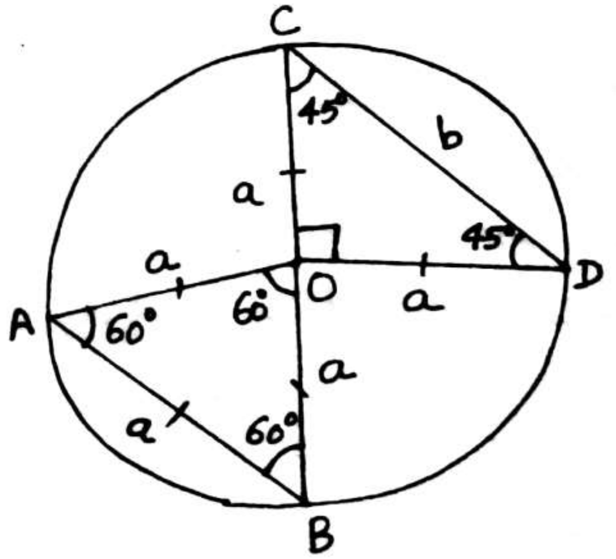
175/177.

In ΔCOD -

$b^2 = a^2 + a^2$

$\boxed{b^2 = 2a^2}$

$$\frac{AB}{CD} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$$

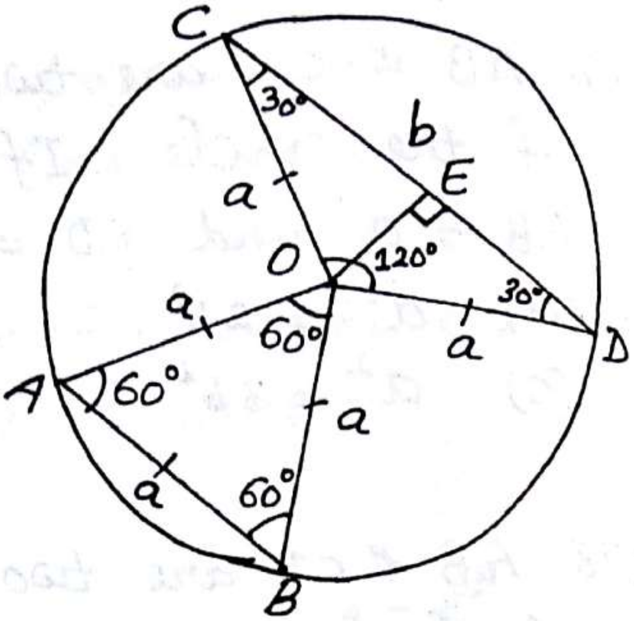


175. $AB : CD = 1 : \sqrt{2}$

177. $2a^2 = b^2$

176/178.

In ΔEOD -



$$\cos 30^\circ = \frac{b/2}{a}$$

$$\frac{\sqrt{3}}{2} = \frac{b}{2a}$$

$$b = \sqrt{3}a$$

176. $\frac{AB}{CD} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$

178. $\sqrt{3}a = b$

$$\boxed{3a^2 = b^2}$$

179. AB & CD are two chords intersect at right angle at point P. The distance between point P & centre of circle is c. If AB = 2a, CD = 2b then find the radius of circle.

180. AB & CD are two chords of circle intersect at right angle at point P. If AP = 2 cm, BP = 6 cm & CP = 3 cm. Find -

(a) radius of the circle

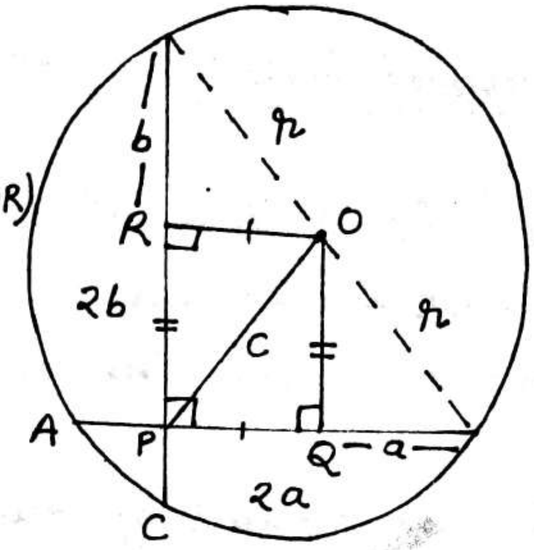
(b) diameter of the circle.

179.

In ΔPOQ -

$$c^2 = PQ^2 + OQ^2$$

$$= OR^2 + OQ^2 \quad (\because PQ = OR)$$



In ΔQOB -

$$r^2 = OQ^2 + a^2 \quad -(i)$$

In ΔROD -

$$r^2 = OR^2 + b^2 \quad -(ii)$$

From equation (i) & (ii) -

$$2r^2 = OQ^2 + OR^2 + a^2 + b^2$$

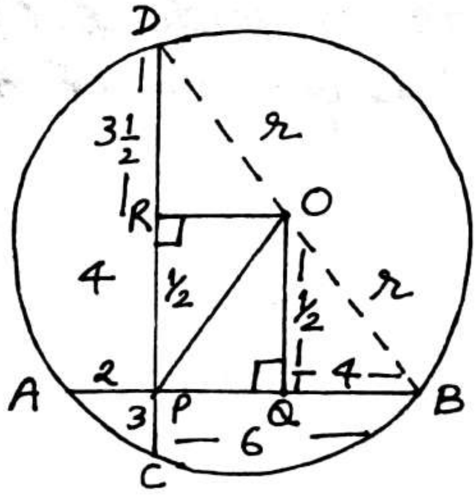
$$2r^2 = a^2 + b^2 + c^2$$

$$r = \sqrt{\frac{a^2+b^2+c^2}{2}}$$

180. $AP \cdot BP = CP \cdot DP$

$$2 \times 6 = 3 \times DP$$

$$DP = 4 \text{ cm}$$



In ΔQOB -

$$r^2 = 16 + \frac{1}{4}$$

$$r^2 = \frac{65}{4}$$

$$r = \frac{\sqrt{65}}{2}$$

$$2r = \sqrt{65}$$

(a) $r = \frac{\sqrt{65}}{2}$ cm

(b) $d = \sqrt{65}$ cm

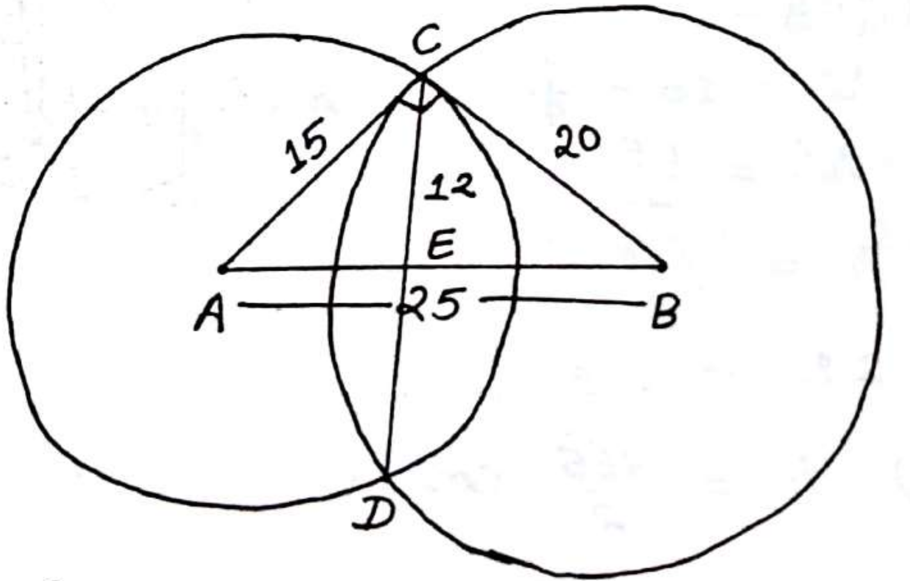
Questions based on length of common chord:-

181. The radius of two circles are 15 cm and 20 cm & the distance between their centres is 25 cm. Find the length of common chord.

182. The radius of two circles are 3 cm and 5 cm & the distance between their centres is 4 cm. Find the length of common chord.

183. Radius of two equal circles is 5 cm passing through each other centres. Find the length of common chord.

Solutions



$$(25)^2 = (15)^2 + (20)^2$$

So, ΔACB is right angle triangle, right angled at C, opposite to max. side.

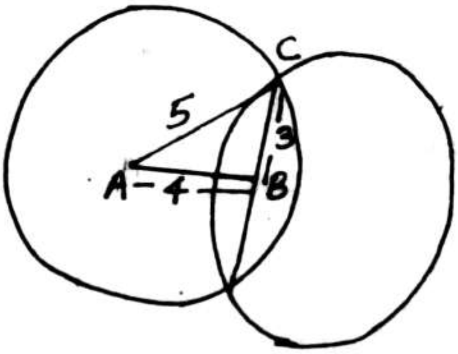
$$EC = \frac{15 \times 20}{25} = 12 \text{cm}$$

$$\boxed{CD = 24 \text{ cm}}$$

182.

$CD = 3+3$

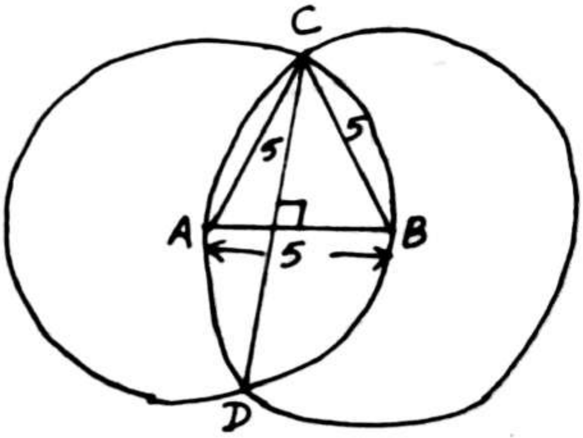
$= \boxed{6 \text{ cm}}$



183.

$$CE = \frac{\sqrt{3}}{2} \times 5$$

$$\boxed{CD = 5\sqrt{3} \text{ cm}}$$

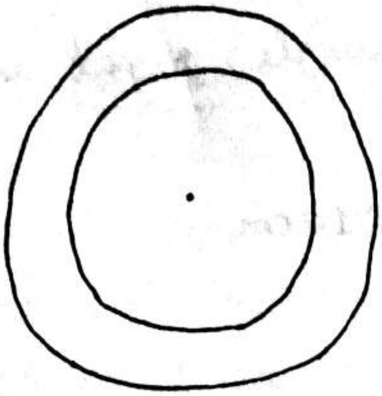


Note:- In equilateral triangle, length of common Chord is $\sqrt{3}r$ always.

Number of common tangents:-

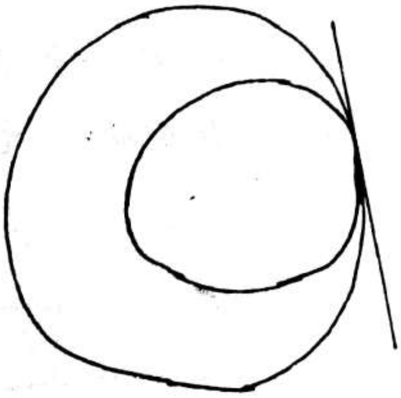
$C_1 \neq C_2$ & $r_1 \neq r_2$

1.



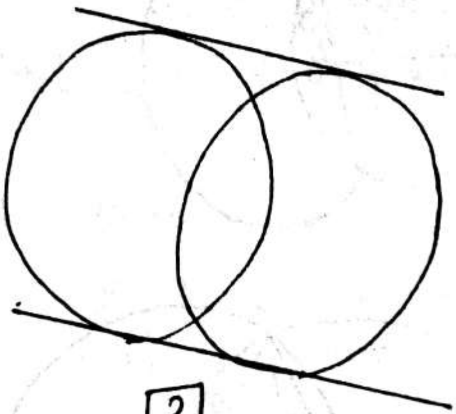
0

2.



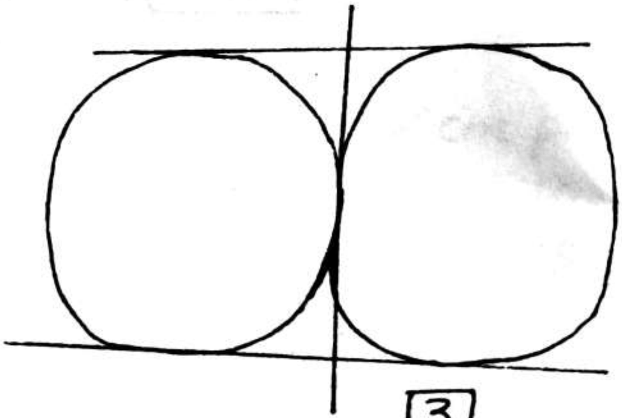
1

3.



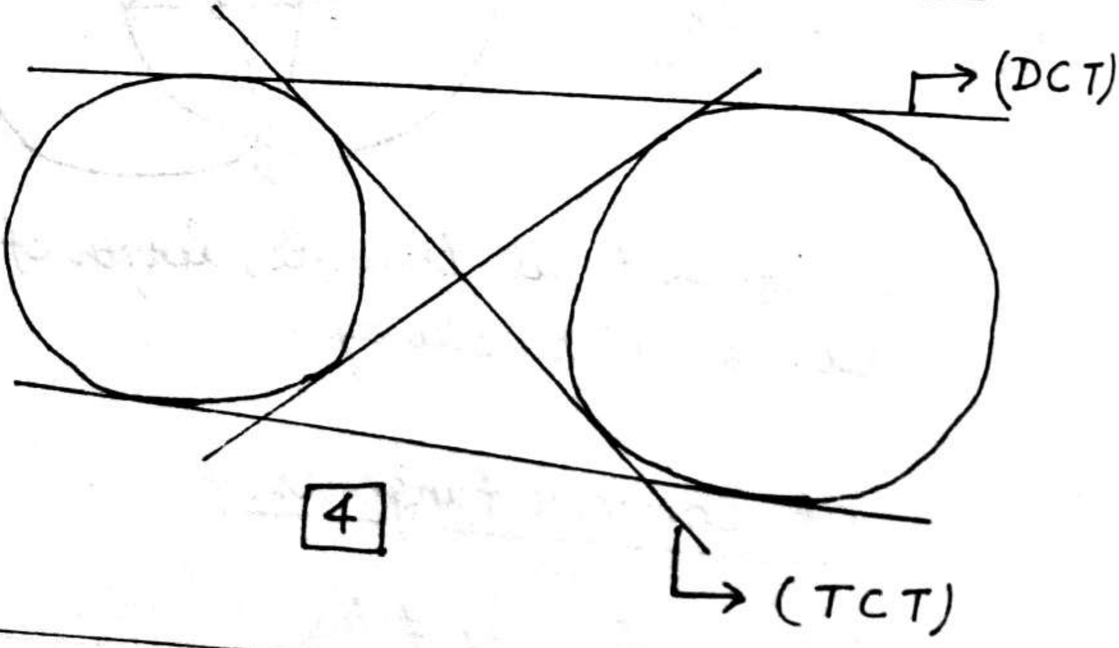
2

4.



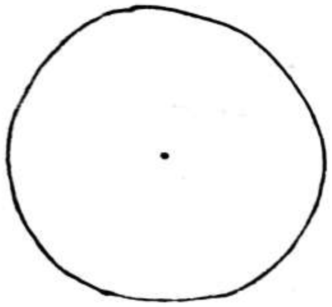
3

5.



4

$C_1 \equiv C_2$, $r_1 = r_2$

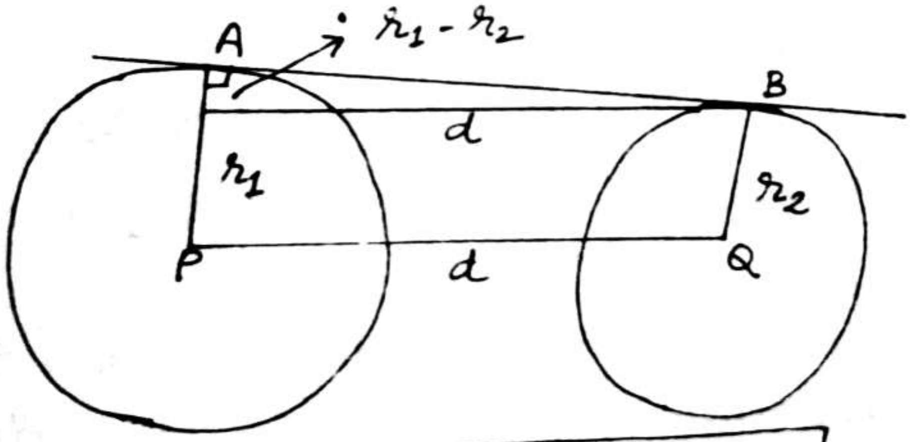


$\boxed{\infty}$

Length of Common tangent

1. Length of direct common tangent (DCT)
2. Length of Transverse Common tangent (TCT)

1. Direct Common tangent :-



$$\boxed{AB = \sqrt{d^2 - (r_1 - r_2)^2}}$$

2. Transverse common tangent:-

$$\boxed{AB = \sqrt{d^2-(r_1+r_2)^2}}$$

184. The radii of two circles are 17 cm & 10 cm and the distance between their centres is 25 cm. Find the length of direct common tangent.

185. The radii of two circles are 14 cm & 10 cm and the distance between their centres is 25 cm. Find the length of transverse common tangent.

186. The radii of two circles are r_1 & r_2 cm touch each other externally. AB is a direct common tangent then AB = ?

(a) $2r_1r_2$
(b) $4r_1r_2$
[✓ (c) $2\sqrt{r_1r_2}$]
(d) $4\sqrt{r_1r_2}$

187. The radii of two circles are 9 cm & 4 cm touch each other externally. AB is a direct common tangent. A small circle touch line AB & both circles. Find the radius of the small circle.

188. The radii of two circles are r_1 cm & r_2 cm touches externally. AB is a direct common tangent. A small circle touches line AB & both circles. If radius of small circle is r_3, then which is true-

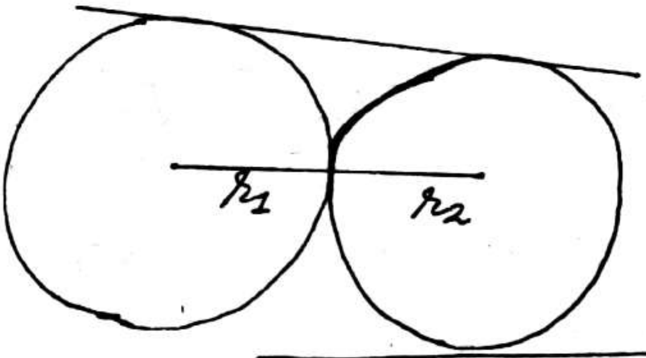
(a) $\sqrt{r_1} + \sqrt{r_2} = \sqrt{r_3}$ [✓ (b)] $\frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}} = \frac{1}{\sqrt{r_3}}$

(c) $\sqrt{r_1} - \sqrt{r_2} = \sqrt{r_3}$ (d) $\frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{r_3}$

184. $AB = \sqrt{625-49} = \sqrt{576} = 24$ cm.

185. $AB = \sqrt{625-576} = \sqrt{49} = 7$ cm

186.



$$AB = \sqrt{d^2 - (r_1 - r_2)^2}$$

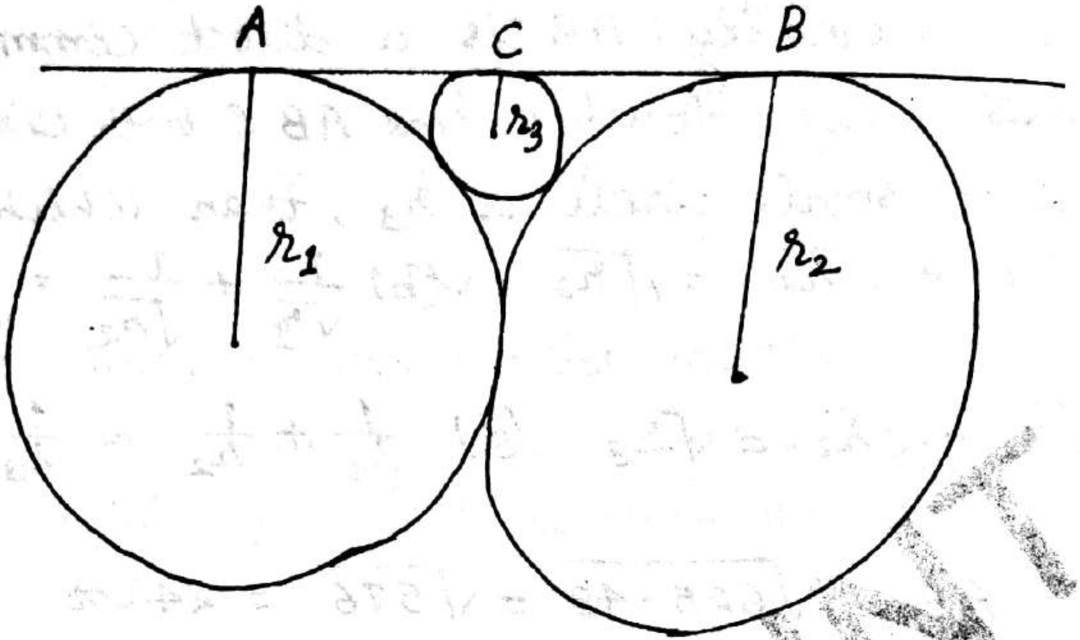
$$= \sqrt{(r_1+r_2)^2 - (r_1-r_2)^2}$$

$$\boxed{AB = 2\sqrt{r_1 r_2}}$$

187/188. $AC = 2\sqrt{r_1 r_3}$

$BC = 2\sqrt{r_2 r_3}$

$$AB = 2\sqrt{r_1 r_2}$$



$$AB = AC + BC$$

$$2\sqrt{r_1 r_2} = 2\sqrt{r_1 r_3} + 2\sqrt{r_2 r_3}$$

$$\sqrt{r_1 r_2} = \sqrt{r_1 r_3} + \sqrt{r_2 r_3}$$

$$\frac{1}{\sqrt{r_3}} = \frac{1}{\sqrt{r_2}} + \frac{1}{\sqrt{r_1}}$$

187. $r_1 = 9$ cm , $r_2 = 4$ cm

$$\frac{1}{\sqrt{r_3}} = \frac{1}{2} + \frac{1}{3}$$

$$\frac{1}{\sqrt{r_3}} = \frac{5}{6} \Rightarrow \frac{1}{r_3} = \frac{25}{36}$$

$$r_3 = \frac{36}{25} \text{ cm}$$

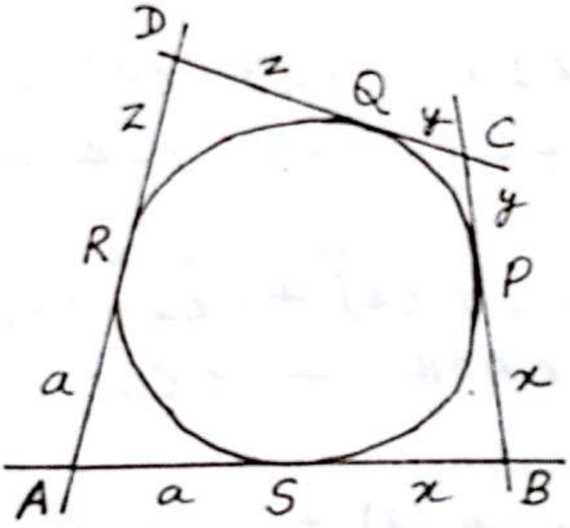
188. $$\boxed{\frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}} = \frac{1}{\sqrt{r_3}}}$$

189. A circle touches all sides of quadrilateral ABCD. AB is 5 cm, BC = 6 cm & CD = 8 cm. Find the length of side AD.

190. A circle touches all sides of quadrilateral ABCD. O is a centre of circle. Find the sum of ∠AOB & ∠COD.

191. A circle touches all sides of quadrilateral. If O is the centre of circle. Find the sum of ∠BOC & ∠AOD.

189.

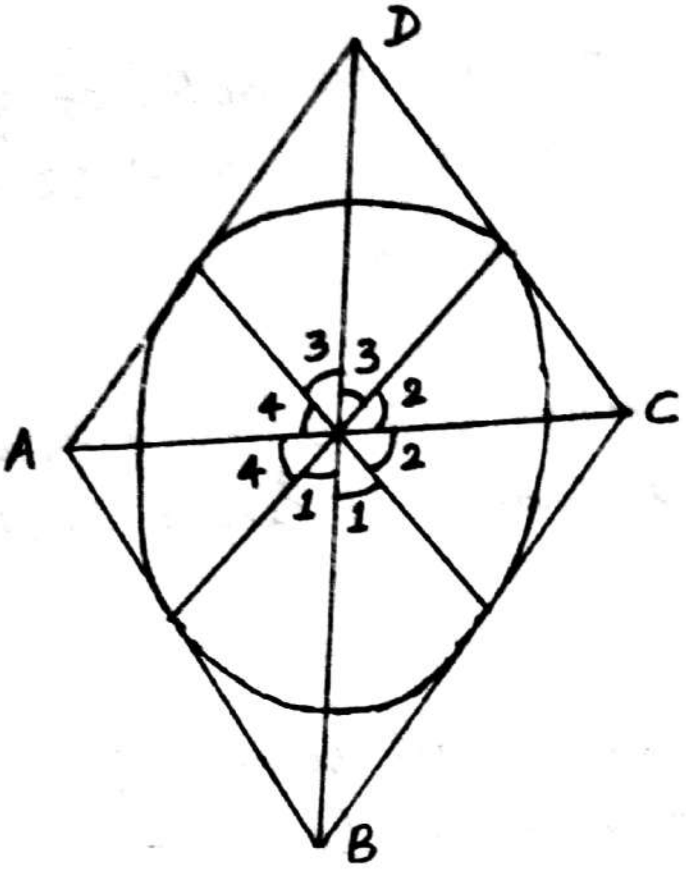


$$\boxed{AB + CD = BC + AD}$$

$$5 + 8 = 6 + AD$$

$$\boxed{AD = 7 \text{ cm}}$$

190/191.



$$2(\angle 1 + \angle 2 + \angle 3 + \angle 4) = 360^\circ$$
$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$$

190 $$(\angle 1 + \angle 4) + (\angle 2 + \angle 3) = 180^\circ$$
$$\angle AOB + \angle COD = 180^\circ$$

191. $$(\angle 1 + \angle 2) + (\angle 3 + \angle 4) = 180^\circ$$
$$\angle BOC + \angle AOD = 180^\circ$$

192. AP & BP are two tangents at point A & B and O is the centre of circle. If OP = 25 cm, AP = 24 cm, find the radius of circle.

193. AP & BP are two tangents at point A & B. If $\angle APB = 80^\circ$, then $\angle PAB = ?$

194. AP & BP are two tangents at point A & B. O is a Centre of circle if $\angle APB = 60^\circ$, find $\angle OAB = ?$

195. AP & BP are two tangents at point A & B and O is a Centre of circle. If OA = 17 cm & AP = 15 cm. Find the area of quadrilateral APBO.

196. AP & BP are two tangents at point A and B. If AP = 20 cm & area of quadrilateral APBO = 180 cm^2. Find the radius of circle.

197. AP & BP are two tangents & O is a centre of circle then quadrilateral APBO will be–

(a) Parallelogram (b) Rectangle
(c) Cyclic (d) None of these

198. AP & BP are two tangents & O is a centre of circle. Find the sum of $\angle APB$ & $\angle AOB$.

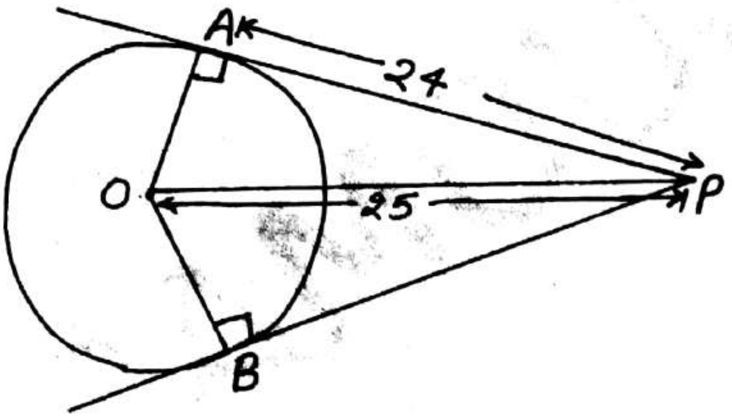
199. AP & BP are two tangents at points A & B and

O is a centre of circle. If $\angle PAB = 40^\circ$, find $\angle AOB = ?$

200. AP & BP are two tangents at points A & B and radius of the circle is 5 cm. If AB = 8 cm, then find the length of AP or BP.

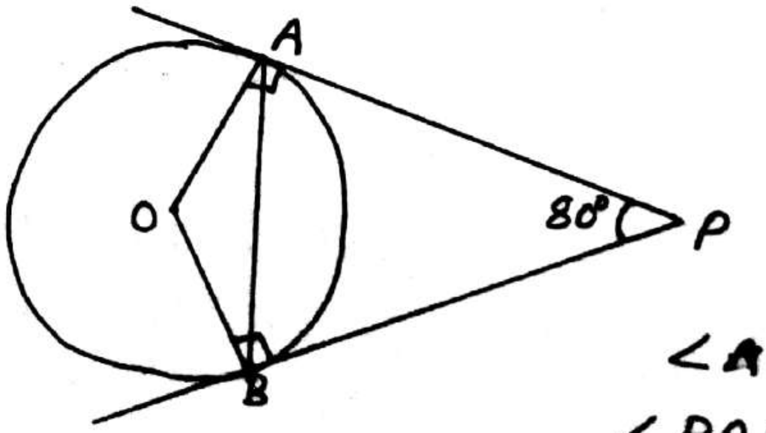
Solutions

192.



$$r = OA = \sqrt{625-576} = \sqrt{49}$$
$$= 7\text{ cm}.$$

193.

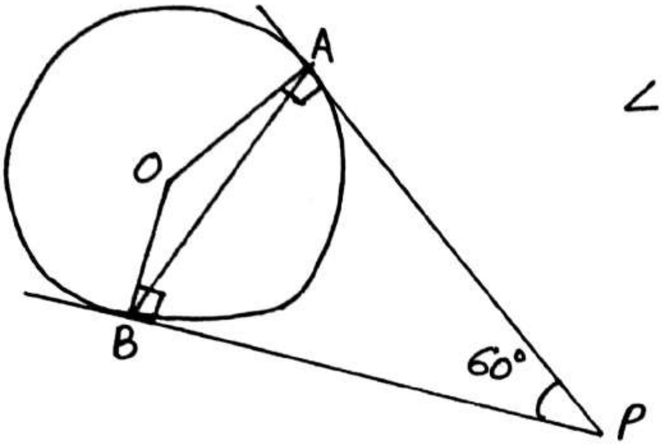


$\angle AOAB = 40^\circ$

$\angle PAB = 90^\circ - 40^\circ$

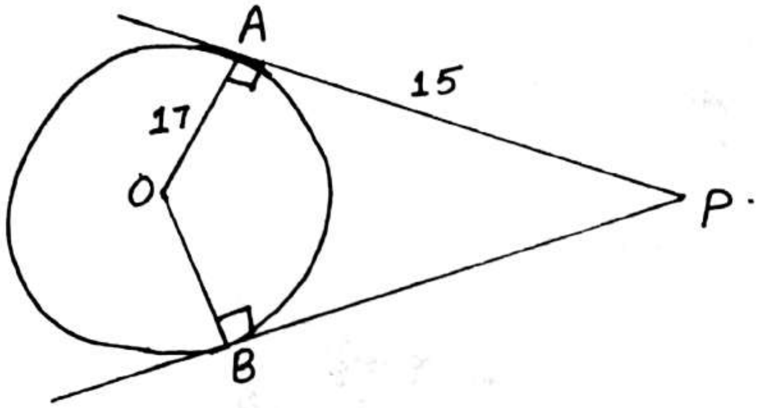
$= 50^\circ$

194.



$\angle OAB = 30^\circ$

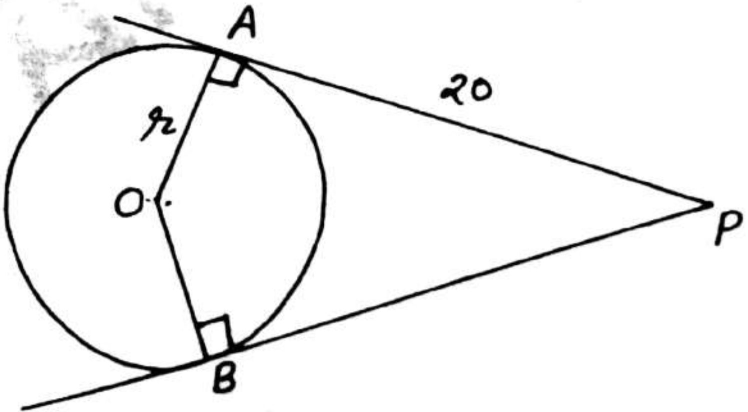
195.



Area of □ APBO $= 17 \times 15$

$= 225 \text{ cm}^2$

196.

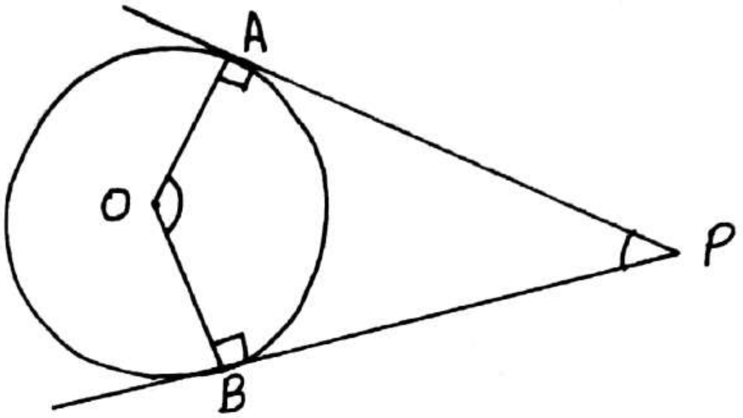


Area of □ APBO $= r \times AP$

$180 = r \times 20$

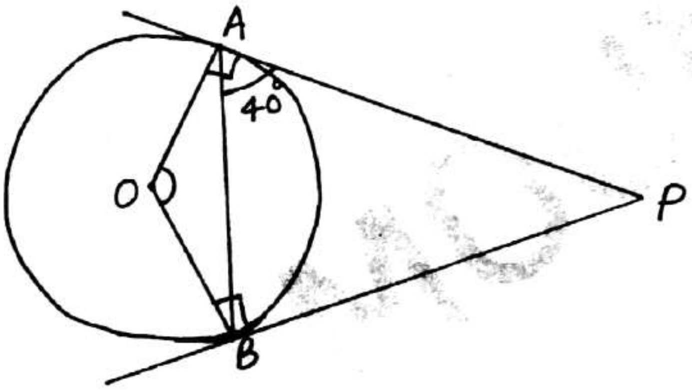
$r = 9$ cm.

198.



$\angle APB + \angle AOB = 180^\circ$

199.



$\angle OAB = 90^\circ - 40^\circ = 50^\circ$

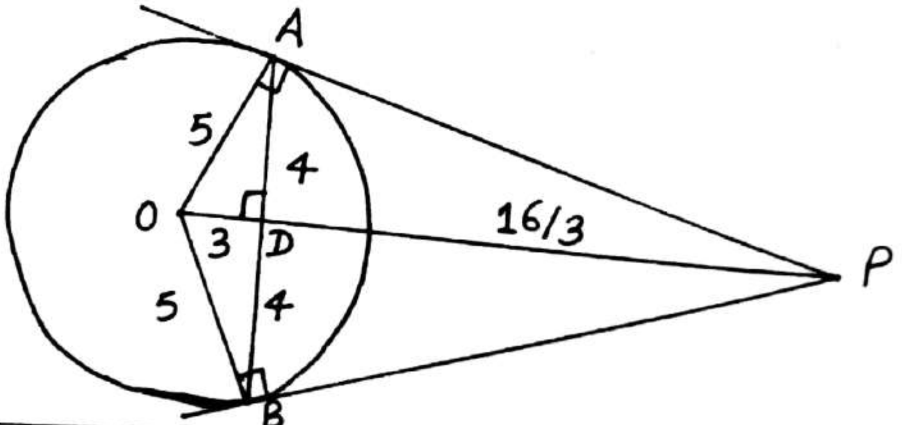
$\angle APB = 100^\circ$

$\Rightarrow \angle AOB = 180^\circ - 100^\circ$

$= 80^\circ$

200.

$\Delta O\overset{90^\circ}{A}P \approx \Delta O\overset{90^\circ}{D}A$



$$\boxed{BD^2 = AD \cdot CD}$$

$$\boxed{BD^2 = OD \quad DP}$$

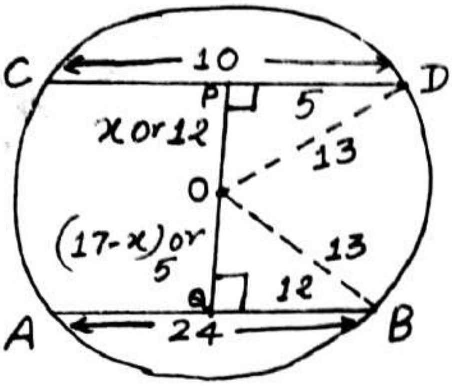
If $\Delta OAP \cong \Delta ODA$

$$\frac{AP}{AD} = \frac{OA}{OD} \qquad \boxed{= \tan\alpha}$$

$$\frac{AP}{4} = \frac{5}{3}$$

$$AP = \frac{20}{3} \text{ cm}$$

201. AB & CD are two chords opposite side of centre & distance between AB & CD is 17 cm. If AB = 24 cm, CD = 10 cm. Find the radius of the circle.



Radius = 13 cm.

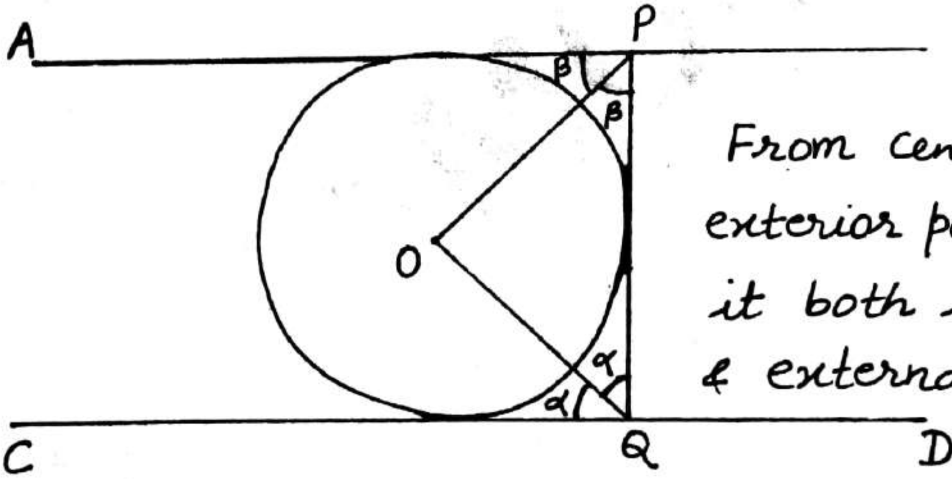
202. AB & CD are two parallel tangents. PQ is a third tangent intersect AB & CD at P & Q.

and O is a centre of circle. Find $\angle POQ = ?$

203. The radii of two concentric circles are 8 cm & 13 cm. AB is the diameter of bigger circle & B is a tangent at point D of smaller circle. Find the length of AD.

204. The radii of two circles are 2 cm & 3 cm touched internally. Find the length of larges chord to be a tangent of smallest circle.

202.



From centre to exterior point bisec it both internall & externally.

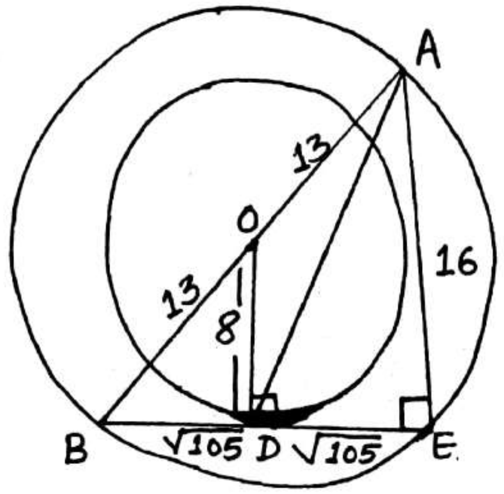
$$2(\alpha+\beta) = 180^\circ$$

$$\alpha+\beta = 90^\circ$$

$$\Rightarrow \angle POQ = 90^\circ$$

203. $\because$ AB is a diameter.

$\therefore \angle AEB = 90^\circ$



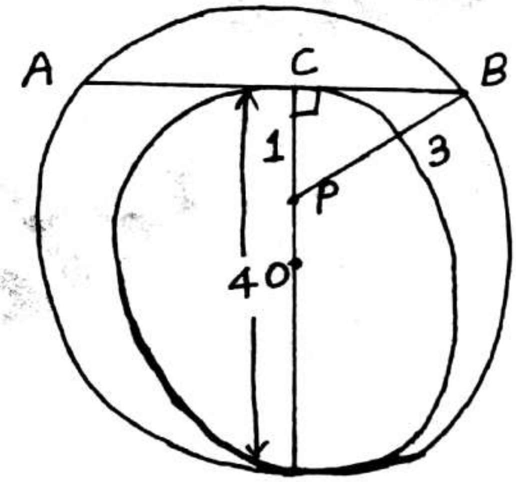
AE = 16 (By mid-point)

In ΔAED

$$AD = \sqrt{256 + 105} = \sqrt{361}$$

$$= 19 \text{ cm}.$$

204.



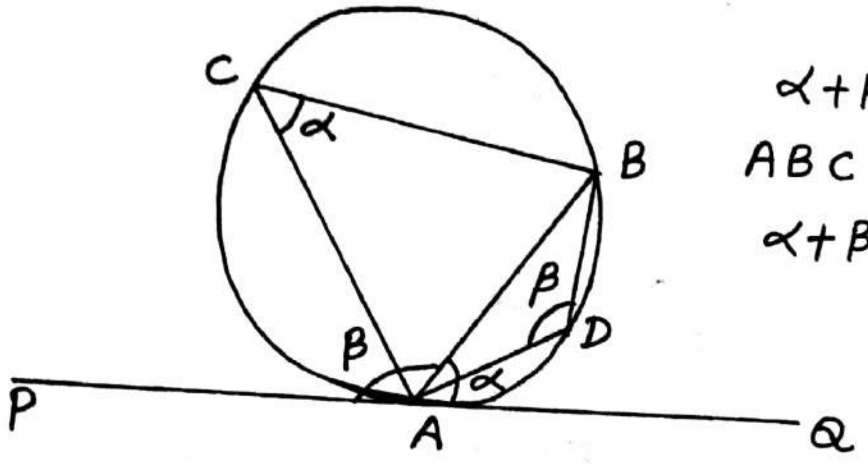
In PCB –

$BC = \sqrt{8}$ cm

$AB = 2\sqrt{8}$

$= 4\sqrt{2}$ cm.

Questions based on Alternate Segment Theorem :-



$\alpha + \beta = 180°$

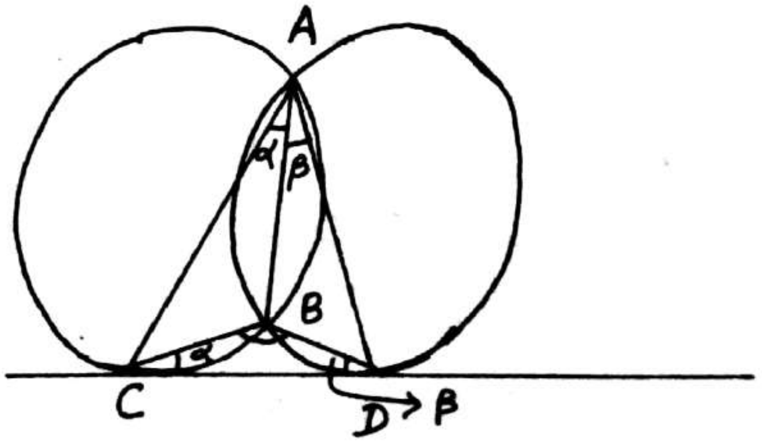
ABCD is cyclic

$\alpha + \beta = 180°$

205. Two circles intersect at two points A & B. CD is a direct common tangent. Find the sum ∠CAD & ∠CBD.

206. ABC is a secant such that points A & B are on circumference of circle. CD is a tangent at point D & O is a centre of circle. If ∠BCD $40°$ & $\angle CDB = 44°$, find $\angle AOB = ?$

205.

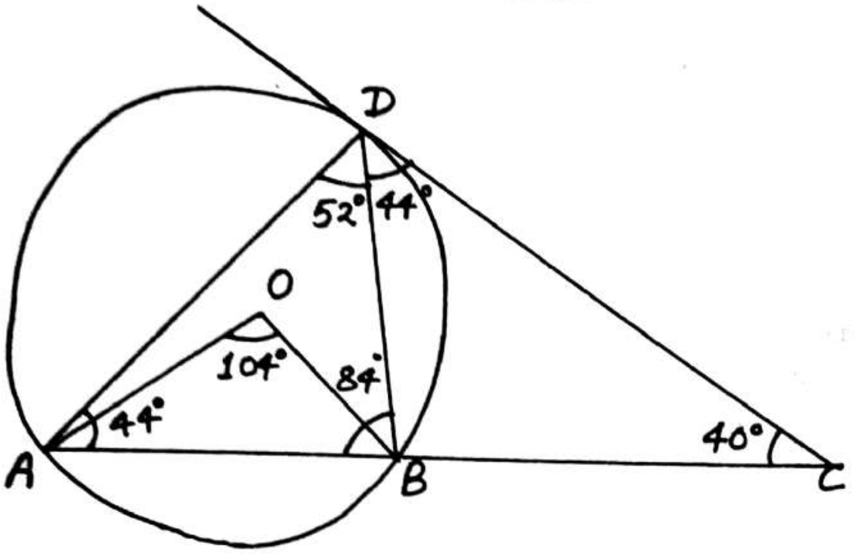


$$\angle CBD = 180^\circ - \alpha - \beta$$

$$\angle CAD + \angle CBD = \alpha + \beta + 180^\circ - \alpha - \beta$$

$$= 180^\circ$$

206.



$\angle ABD = 40^\circ + 44^\circ$ (Sum of exterior angles)

$= 84^\circ$

$\angle ADB = 180^\circ - 128^\circ$

$= 52^\circ$

$\angle AOB = 104^\circ$

Quadrilateral :- A closed figure by four lines or four sides is called quadrilateral.

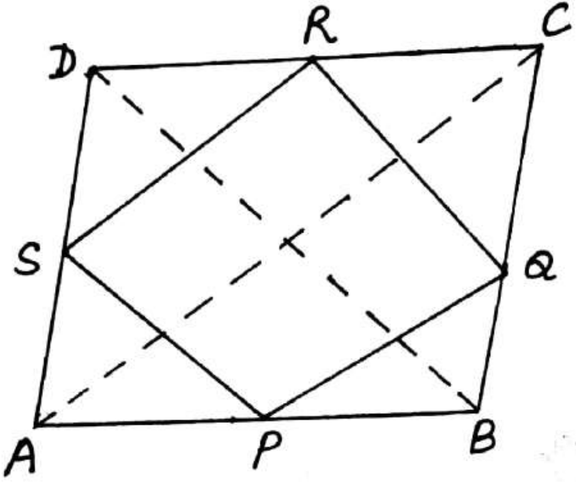
Types of Quadrilateral :-

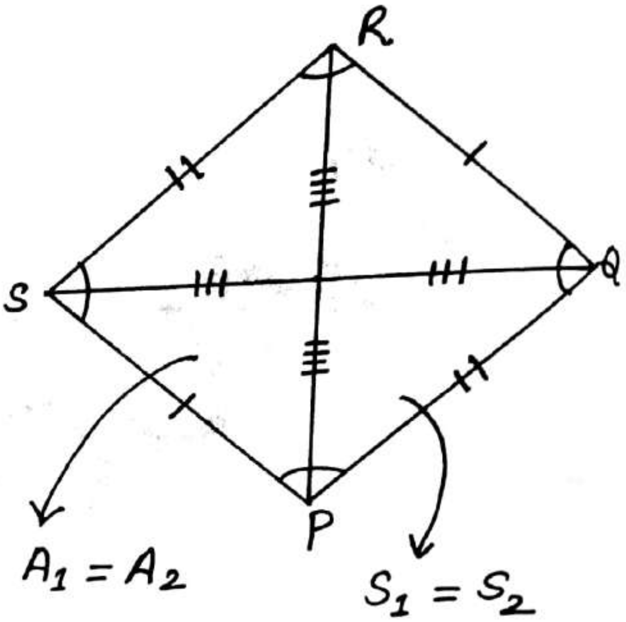
1. Parallelogram

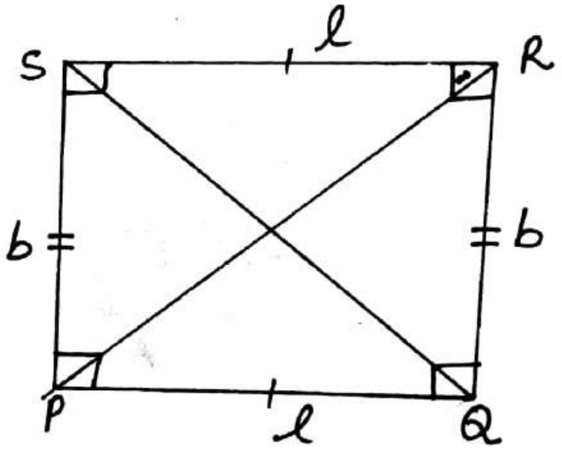
2. Rectangle
3. Rhombus
4. Square

} Parallelogram

5. Trapezium
6. Kite

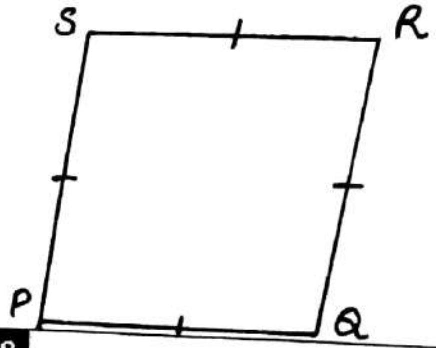






$90^\circ \Rightarrow d_1 = d_2$

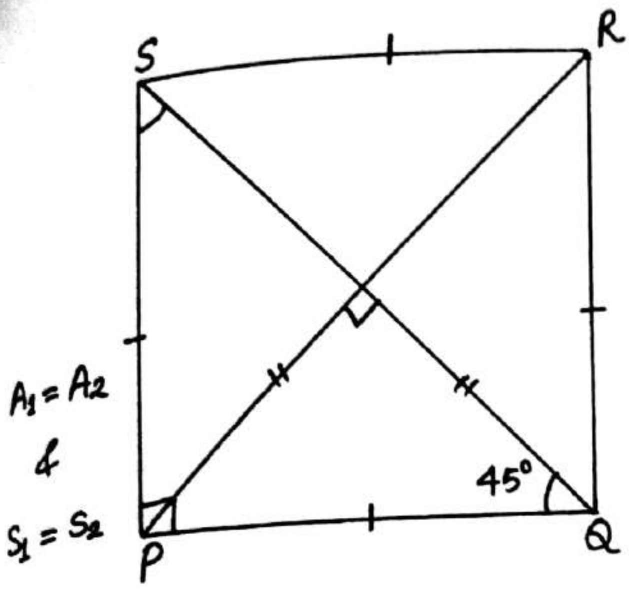
Rectangle



Rhombus

$S_1 = S_2$

$d_1 \perp d_2$



Square

If $S_1 = S_2$

Diagonal bisect vertex angle

Area of parallelogram PQRS $= 2 \times \frac{1}{2} \times ab\sin\theta$

$= ab\sin\theta$

'or'

Area of ▱ PQRS = Base × Height

Area of ▭ PQRS $= lb\sin 90^\circ = lb$

Area of ▱ PQRS $= a^2\sin\theta$ or $\frac{1}{2} d_1 d_2$
(Rhombus)

Area of □ PQRS $= a^2\sin 90^\circ = a^2$
(square)

||gm PQRS

$$PR^2 + SQ^2 = PQ^2 + QR^2 + RS^2 + PS^2$$

$$d_1^2 + d_2^2 = 2(a^2 + b^2)$$

▭ PQRS

$$d_1^2 + d_2^2 = 2(l^2 + b^2)$$

□ (Rh.) PQRS

$$d_1^2 + d_2^2 = 4a^2$$

□ (Square) PQRS

$$d_1^2 + d_2^2 = 4a^2$$

207. ABCD is a ||gm. E is a mid-point of BC. Find the ratio of area of ||gm ABCD & ΔABE.

208. ABCD is a ||gm. E & F are mid-points of BC & CD. Find the ratio of area of ||gm ABCD & ΔEFC.

209. ABCD is a ||gm. Points P, Q, R and S are mid-points of AB, BC, CD & AD. Find the ratio of area of ||gm ABCD & PQRS.

210. ABCD is a ||gm. Points E & F are mid-points of BC & CD. Find the ratio of area of ||gm ABCD & ΔAEF.

211. ABCD is a ||gm. E & F are mid-points of sides BC & CD. Find the ratio of area of ΔABC & ΔAEF.

212. ABCD is a ||gm. AC & BD intersect at O. Points P, Q, R and S are mid-points of AO, BO, CO & DO. Find the ratio of area of ||gm ABCD & PQRS.

213. ABCD is a ||gm. Points P & Q are Centroid of ΔBAD & ΔBCD. If AC = 18 cm, find the length of PQ.

214. ABCD is a ||gm. AC & BD intersect at O. E is the mid-point of AD. Find ratio of area of ||gm ABCD & Quadrilateral OCDE.

215. ABCD is a ||gm. Points E & F are mid-points of sides BC & AD. If Area of ΔAEF is 12 cm^2, then find the area of ||gm ABCD.

216. ABCD is a ||gm. AB = 30 cm, BC = 20 cm. The distance between AB & DC is 10 cm. Find the distance between BC & AD.

217. ABCD is a Cyclic ||gm. Find ∠ABC.

218. ABCD is a ||gm. AB is extended to any

point E. BC & DE intersect at P. If AB = BE, then find the ratio of BP & CP.

219. ABCD is a rhombus and ∠ABC = 60°. If AB = 10 cm, find the length of AC.

220. ABCD is a rhombus. If ∠ABC = 120° & AB = 20 cm, then find the length of AC.

221. ABCD is a rhombus & ∠ABC = 60°. Find the ratio of AC & BD.

222. ABCD is a rhombus and ∠ABC = 60°. If AB = 40 cm, then find area of rhombus ABCD.

223. ABCD is a cyclic rhombus. Find ∠ABC = ?

224. The perimeter of rhombus ABCD is 146 cm. If length of one diagonal is 55 cm. Find the length of other diagonal.

225. ABCD is a rhombus. AB & BA are extended to any points E & F such that AB = BE = AF. FD & EC meet at G. Find ∠EGF = ?

226. ABCD is a rhombus. AC = 12 cm and BD = 16 cm. Find the side of rhombus.

227. ABCD is a rhombus. AC & BD intersect at O. The points P, Q, R and S are mid-points of AO, BO, CO & DO. Find the ratio of area of rhombus ABCD & PQRS.

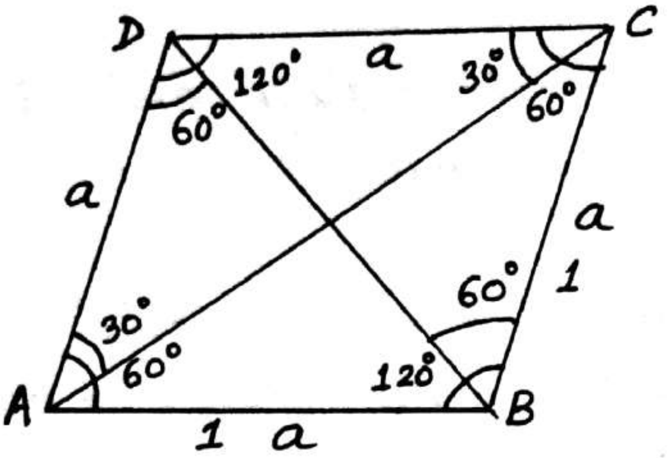
228. ABCD is a square. ΔCDE is an equilateral triangle and E is outside of triangle. Find $\angle DEA = ?$

229. In a square ABCD, ΔAOD is an equilateral triangle. Find $\angle COD$.

230. In a square ABCD, ΔAOB is an equilateral triangle. AC & BO intersect at P. Find $\angle APB$.

231. ABCD is a rectangle. Points P, Q & R are on side AB such that AP = QR = PQ = RB. Find the ratio of area of rectangle ABCD & ΔCPQ.

Concept of Rhombus at 60°:-



$1 \equiv a$

$\sqrt{3} \equiv a\sqrt{3}$

$60° \Rightarrow \boxed{d_1 = a}$

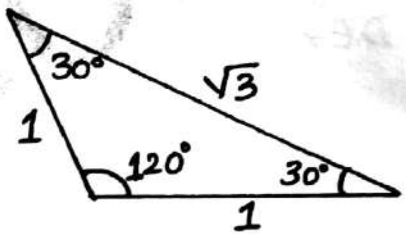
$120° \Rightarrow \boxed{d_2 = a\sqrt{3}}$

$\sin(90° + 30°) = \cos 3$

$\sin 30° : \sin 30° : \sin 120°$

$\frac{1}{2} : \frac{1}{2} : \frac{\sqrt{3}}{2}$

$1 : 1 : \sqrt{3}$



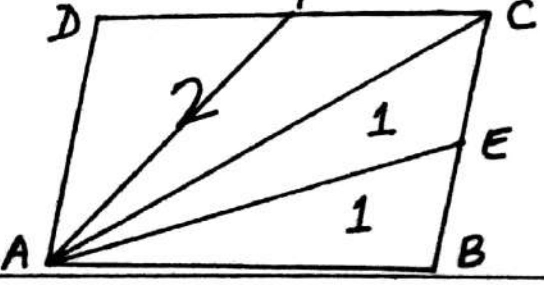
$$\text{Area} = \frac{1}{2} d_1 d_2$$

$$= \frac{a^2 \times \sqrt{3}}{2}$$

$$\boxed{\text{Area} = \frac{\sqrt{3}}{2} a^2}$$

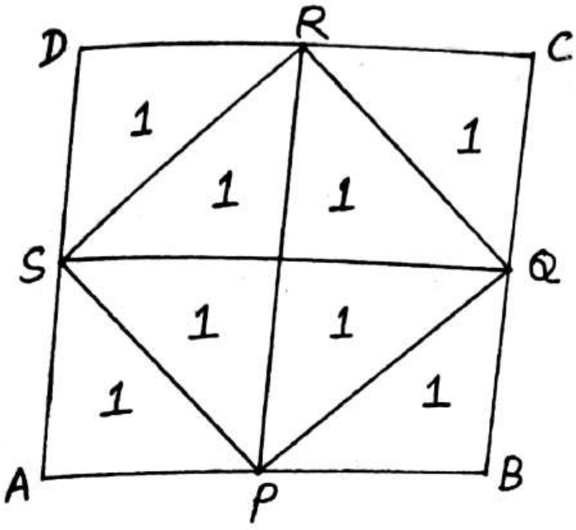
Concept of ||gm based on Area :-

1.



ΔADE
or
□ABCD : ΔABE
4 : 1

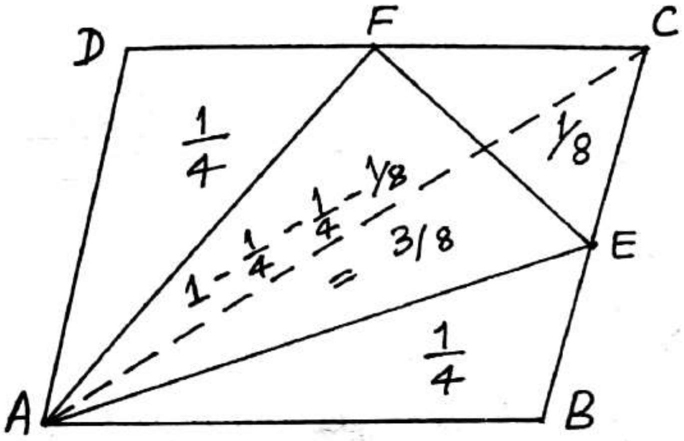
2.



□ABCD : ΔQRC
8 : 1

□ABCD : □PQRS
= 8 : 4
= 2 : 1

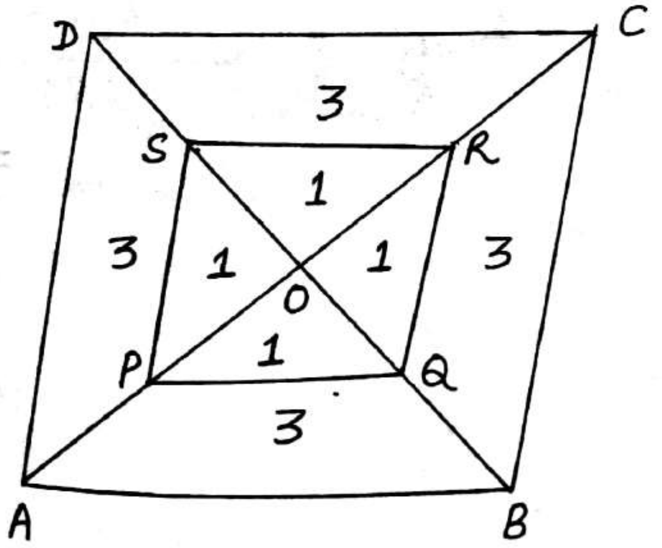
3.

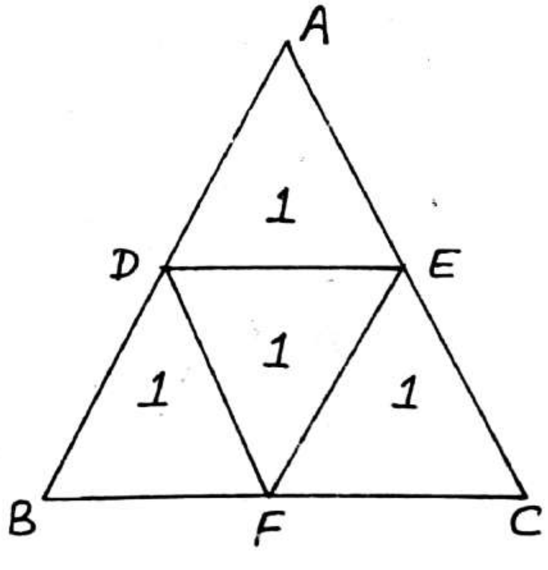


□ABCD : ΔAEF
8 : 3

ΔABC : ΔAEF
4 : 3

4.





□ABCD : □PQRS
= 16 : 4
= 4 : 1