Chapter 1 Number System

1. Important Terminology

1.1 Digits—0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are defined as digits is Mathematics. We can create many numbers by using these digits. For example : 10, 123, 456, 789 etc.

1.2 Number System—There are mainly two types defined in the number systems. These are :

I. Decimal Number System—it contains 0 to 9 digits. That's why it is called *decimal number system*. In this system, the numbers is read and written in two ways—Indian number system and International number system.

In the Indian number system or Hindi-Arabic system, the numbers are read and written as per their place values. These numbers are read as per the following table.

Periods	Crores		Lakhs		Thousands		Ones		
Values	10,00,00,000 (Ten Crores)	1,00,00,000 (Crore)	10,00,000 (Ten Lakhs)	1,00,000 (Lakh)	10,000 (Ten Thousands)	1,000 (Thousand)	100 (Hundred)	10 (Ten)	1 (One)
	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0

Example : Number 51,45,42,786 can be read as Fifty-one Crores Forty-five Lakhs Forty-two Thousands Seven Hundred and Eighty-six. It is also called number name.

Unit Conversions :

- 1 tens = 10 units
- 1 Hundred = 10 tens = 100 units
- 1 Thousand = 10 Hundreds = 100 tens = 1000 units
- 1 Lakh = 10 Thousands = 100 Hundreds = 1000 tens
- 1 Crore = 10 Lakhs = 100 Thousands = 1000 Hundreds

In International number system, the numbers are read and written as per the following table.

Periods	Millions			Thousands			Ones		
Values	100,000,000 (Hundred Millions)	10,000,000 (Ten Millions)	10,00,000 (Milllon)	100,000 (Hundred Thousands)	10,000 (Ten Thousands)	1,000 (Thousand)	100 (Hundred)	10 (Ten)	1 (One)
	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0

Example : Number 14,542,786 can be read as Fourteen Millions Five Hundred Forty-two Thousand Seven Hundred Eighty-six.

II. Roman Number System—In this system, numbers are represented by Latin alphabets. The Roman numerals used in, are based on seven symbols or letters.

Roman System	I	V	X	L	C	D	M
Hindu-Arabic System	1	5	10	50	100	500	1000

Example : 25 can be written as XXV and 101 as CI.

Note

- A letter repeats its value that many times (XXX = 30, CC = 200 etc.). A letter can only be repeated three times.
- If one or more letters are placed after another letter of greater value, add that amount.
  For example,
  VII = 7 (5 + 1 + 1); LXI = 61 (50 + 10 + 1); MCC = 1200 (1000 + 100 + 100)
- If a letter is placed before another letter of greater value, subtract that amount.
  For example,
  IV = 4 (5 – 1); XIV = 14 (10 + 5 – 1); CM = 900 (1000 – 100)
- Only subtract powers of ten (I, X, or C, but not V or L).
- Only subtract one number from another.
- Do not subtract a number from one that is more than 10 times greater (that is, you can subtract 1 from 10 [IX] but not 1 from 20—there is no such number as IXX.)
- A bar placed on top of a letter or string of letters increases the numeral's value by 1,000 times.
  For example, XII = 12 whether $\overline{\text{XII}}$ = 12000.

2. Digits of Number

- **Units**—Digit 0 to 9 are called Unit digits. The smallest and the largest number of 1-digit are 0 and 9 respectively.
- **Tens**—The numbers from 10 to 99 are called ten numbers. The smallest and the largest number of 2-digits are 10 and 99 respectively.
- **Hundred**—The numbers from 100 to 999 are called hundred numbers. The smallest and the largest number of 3-digits are 100 and 999 respectively.

- **Thousand—**The numbers from 1,000 to 9,999 are called thousand numbers. The smallest and the largest number of 4-digits are 1000 and 9999 respectively.
- **Ten thousand—**The numbers from 10,000 to 99,999 are called ten thousand numbers. The smallest and the largest number of 5-digits are 10,000 and 99,999 respectively.
- **Lakh—**The numbers from 1,00,000 to 9,99,999 are called lakh numbers. The smallest and the largest number of 6-digits are 1,00,000 and 9,99,999 respectively.
- **Ten Lakh—**The numbers from 10,00,000 to 99,99,999 are called ten lakh numbers. The smallest and the largest number of 7-digits are 10,00,000 and 99,99,999 respectively.
- **Crore—**The numbers from 1,00,00,000 to 9,99,99,999 are called crore numbers. The smallest and the largest number of 8-digits are 1,00,00,000 and 9,99,99,999 respectively.

3. Value of Digits

- **Place Value—**Place value helps us determine the value of numbers. Our (base-10) number system contains numerals or digits only from 0 to 9, but we often need to use numbers greater than 9. We show numbers greater than 9 by using place value. Place value refers to the value of each digit in a number.

  Example : In a number 489765, place value of 7 will be 7×100 units, *i.e.*, 700. Similarly, the place value of 8 will be $8 \times 10{,}000 = 80{,}000$.

- **Face Value—**The actual value of a digit in a number is the digit itself. The place value of the digit is ignored in the number.

  Example : In a number 59,438, the face value of 4 is 4, face value of 9 is 9 etc.

Note

If x and y be the tens digit and unit digit respectively, then the 2-digit number formed by these digits will be $10x + y$.

4. Comparison of Numbers

- **When both numbers have unequal number of digits**

  The number having more digits is greater. It means

  ..... 5-digit number > 4-digit number > 3-digit number .....

  Example : Find out which is greater 5429683 or 65245893 ?

  Solution : Since, the first number 5429683 is of 7-digit number whether the second number 65245893 is of 8-digit. Therefore, the second number is greater than the first number.

- **When both numbers have equal number of digits**

  In case of the equal number of digits, we have to check the place value of the left-most digit of both numbers. If the digits of both numbers are also equal, then we move to its next digit placed on the right side and repeat the process until we get the desired result.

  Example : Arrange the following numbers in ascending order.

  5403100, 5460860, 5458087, 5420378

  Solution : At first, we check the place value of the leftmost digit of each number. Then repeat the same process until we get the answer. Here, in each number, two leftmost digits are equal. After that, we check ten thousand place values and then arrange the digits in ascending order. Hence, we get

  5403100 < **542**0378 < **545**8087 < **546**0860

5. Classification of Numbers

There are several types of numbers exist in the number system for different purposes. These numbers are classified into different groups according to their properties. These are :

- **Natural Numbers—**Counting numbers starting from 1, 2, 3..., etc. are called natural numbers. It is represented by capital letter **N**. Its set is shown as

  $$N = \{1, 2, 3, 4, 5\ldots\}$$

- **Whole Numbers—**All natural numbers along with 0 is known as whole numbers. It is represented by capital letter **W**. Its set is shown as

  $$W = \{0, 1, 2, 3, 4\ldots\}$$

- **Even and Odd Numbers—**A number is even if it is a multiple of two, and is odd otherwise. Even numbers are denoted by capital letter **E** and odd numbers are denoted by capital letter **O**.

  $E = \{2, 4, 6, 8\ldots\}$ and $O = \{1, 3, 5, 7\ldots\}$

- **Integers—**Positive and negative counting numbers, as well as zero are called integers. Integers are denoted by capital letter Z.

  $$Z = \{\ldots -3, -2, -1, 0, 1, 2, 3\ldots\}$$

- **Prime Numbers—**An integer with exactly two positive divisors : itself and 1, is called prime number. For example : 2, 3, 5, 7, 11, 13...etc. are few prime numbers. 2 is the smallest prime number.
- **Composite Numbers—**All those numbers greater than 1 that are not prime are called composite numbers. For example : 4, 6, 8, 9, 10 etc. are few composite numbers.
- **Rational Numbers—**Numbers that can be expressed as a ratio of an integer to a non-zero integer. Rational numbers are denoted by capital letter **Q**. All integers are rational, but the converse is not true.

  $$Q = \left\{\ldots -\frac{1}{2} \ldots 1 \ldots \frac{1}{4} \ldots \frac{1}{2} \ldots\right\}$$

- **Irrational Numbers—**All the real numbers that are not rational are called irrational numbers. Irrational numbers are denoted by **I**.

  $$I = \left\{\ldots \frac{2}{3}, \sqrt{2}, \sqrt{3} \ldots\right\}$$

- **Real Numbers—**They can be positive, negative or zero. All rational numbers are real, but the converse is not true.

6. Approximate Values of Numbers

Place values are considered to be the base to find approximation values in numbers. Approximation value of few place values is determined by the following methods.

- **Approximate value nearest tens place—**If the number at units place is less than 5 then it is rounded of zero otherwise add 1 to the tens place and keeps unit place as zero.

  Example : 73 can be rounded off to 70, 156 can be rounded off to 160 and 4265 can be rounded off to 4270.

- **Approximate value nearest hundred place—**If the number at tens place is less than 5 then it is rounded of zero otherwise add 1 to the hundred place and keeps tens place and unit place as zero.

  Example : 510 can be rounded off to 500, 9573 can be rounded off to 9600 and 53650 can be rounded off to 53700.

- **Approximate value nearest thousand place—**If the number at hundred place is less than 5 then it is rounded of zero otherwise add 1 to the thousand place and keeps hundred place, tens place and unit place as zero.

  Example : 6240 can be rounded off to 6000, 17573 can be rounded off to 18000 and 553650 can be rounded off to 554000.

7. Divisibility Test of Numbers

- **Divisibility by 2 :**

  If the unit digit of a number is any of 0, 2, 4, 6, 8, then the given number is divisible by 2.

  Example : 84, 786, 282, 1008, 5000....., etc. are divisible by 2.

- **Divisibility by 3 :**

  A number is divisible by 3, if the sum of all digits of the number is divisible by 3.

  Example : 786, here 7 + 8 + 6 = 21 (completely divisible by 3)

  So, the number 786 will be divisible by 3.

- **Divisibility by 4 :**

  A number is divisible by 4, if the last two-digits of the number is divisible by 4.

  Example : 3464, here 64 is the last two-digit number which is divisible by 4.

  So, the number 3464 will be divisible by 4.

- **Divisibility by 5 :**

  A number is divisible by 5, if the unit digit of the number is either 0 or 5.

  Example : 3125, 2010, 2015, 6580....., etc. are divisible by 5.

- **Divisibility by 6 :**

  A number is divisible by 6, if the number is divisible by the numbers 2 and 3.

  Example : Test whether number 8202 is divisible by 6.

  Solution : (i) the unit digit of the number is 2 which is divisible by 2.

  (ii) the sum of digits of the number = 8 + 2 + 0 + 2 = 12 (divisible by 3)

  Since, it is clear from (i) and (ii) that the number 8202 is divisible by both 2 and 3. So, the number will be divisible by 6.

- **Divisibility by 7 :**

  Take the last digit of the given number and double it. Subtract this number from the rest of the digits in the original number. If this new number is either 0 or if it is a number that is divisible by 7, then the given number is also divisible by 7.

  Example : Test whether number 2492 is divisible by 7.

  Solution : Here, the unit digit of the number = 2

  $249 - 2 \times 2 = 245$ (divisible by 7). So, the number will be divisible by 7.

- **Divisibility by 8 :**

  A number is divisible by 8, if the last three-digits of the number is divisible by 8.

  Example : Test whether number 6288 is divisible by 8.

  Solution : Here, in the given number, 288 is the last three-digit number which is completely divisible by 8.

  So, the number 6288 will be divisible by 8.

- **Divisibility by 9 :**

  A number is divisible by 9, if the sum of its digits is divisible by 9.

  Example : Test whether number 7074 is divisible by 9.

  Solution : Sum of all digits of the number = 7 + 0 + 7 + 4 = 18 (divisible by 9).

  So, the number 7074 will be divisible by 9.

- **Divisibility by 11 :**

  A number is divisible by 11, if difference between the sum of digits at odd places and the sum of digits at even places, is divisible by 11.

  Example : Test whether number 86460 is divisible by 11.

  Solution : Sum of the all digits at even places in the number = 6 + 6 = 12

  Sum of the all digits at odd places in the number = 8 + 4 + 0 = 12

  Their difference = 12 – 12 = 0. So, the number 86460 will be divisible by 11.

8. Division Algorithm

The number which we divide is called the dividend. The number by which we divide is called the divisor. The result obtained is called the quotient. The number left over is called the remainder. Some formula are given below for Division based questions.

- Dividend = Divisor × Quotient + Remainder
- $\text{Divisor} = \frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$
- $\text{Quotient} = \frac{\text{Dividend} - \text{Remainder}}{\text{Divisor}}$

Example : In a question, the divisor is 4 times the quotient and 2 times the remainder. If the remainder is 20, then find the value of remainder.

Solution : According to Question,

$$\text{Divisor} = 2 \times \text{Remainder} = 2 \times 20 = 40 \quad \ldots(1)$$

And, $\text{Divisor} = 4 \times \text{Quotient}$

$\Rightarrow 4 \times \text{Quotient} = 40$ [from eq.(1)]

$\Rightarrow \text{Quotient} = 40/4 = 10$

$\therefore \text{Dividend} = 40 \times 10 + 20 = 400 + 20 = 420$

9. Whole Numbers

We start counting from the number 1. Hence 1 is the first natural number and the next natural number is 2 which is obtained by adding 1 to the first number. Hence, numbers are represented in two ways according to their orderliness :

- **Antecedent Number—**The natural number immediately preceding a natural number is its predecessor.

  Example : Predecessor number of 65 = 65 – 1 = 64

  Predecessor number of 127 = 127 – 1 = 126

- **Subsequent Number—**The natural number immediately next to any natural number is its successor.

  Example : Successor number of 785 = 785 + 1 = 786

  Successor number of 109 = 109 + 1 = 110

9.1 Whole Numbers—Natural numbers combine with zeroes to form whole numbers. When operations (addition, subtraction, multiplication, division) are used on whole numbers, many properties are revealed.

9.2 Characteristics of Whole Numbers

- All properties of natural numbers are true for the whole numbers.
- The smallest whole number is '0' (zero).

9.3 Properties of Whole Numbers

I. Closure property – If a and b be two whole numbers, then $a + b$ and $a * b$ will also be whole numbers.

Example :

- $4 + 5 = 9$, a whole number
- $4 \times 5 = 20$, a whole number
- $4 - 5 = -1$, not a whole number
- $4 \div 5 = 0.8$, not a whole number

Hence, whole numbers don't follow the subtraction and division operations for closure property.

II. Communicative property—Addition and multiplication operations are both communicative for whole numbers.

Example :

- $4 + 5 = 9 = 5 + 4$, a whole number
- $4 \times 5 = 20 = 5 \times 4$, a whole number
- $4 - 5 = -1 \neq 5 - 4 = 1$, not a whole number
- $4 \div 5 = 0.8 \neq 5 \div 4 = 1.25$, not a whole number

Hence, whole numbers don't follow the subtraction and division operations for communicative property.

III. Associative property—Addition and multiplication operations are both associative for whole numbers.

Example : $4 + (5 + 6) = 4 + 11 = 15$

$(4 + 5) + 6 = 9 + 6 = 15$

$\therefore \quad 4 + (5 + 6) = (4 + 5) + 6$

IV. Distributive property—

$$a \times (b + c) = a \times b + a \times c$$

or $(a + b) \times c = a \times c + b \times c$

Example : $4 \times (5 + 8) = 4 \times 5 + 4 \times 8$

$4 \times 13 = 20 + 32$

$52 = 52$

It is clear from the example that it is called distribution property of multiplication on addition operation.

V. Identity element—

- **Additive identity—**'0' is called additive identity because it is only the element its addition with any number gives the same number.

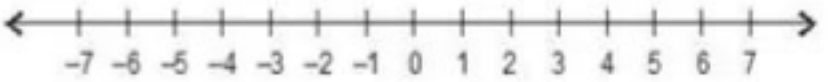
  Example : $5 + 0 = 5$, and $7 + 0 = 7$ etc.

- **Multiplicative identity—**'1' is called multiplicative identity because it is only the element its multiplication with any number gives the same number.

  Example : $6 \times 1 = 6$, and $7 \times 1 = 7$ etc.

9.4 Integers—The set of all negative numbers and positive numbers on either side of the zero marked on the number line is called an integer.

–5, – 4, – 3, –2, –1, 0, 1, 2, 3, 4, and 5 all are the integers. On the number line, integers are represented as follows :



I. Properties of integers

- **Closure property—**If a and b be two whole numbers, then $a + b$ and $a * b$ will also be whole numbers.

  Example :
  - $(+4) + (+5) = +9$, an integer
  - $(-4) \times (+5) = -20$, an integer

- **Communicative property—**Addition and multiplication operations are both communicative for whole numbers.

  Example :
  - $(+4) + (+5) = +9 = (+5) + (+4)$, an integer
  - $(-4) \times (+5) = -20 = (+5) \times (-4)$, an integer

- **Associative property**—Addition and multiplication operations are both associative for whole numbers.

  Example : • $7+(5-3)=7+2=9$
  • $(7+5)-3=12-3=9$
  • $7+(5-3)=(7+5)-3$

- **Distributive property**—$a\times(b+c)=a\times b+a\times c$ **or** $(a+b)\times c=a\times c+b\times c$
- **Identity element**—'0' is called additive identity and '1' is called multiplicative identity.

Important Questions

1. The sum of the squares of 3 consecutive positive numbers is 365. Accordingly, what is the sum of those numbers ? **(NCERT)**

(A) 30 (B) 33
(C) 36 (D) 45

2. In a division question, the denominator is 10 times its quotient and 5 times the remainder. Accordingly, if the remainder is 46, what will be the dividend ?

(A) 4236 (B) 4306
(C) 4336 (D) 5336

3. 30% of one number is equal to 40% of the other number. 25% of the sum of both those numbers is equal to 420. Find the smaller number.

(A) 780 (B) 760
(C) 720 (D) 700

4. Four prime numbers are in the ascending order. Product of the first three numbers is 455 and product of the last three numbers is 1729. Find the largest prime number of them. **(NCERT)**

(A) 7 (B) 13
(C) 19 (D) 23

5. Sum and product of two numbers are 5 and 6 respectively. Find the sum of reciprocals of their squares.

(A) $\frac{13}{36}$ (B) $\frac{36}{13}$
(C) $\frac{6}{5}$ (D) $\frac{5}{6}$

6. The maximum score of runs in an innings was 3/11 of the total score. The second score in the same innings was the maximum score of 3/11 of the remaining runs. If the difference of both scores is 9, then what was the total score ?

(A) 106 (B) 146
(C) 118 (D) 121

7. Find such three numbers that the twice the first number, thrice the second and 4 times of the third number make a sum of 191.

(A) 19, 20, 21 (B) 21, 22, 23
(C) 20, 21, 22 (D) 22, 23, 24

8. There are 108 tables and 132 chairs in an office. If 1/6 tables and 1/4 chairs got broken, then how many people can work in the office as per the need of one table and chair for each ? **(NCERT)**

(A) 86 (B) 90
(C) 92 (D) 99

9. Divide 37 in such two parts that the sum of 5 times of the first part and 11 times of the second part is 227.

(A) 15, 22 (B) 20, 17
(C) 25, 12 (D) 30, 7

10. If 510 is divided between A, B, C in such a way that A gets $\frac{2}{3}$ of what B gets, and C gets $\frac{1}{4}$ of what B gets, then their share in rupees (shares) respectively ? **(NCERT)**

(A) 120, 240, 150 (B) 60, 90, 360
(C) 150, 300, 60 (D) 110, 250, 150

11. If in 24 coins, there are 1 coins, 50 paise coins and 25 paise coins. The value of these coins is 13.75. If the number of 1 coins and the number of 25 paise coins is equal, then how many 25 paise coins are there ?

(A) 16 (B) 5
(C) 7 (D) 12

12. There are 12000 soldiers in an army, some of whom are Indians and the rest are European. The average height of a European soldier is 1.80 m, that of an Indian soldier is 1.75 m, and the length of the entire army is $1\frac{47}{60}$ m. So, how much are the Indian soldiers ?

(A) 6000 (B) 8000
(C) 1000 (D) 4000

13. A teacher wants to keep his students in an equal number in rows and columns. If the total number of students is 1369, then find the number of students in the last row ? **(NCERT)**

(A) 37 (B) 33
(C) 63 (D) 47

14. After his death, Ram left 1/3 of his asset in the name of his widowed wife and left the remaining 3/5 in the name of his daughter and gave the remaining property to the son. If the son got 6400, then what was the original asset of Rama ?

(A) ₹ 16000 (B) ₹ 32000
(C) ₹ 24000 (D) ₹ 1600

15. The sum of perfect squares of numbers between 120 and 300 is : **(NCERT)**

(A) 1204 (B) 1024
(C) 1296 (D) 1400

16. Average age of P, Q and R is 5 more than the age of R. If sum of ages of P and Q is 39 years, then find the age of R.

(A) 16 yr (B) 14 yr
(C) 12 yr (D) 24 yr

17. The sum of two numbers is 8 and their product is 15. Find the sum of their reciprocals.

(A) $\frac{8}{15}$ (B) $\frac{15}{8}$
(C) 23 (D) 7

18. How many $\frac{1}{6}$'s together make $41\frac{2}{3}$?

(A) 125 (B) 150
(C) 250 (D) 350

19. Which of the following number is the largest number ?

$0.9, 0.\bar{9}, 0.0\bar{9}, 0.\overline{09}$

(A) 0.9 (B) $0.\bar{9}$
(C) $0.0\bar{9}$ (D) $0.\overline{09}$

20. A fraction with denominator 30 and exist between $\frac{5}{8}$ and $\frac{7}{11}$, is :

(A) $\frac{18}{30}$ (B) $\frac{19}{30}$
(C) $\frac{20}{30}$ (D) $\frac{21}{30}$

21. Ratio between a father's age and his son's age is 5 : 2. If product of their ages 1000, then fins the father's age after 10 years :

(A) 50 (B) 60

(C) 80 (D) 100

22. The product of all prime numbers between 80 and 90 is : **(NCERT)**

(A) 83 (B) 89

(C) 7387 (D) 7200

23. If the sum of 5 consecutive integers is S, then how will the largest integer S be related to S ?

(A) $\frac{S-10}{5}$ (B) $\frac{S+4}{4}$

(C) $\frac{S+5}{4}$ (D) $\frac{S+10}{5}$

24. x is the average of the numbers y and y is the average of the numbers x. What will be the total average of all the numbers ?

(A) $\frac{x+y}{2xy}$ (B) $\frac{2xy}{x+y}$

(C) $\frac{x^2+y^2}{x+y}$ (D) $\frac{xy}{x+y}$

25. Which statement among the following is not true ? **(NCERT)**

(A) Every natural number is an integer.

(B) Every natural number is a real number.

(C) Every real number is a rational number.

(D) Every integer in a rational number.

26. Sum of a positive number and its square is equal to the product of first three prime numbers. Find the positive number.

(A) 2 (B) 3

(C) 5 (D) 6

27. What least integer should be added to 1000 to make it a perfect square ? **(NCERT)**

(A) 10 (B) 18

(C) 24 (D) 89

28. A man has some hens and cows. If heads : feet = 2 : 35 and total number of heads is 48, then find the number of hens.

(A) 28 (B) 26

(C) 24 (D) 22

29. Find the sum of all those prime numbers that are not larger than 17.

(A) 59 (B) 58

(C) 41 (D) 42

30. If p and q are two odd positive integers and $p + q = 10$, $p < q$, then find the possible values of p ?

(A) 2 (B) 3

(C) 4 (D) 1

31. I multiplied a positive integer by 18 and another number by 21, then add both the results. Find the sum.

(A) 2007 (B) 2008

(C) 2006 (D) 2002

32. Product of two numbers is 45 and their difference is 4. What will be the sum of squares of the numbers ? **(NCERT)**

(A) 135 (B) 240

(C) 73 (D) 106

33. Find the unit digit in the product $(122)^{173}$

(A) 2 (B) 4

(C) 6 (D) 8

34. I am three times older of my son. after 15 years, I will be twice the age of the son. Find the sum of our ages.

(A) 48 years (B) 60 years

(C) 64 years (D) 72 years

35. If $\sqrt{1+\frac{x}{9}} = \frac{13}{3}$, then find x :

(A) $\frac{1439}{9}$ (B) 160

(C) $\frac{1443}{9}$ (D) 169

36. Sum of two numbers is 24 and their product is 143. Find the sum of their squares.

(A) 296 (B) 295

(C) 290 (D) 228

37. A man bought some eggs for 5 at 3 per each and sold them for 12 at 5 per each. If he received 143 in total, then find the number of eggs he bought. **(NCERT)**

(A) 210 (B) 200

(C) 195 (D) 190

38. Find the least number which is added to the largest 4-digit number and the obtained result is divisible by 345.

(A) 50 (B) 6

(C) 60 (D) 5

39. Find the unit digit in the sum of $(124)^{372} + (124)^{373}$.

(A) 5 (B) 4

(C) 2 (D) 0

40. If $\sqrt{0.03\times0.3\times a} = 0.3\times0.3\times\sqrt{b}$, then find $\frac{a}{b}$.

(A) 0.009 (B) 0.03

(C) 0.9 (D) 0.08

41. If sum of $\frac{a}{b}$ and its reciprocal is 1 and $a \neq 0, b \neq 0$, then find $a^3 + b^3$.

(A) 2 (B) –1

(C) 0 (D) 1

SOLUTIONS

1. (B) Let, three consecutive positive numbers are $(x - 1)$, x and $(x + 1)$ respectively.

According to question,

$$(x-1)^2 + (x)^2 + (x+1)^2 = 365$$

$$\Rightarrow x^2 + 1 - 2x + x^2 + x^2 + 1 + 2x = 365$$

$$\Rightarrow 3x^2 + 2 = 365$$

$$\Rightarrow 3x^2 = 365 - 2 = 363$$

$$\Rightarrow x^2 = \frac{363}{3} = 121 = (11)^2$$

$$\therefore x = 11$$

$$\therefore x - 1 = 11 - 1 = 10$$

and $x + 1 = 11 + 1 = 12$

$\therefore$ Required sum $= 10 + 11 + 12 = 33$

2. (D) According to the question,

Divisor = Quotient × 10

Divisor = Remainder × 5

Divisor = 46 × 5 = 230

$$\text{Quotient} = \frac{\text{Divisor}}{10} = \frac{230}{10} = 23$$

Dividend = Divisor × Quotient + Remainder

$= 230 \times 23 + 46$

$= 5290 + 46 = 5336$

3. (C) Let the numbers be x and y.

According to the question,

$$\frac{x\times 30}{100} = \frac{y\times 40}{100}$$

$$\Rightarrow 3x = 4y$$

$$\therefore x = \frac{4y}{3}$$

$$\frac{(x+y)\times 25}{100} = 420$$

$$\therefore x + y = 1680$$

$$\Rightarrow \quad \frac{4y}{3}+y = 1680$$

$$\Rightarrow \quad \frac{4y+3y}{3} = 1680$$

$$\Rightarrow \quad \frac{7y}{3} = 1680$$

$$\therefore \quad y = \frac{1680\times3}{7}$$

$$= 720$$

So, the smaller number = 720

4. (C) Let, 4 prime numbers a,b,c,d are in ascending order.

According to question,

$$a \times b \times c = 455$$

and $$b \times c \times d = 1729$$

$$\therefore \quad \frac{a\times b\times c}{b\times c\times d} = \frac{455}{1729}$$

$$\Rightarrow \quad \frac{a}{d} = \frac{5}{19}$$

So, the smallest number = 5, and the largest number = 19

5. (A) Let, the numbers be x and y.

According to question,

$$x + y = 5 \qquad ...(i)$$

and $$x \times y = 6$$

$$(x-y)^2 = (x+y)^2 - 4xy$$

$$= (5)^2 - 4\times6$$

$$= 25 - 24$$

$$= 1$$

$$x - y = 1 \qquad ...(ii)$$

From eq. (i) and (ii),

$$x + y = 5$$

$$x - y = 1$$

$$2x = 6$$

$$\therefore \quad x = \frac{6}{2} = 3$$

From eq. (i), $x + y = 5$

$$\Rightarrow \quad 3 + y = 5$$

$$\therefore \quad y = 5 - 3 = 2$$

According to the question,

$$\left(\frac{1}{x}\right)^2+\left(\frac{1}{y}\right)^2 \Rightarrow \left(\frac{1}{3}\right)^2+\left(\frac{1}{2}\right)^2$$

$$= \frac{1}{9}+\frac{1}{4} = \frac{4+9}{36} = \frac{13}{36}$$

6. (D) Let, the total score = x

$$\text{Maximum score} = \frac{3x}{11}$$

and maximum number of 2nd number

$$= \frac{3}{11} \times \text{Remaining runs}$$

$$\frac{3}{11}\times\left(x-\frac{3x}{11}\right) = \frac{3}{11}\times\frac{8x}{11} = \frac{24x}{121}$$

According to the question,

$$\frac{3x}{11}-\frac{24x}{121} = 9$$

$$\Rightarrow \quad \frac{33x-24x}{121} = 9$$

$$\Rightarrow \quad \frac{9x}{121} = 9$$

$$\therefore \quad x = \frac{9\times121}{9} = 121$$

7. (C) The numbers are x, $x + 1$ and $x + 2$.

$$\therefore \quad 2x + 3x + 3 + 4x + 8 = 191$$

$$\Rightarrow \quad 9x = 191 - 11 = 180$$

$$\Rightarrow \quad x = 20$$

$$\therefore \quad \text{Answer} = 20, 21 \text{ and } 22$$

8. (B) Safe tables $= \frac{5}{6}\times108 = 90$

$$\text{Safe chairs} = \frac{3}{4}\times132 = 99$$

$$\therefore \quad \text{Safe pairs} = 90$$

9. (D) If 1st part be x.

Then, the 2nd number $= 37 - x$

$$\therefore \quad x\times5 + (37-x)\times11 = 227$$

$$\Rightarrow \quad 5x + 407 - 11x = 227$$

$$\Rightarrow \quad 6x = 407 - 227 = 180$$

$$\Rightarrow \quad x = 30$$

$$\therefore \quad \text{the 2nd number} = 7$$

10. (B)

A	B	C
$\frac{2x}{3}$	x	$4x$

$$\Rightarrow \quad \frac{2x}{3} + x + 4x = 510$$

$$\Rightarrow \quad 17x = 510\times3$$

$$\Rightarrow \quad x = 90$$

$$\therefore \quad A = \frac{2\times90}{3} = ₹\,60$$

$$B = ₹\,90,$$

$$C = ₹\,360$$

11. (C) Let, the number of coins of 25p = x.

$$\therefore \quad \text{The number of coins of 50p} = 24 - 2x$$

$$\therefore \quad x + \frac{24-2x}{2}+\frac{x}{4} = 13.75$$

$$\Rightarrow \quad x + 12 - x + \frac{x}{4} = 13.75$$

$$\Rightarrow \quad \frac{x}{4} = 13.75 - 12 = 1.75$$

$$\therefore \quad x = 1.75\times4 = 7$$

12. (D) Let, the number of Indian soldiers be x.

$$\therefore \quad \text{European soldiers} = 12000 - x$$

$$\therefore \quad (12000-x)\times1.8 + x\times1.75$$

$$= 12000\times\left(1\frac{47}{60}\right)$$

$$\Rightarrow \quad 21600 - 1.8x + 1.75x = 21400$$

$$\Rightarrow \quad -0.05x = -200$$

$$\therefore \quad x = \frac{200}{0.05} = 4000$$

13. (A) Required numbers of students in the last row $= \sqrt{1369} = 37$

14. (C) Let, Ram's assest = x.

According to question,

$$\frac{x}{3}+\left(x-\frac{x}{3}\right)\frac{3}{5}+6400 = x$$

$$\Rightarrow \quad \frac{x}{3}+\frac{2x}{3}\times\frac{3}{5}+6400 = x$$

$$\Rightarrow \quad x-\left(\frac{x}{3}+\frac{2x}{5}\right) = 6400$$

$$\Rightarrow \quad x-\left(\frac{5x+6x}{15}\right) = 6400$$

$$\Rightarrow \quad \frac{15x-11x}{15} = 6400$$

$$\Rightarrow \quad 4x = 6400\times15$$

$$\therefore \quad x = 1600\times15$$

$$x = ₹\,24000$$

15. (D) Perfect square between120 and 300 are : $(11)^2$, $(12)^2$, $(13)^2$, $(14)^2$, $(15)^2$, $(16)^2$, $(17)^2$

Required sum = 121 + 144 + 169 + 196 + 225 + 256 + 289 = 1400

16. (C) According to question,

$$\frac{P+Q+R}{3} = (R+5)$$

$$\Rightarrow \quad P + Q + R = 3R + 15$$

$$\Rightarrow \quad P + Q = 2R + 15$$

$$\Rightarrow \quad 2R + 15 = 39 \quad [\because P + Q = 39]$$

$$\Rightarrow \quad 2R = 39 - 15$$

$$\Rightarrow \quad 2R = 24$$

$$\therefore \quad R = 12 \text{ yr}$$

17. (A) Let the two numbers be x and y.

According to question,

$$x + y = 8$$

and $$x \times y = 15$$

$$\frac{1}{x}+\frac{1}{y} = \frac{y+x}{xy}$$

$$= \frac{8}{15}$$

18. (C) $41\frac{2}{3} \div \frac{1}{6} = \frac{125}{3} \div \frac{1}{6} = \frac{125}{3} \times \frac{6}{1}$

$= 250$

19. (B) $0.9 = 0.9000$

$0.\bar{9} = 0.9999$

$0.0\bar{9} = 0.0999$

$0.\overline{09} = 0.0909$

So, the largest number = 0.9999

$= 0.\bar{9}$

20. (B) $\frac{5}{8} = 0.625$

$\frac{7}{11} = 0.6\overline{36}$

And $\frac{19}{30} = 0.6\bar{3}$

So, required fraction = $\frac{19}{30}$

21. (B) Let, the present age of father and his son are $5x$ *yr.* and $2x$ *yr.*

According to question,

$5x \times 2x = 1000$

$\Rightarrow \quad 10x^2 = 1000$

$\Rightarrow \quad x^2 = 100$

$\therefore \quad x = 10$

Father's age after 10 years $= 5x + 10$

$= 5 \times 10 + 10$

$= 50 + 10 = 60$ *yr.*

22. (C) Prime numbers between 80 and 90 = 83 and 89

$\therefore$ Required product $= 83 \times 89 = 7387$

23. (D) Let, five consecutive numbers be a, $(a + 1)$, $(a + 2)$, $(a + 3)$ and $(a + 4)$ respectively.

$\therefore \quad S = (a) + (a + 1) + (a + 2) + (a + 3) + (a + 4)$

$= 5a + 10$

$\Rightarrow 5a = S - 10 \quad \therefore \quad a = \frac{S-10}{5}$

$\therefore$ Largest integer $= a + 4$

$= \frac{S-10}{5} + 4$

$= \frac{S-10+20}{5}$

$= \frac{S+10}{5}$

24. (B) According to question,

Sum of x numbers $= x \times y = xy$

Sum of y numbers $= y \times x = xy$

$\therefore$ Average of $(x + y) = \frac{xy+xy}{x+y}$

$= \frac{2xy}{x+y}$

25. (C) Every real number is a rational number.

26. (C) Let, the required number $= x$

According to question,

$x + (x)^2 = 2 \times 3 \times 5$

(three prime numbers)

$\Rightarrow \quad x + x^2 = 30$

$\Rightarrow \quad x^2 + x - 30 = 0$

$\Rightarrow \quad x^2 + 6x - 5x - 30 = 0$

$\Rightarrow \quad x(x + 6) - 5(x + 6) = 0$

$\Rightarrow \quad (x + 6)(x - 5) = 0$

$\Rightarrow \quad (x - 5) = 0$

$\therefore \quad x = 5$

27. (C) $(31)^2 < 1000 < (32)^2$

$\Rightarrow \quad (32)^2 = 1024$

$\therefore$ Required number $= 1024 - 1000$

$= 24$

28. (B) Let, total hens $= x$, then

number of cows $= 48 - x$

$\therefore \quad 2x + (48 - x) \times 4 = 35 \times 4$

$\Rightarrow \quad 2x + 192 - 4x = 140$

$\Rightarrow \quad 2x = 192 - 140 = 52$

$x = \frac{52}{2} = 26$

$\therefore$ total hens = 26

29. (B) Prime numbers up to 17

$= 2, 3, 5, 7, 11, 13, 17$

$\therefore$ Required sum

$= 2 + 3 + 5 + 7 + 11 + 13 + 17$

$= 58$

30. (A) $p + q = 10$

Possible pairs = (1, 9); (3, 7)

31. (A) Let, two positive integers are x and y. Then, according to question

$18x + 21y = 3(6x + 7y)$

So, only 2007 is divisible by 3. Hence, it is only the possible.

32. (D) Let the numbers be x and y.

According to question,

$x \times y = 45$

and $x - y = 4$

$\therefore \quad x^2 + y^2 = (x - y)^2 + 2xy$

$x^2 + y^2 = (4)^2 + 2 \times 45$

$= 16 + 90 = 106$

33. (A) Unit digit in $(122)^{173}$

= Unit digit in $(2)^{173}$

= Unit digit in $(2)^{172+1}$

= Unit digit in $(2)^1 = 2$

34. (B) Let, Son's age $= x$ *yr*

My present age $= 3x$ *yr*

According to question,

$\frac{3x+15}{x+15} = \frac{2}{1}$

$\Rightarrow \quad 3x + 15 = 2x + 30$

$\Rightarrow \quad 3x - 2x = 30 - 15$

$\therefore \quad x = 15$

Sum of the ages

$= x + 3x$

$= 4x = 4 \times 15 = 60$ *yr*

35. (B) According to question,

$\sqrt{1+\frac{x}{9}} = \frac{13}{3}$

$\Rightarrow \quad \left(\sqrt{1+\frac{x}{9}}\right)^2 = \left(\frac{13}{3}\right)^2$

$\Rightarrow \quad 1+\frac{x}{9} = \frac{169}{9}$

$\Rightarrow \quad \frac{9+x}{9} = \frac{169}{9}$

$\Rightarrow \quad 9 + x = 169$

$\therefore \quad x = 169 - 9 = 160$

36. (C) Let, the numbers be x and y.

According to question

$x + y = 24$

and $x \times y = 143$

$\therefore \quad x^2 + y^2 = (x + y)^2 - 2xy$

$= (24)^2 - 2 \times 143$

$= 576 - 286 = 290$

37. (C) Let, the total no. of eggs $= x$

C.P of total eggs $= x \times \frac{5}{3}$

$= ₹ \frac{5x}{3}$

S.P of total eggs $= x \times \frac{12}{5}$

$= ₹ \frac{12x}{5}$

$$\therefore \quad \frac{12x}{5} - \frac{5x}{3} = 143$$

$$\Rightarrow \quad \frac{36x - 25x}{15} = 143$$

$$\Rightarrow \quad \frac{11x}{15} = 143$$

$$\therefore \quad x = \frac{143 \times 15}{11} = 195$$

Hence, total purchased eggs = 195

38. (B) प्र 10005 is exactly divisible by 345.

Hence, the smallest number

$= 10005 - 9999 = 6$

39. (A) $(124)^{372} + (124)^{373}$

$= (124)^{372}\,[1 + 124]$

$= (124)^{372} \times 125$

$\therefore$ Required Unit digit

= Unit digit in $(4)^0 \times 125$

$= 5$

40. (C) प्र $\sqrt{0.03 \times 0.3 \times a} = 0.3 \times 0.3 \times \sqrt{b}$

$$\Rightarrow \quad \sqrt{0.009 \times a} = 0.09 \times \sqrt{b}$$

$$\Rightarrow \quad 0.009 \times a = 0.0081 \times b$$

$$\therefore \quad \frac{a}{b} = \frac{0.0081}{0.009} = \frac{81}{90}$$

$$= 0.9$$

41. (C) According to question

$$\Rightarrow \quad \frac{a}{b} + \frac{b}{a} = 1$$

$$\Rightarrow \quad \frac{a^2 + b^2}{ab} = 1$$

$$\Rightarrow \quad a^2 + b^2 = ab$$

$$\Rightarrow \quad a^2 + b^2 - ab = 0$$

$$\therefore \quad a^3 + b^3 = (a + b)\,(a^2 + b^2 - ab)$$

$$= (a + b) \times 0 = 0$$

Chapter 2 Square Root and Cube Root

1. Introduction

1.1 Square

If a number is multiplied by itself, then the result we get is the square of the number.

Example : Square of $5 = 5 \times 5 = 25$

Square of $4 = 4 \times 4 = 16$

1.2 Perfect Square

It is a number that can be expressed as the product of two equal integers.

Example : 16 is a perfect square because it expressed as 4×4.

1.3 Sqaure of Decimal Numbers

First find the square of a number without a decimal point. After that place the decimal point on the twice place of the position of decimal point in the given number.

Example : $(1.5)^2 = ?$

Step 1 : Take square of 15, *i.e.* $15 \times 15 = 225$

Step 2 : Place the decimal point on the twice place of the position of decimal point, i.e. after two digits from right side.

$\therefore \quad (1.5)^2 = 2.25$

1.4 Methods for Finding Square of a Number

There two methods as follows :

- A number is multiplied by itself and get the square of given number.

  Example : $(25)^2 = 25 \times 25 = 625$

- We can also use the algebraic formulas to find the square root of the given number. These are

  (i) $(a+b)^2 = a^2 + 2ab + b^2$

  (ii) $(a-b)^2 = a^2 - 2ab + b^2$

Example 1 : $(25)^2 = (20+5)^2$

$= (20)^2 + 2 \times 20 \times 5 + (5)^2$

$= 400 + 200 + 25$

$= 625$

Example 2 : $(45)^2 = (50-5)^2$

$= (50)^2 - 2 \times 50 \times 5 + (5)^2$

$= 2500 - 500 + 25$

$= 2000 + 25 = 2025$

I. Rememberable Squares of Few Numbers

Number	Description	Square
1	$1^2 = 1 \times 1$	1
2	$2^2 = 2 \times 2$	4
3	$3^2 = 3 \times 3$	9
4	$4^2 = 4 \times 4$	16
5	$5^2 = 5 \times 5$	25
6	$6^2 = 6 \times 6$	36
7	$7^2 = 7 \times 7$	49
8	$8^2 = 8 \times 8$	64
9	$9^2 = 9 \times 9$	81
10	$10^2 = 10 \times 10$	100
11	$11^2 = 11 \times 11$	121
12	$12^2 = 12 \times 12$	144
13	$13^2 = 13 \times 13$	169
14	$14^2 = 14 \times 14$	196
15	$15^2 = 15 \times 15$	225
16	$16^2 = 16 \times 16$	256
17	$17^2 = 17 \times 17$	289
20	$20^2 = 20 \times 20$	400
21	$21^2 = 21 \times 21$	441
25	$25^2 = 25 \times 25$	625
30	$30^2 = 30 \times 30$	900
40	$40^2 = 40 \times 40$	1600
50	$50^2 = 50 \times 50$	2500
100	$100^2 = 100 \times 100$	10000

II. Unit digit and last two digits in the square of any number

S. No.	Unit Digit of the Number	Unit Digit of Square of Number	Last Two Digit of Square of Number	If the last two-digit in square of a number then such number will not perfect squares
•	01 or 09	1	01, 21, 41, 61, 81	11, 31, 51, 71, 91
•	02 or 08	4	04, 24, 44, 64, 84	14, 34, 54, 94
•	03 or 07	9	09, 29, 49, 69, 89	19, 36, 59, 99
•	04 or 06	6	16, 36, 56, 76, 96	06, 26, 46, 66, 86

S. No.	Unit Digit of the Number	Unit Digit of Square of Number	Last Two Digit of Square of Number	If the last two-digit in square of a number then such number will not perfect squares
•	05	5	25	.....
•	0	0	0	0

1.5 Square Root of a Number

Square root of a non-negative number is a non-negative number that when multiplied by itself results in the original number.

Example : Square root of 25 = ?

Solution : 25 is a non-negative number.

$25 = 5 \times 5$

It means, $\sqrt{25} = \sqrt{5 \times 5} = 5$

It is denoted by the symbol '$\sqrt{\ }$' which means raise to the power $\frac{1}{2}$.

$\sqrt{25} = (5 \times 5)^{\frac{1}{2}} = 5^{2 \times \frac{1}{2}} = 5$

I. Methods to find Square Root

(i) Factor Method—Follow the following steps :

Step 1 : Find all prime factors of the number.

Step 2 : Make pairs of all prime factors

Step 3 : Collect one factor from each pair of prime factor and multiply them.

Step 4 : Obtained result after multiplication, is the required square root of the given number.

Example : Find the value of $\sqrt{50625}$

Solution :

Step 1 :

3	50625
3	16875
3	5625
3	1875
5	625
5	125
5	25
5	5
	1

$\therefore$ Prime factors $= 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5$

Step 2 : Make pairs = $\overline{3 \times 3} \times \overline{3 \times 3} \times \overline{5 \times 5} \times \overline{5 \times 5}$

Step 3 : $3 \times 3 \times 5 \times 5$

Step 4 : $\sqrt{50625} = 3 \times 3 \times 5 \times 5 = 225$

(ii) Division Method

Step 1 : Make the pairs starting from unit digit.

Step 2 : Find such number whose square is either equal to or less than the first pair.

Step 3 : Consider this number as the divisor and the quotient. Now, subtract its square from the first pair. Along with the remainder we keep the next pair adjacent to it.

Step 4 : For the new divisor, twice the quotient part and enter new digit in the unit place for further calculation.

Step 5 : Again, repeat these steps untill the last pair is in used.

Example : Find the square root of 50625 using division method.

Solution :

	225
2	50625
+ 2	– 4
42	106
+2	– 84
445	2225
+5	– 2225
	0

$\therefore \sqrt{50625} = 225$

II. Rememberable Square Root of Few Numbers

Number	Description	Square-Root
1	$\sqrt{1} = \sqrt{1 \times 1}$	1
4	$\sqrt{4} = \sqrt{2 \times 2}$	2
9	$\sqrt{9} = \sqrt{3 \times 3}$	3
16	$\sqrt{16} = \sqrt{4 \times 4}$	4
25	$\sqrt{25} = \sqrt{5 \times 5}$	5
36	$\sqrt{36} = \sqrt{6 \times 6}$	6
49	$\sqrt{49} = \sqrt{7 \times 7}$	7
64	$\sqrt{64} = \sqrt{8 \times 8}$	8
81	$\sqrt{81} = \sqrt{9 \times 9}$	9

Number	Description	Square-Root
100	$\sqrt{100} = \sqrt{10 \times 10}$	10
121	$\sqrt{121} = \sqrt{11 \times 11}$	11
144	$\sqrt{144} = \sqrt{12 \times 12}$	12
169	$\sqrt{169} = \sqrt{13 \times 13}$	13
196	$\sqrt{196} = \sqrt{14 \times 14}$	14
225	$\sqrt{225} = \sqrt{15 \times 15}$	15
256	$\sqrt{256} = \sqrt{16 \times 16}$	16
289	$\sqrt{289} = \sqrt{17 \times 17}$	17
400	$\sqrt{400} = \sqrt{20 \times 20}$	20
441	$\sqrt{441} = \sqrt{21 \times 21}$	21
625	$\sqrt{625} = \sqrt{25 \times 25}$	25
900	$\sqrt{900} = \sqrt{30 \times 30}$	30
1600	$\sqrt{1600} = \sqrt{40 \times 40}$	40
2500	$\sqrt{2500} = \sqrt{50 \times 50}$	50
10000	$\sqrt{10000} = \sqrt{100 \times 100}$	100

III. Square Root of Decimal Numbers

To find the square root of a decimal number, we ignore the decimal point and find the square root of the number as did previously. Then put the decimal point after the half of the number of digits of decimal point in the given number from the unit place.

Example : $\sqrt{6.25} = \sqrt{2.5 \times 2.5} = 2.5$

2. Cube of Numbers

If we multiply a number by itself three times, the product so obtained is called the perfet cube of that number. It is shown by power '3'. If y is the cube of x, then

$$x^3 = y$$

2.1 Methods for finding Cube of a Number

I. Multiplication method—In this method, a number is multiplied by itself three times.

Example : $(25)^3 = 25 \times 25 \times 25$

$= 625 \times 25$

$= 15625$

II. Algebraic method—In this method, the following formulae are used :

(i) $(a + b)^3 = a^3 + b^3 + 3ab\,(a + b)$

(ii) $(a - b)^3 = a^3 - b^3 - 3ab\,(a - b)$

Example : Find $(15)^3$

Solution : $(15)^3 = (10 + 5)^3$

$\because (a + b)^3 = a^3 + b^3 + 3ab\,(a + b)$

$= (10)^3 + (5)^3 + 3 \times 10 \times 5\,(10 + 5)$

$= 1000 + 125 + 150 \times 15$

$= 1125 + 2250 = 3375$

or

$(15)^3 = (20 - 5)^3$

$= (20)^3 - (5)^3 - 3 \times 20 \times 5\,(20 - 5)$

$= 8000 - 125 - 300 \times 15$

$= 8000 - 125 - 4500$

$= 3375$

Note

To find the cube of decimal numbers, first find the cube of give number without decimal point, then in the result put the decimal point three times the given digit from right side.

Example : $(1.5)^3 = ?$

Solution :

- First find the cube of 15

  $\Rightarrow (15)^3 = 15 \times 15 \times 15$

  $= 3375$

- since, decimal point is place after one digit from the right side. Take its three time, *i.e.* $1 \times 3 = 3$ and place the decimal after 3 digits from the right side.

$\Rightarrow (1.5)^3 = 3.375$

2.2 Rememberable Cubes of the Numbers

Number	Details	Cube
1	$1^3 = 1 \times 1 \times 1$	1
2	$2^3 = 2 \times 2 \times 2$	8
3	$3^3 = 3 \times 3 \times 3$	27
4	$4^3 = 4 \times 4 \times 4$	64
5	$5^3 = 5 \times 5 \times 5$	125
6	$6^3 = 6 \times 6 \times 6$	216
7	$7^3 = 7 \times 7 \times 7$	343
8	$8^3 = 8 \times 8 \times 8$	512
9	$9^3 = 9 \times 9 \times 9$	729
10	$10^3 = 10 \times 10 \times 10$	1000
11	$11^3 = 11 \times 11 \times 11$	1331
15	$15^3 = 15 \times 15 \times 15$	3375
20	$20^3 = 20 \times 20 \times 20$	8000

3. Cube Root

Cube root of a number is the number whose cube is given number. Cube root is denoted by expression $\sqrt[3]{\ }$.

Example : Find $\sqrt[3]{15625}$

Solution : $\sqrt[3]{1\,5625} = (1\,5625)^{\frac{1}{3}}$

$= (25\times25\times25)^{\frac{1}{3}}$

$= (25)^{3\times\frac{1}{3}} = 25$

3.1 Methods to find the Cube Root

I. **Factor method–**Follow the steps given below :

- Find all possible prime factors.
- Make triplets of same factors.
- One factor from each triple is multiplied.
- This product is the required cube root of the number

Example : $\sqrt[3]{1331} = ?$

Solution : $\sqrt[3]{1331} = \sqrt[3]{11\times11\times11} = 11$

Example : Find cube root of 5832

Solution : $\sqrt[3]{5832} = \sqrt[3]{2\times2\times2\times3\times3\times3\times3\times3\times3}$

$= \sqrt[3]{(2\times2\times2)\times(3\times3\times3)\times(3\times3\times3)}$

$= 2\times3\times3 = 18$

3.2 Rememberable Cube Roots

Number	Details	Cube Roots
1	$\sqrt[3]{1} = \sqrt[3]{1\times1\times1}$	1
8	$\sqrt[3]{8} = \sqrt[3]{2\times2\times2}$	2
27	$\sqrt[3]{27} = \sqrt[3]{3\times3\times3}$	3
64	$\sqrt[3]{64} = \sqrt[3]{4\times4\times4}$	4
125	$\sqrt[3]{125} = \sqrt[3]{5\times5\times5}$	5
216	$\sqrt[3]{216} = \sqrt[3]{6\times6\times6}$	6
343	$\sqrt[3]{343} = \sqrt[3]{7\times7\times7}$	7
512	$\sqrt[3]{512} = \sqrt[3]{8\times8\times8}$	8
729	$\sqrt[3]{729} = \sqrt[3]{9\times9\times9}$	9
1000	$\sqrt[3]{1000} = \sqrt[3]{10\times10\times10}$	10
1331	$\sqrt[3]{1331} = \sqrt[3]{11\times11\times11}$	11
3375	$\sqrt[3]{3375} = \sqrt[3]{15\times15\times15}$	15
8000	$\sqrt[3]{8000} = \sqrt[3]{20\times20\times20}$	20

3.3 Cube Root of Decimal Numbers

To find the cube root of decimal numbers, first find the cube root of given number without decimal point and then place the decimal at $\frac{1}{3}^{rd}$ of previous position of the decimal point.

Example : Find $\sqrt[3]{0.008} = ?$

Solution : $\sqrt[3]{0.008} = \sqrt[3]{\frac{8}{1000}}$

$= \sqrt[3]{\frac{2\times2\times2}{10\times10\times10}} = \frac{2}{10} = 0.2$

4. Important Rules

Rule 1 : Find the square of a two-digits number.

$(xy)^2 = x^2/2xy/y^2$

Example : $(25)^2 - (24)^2$

Solution : $(25)^2 - (24)^2$

$= (2)^2/2\times2\times5/(5)^2 - (2)^2/2\times2\times4/(4)^2$

$= 4/20/\overset{+}{\underset{2}{\circlearrowleft}}5 - 4/16/\overset{+}{\underset{1}{\circlearrowleft}}6$

$= 4/\overset{+}{\underset{2}{\circlearrowleft}}2/5 - 4/\overset{+}{\underset{1}{\circlearrowleft}}7/6$

$= 625 - 576$

$= 49$

Rule 2 : Find the square of a 3-digits number.

$(xyz)^2 = x^2/2xy/2xz + y^2/2yz/z^2$

Example : $(235)^2 = ?$

Solution : $(235)^2 = (2)^2/2\times2\times3/2\times2\times5 + (3)^2/2\times3\times5/(5)^2$

$= 4/12/\overset{+}{\underset{2}{\circlearrowleft}}0 - 9/30/\overset{+}{\underset{2}{\circlearrowleft}}5$

$= 4/\overset{+}{\underset{1}{\circlearrowleft}}4/0 - 9/\overset{+}{\underset{3}{\circlearrowleft}}2/5$

$= 5\overset{+}{\underset{40}{\circlearrowleft}}\overset{+}{\underset{12}{\circlearrowleft}}25$

$= 55225$

Rule 3 : Find the square of a number formed by repeatitive 1's.

Example : $(1111)^2 = ?$

Solution : There are 4's. So, the counting will start from 1 to 4 then reverse counting will start.

$(1111)^2$

$= \underset{\text{Counting Forward}}{\overset{1\,2\,3\,4}{\longrightarrow}}\ \underset{\text{Counting Backward}}{\overset{3\,2\,1}{\longrightarrow}}$

Example : $(1111)^2-(111)^2+(11)^2-1^2$

Solution : $(1111)^2-(111)^2+(11)^2-(1)^2$

$=1234321-12321+121-1$

$=1222120$

Rule 4 : $(AAAA)^2=A^2.(1111)^2=A^2\times(1234321)$

Example : Find $(444)^2$

Solution : $(444)^2=(4)^2\times(111)^2$

$=16\times12321$

$=197136$

Rule 5 : $(A/5)^2=A(A+1)/25$

Example : Find $(125)^2$

Solution : $(12/5)^2=12(12+1)/25$

$=12\times13/25=15625$

Rule 6 : $(A\pm B)^2=A^2\pm2AB+B^2$

Example : Find $(105)^2$

Solution : $(105)^2=(100+5)^2$

$=(100)^2+2\times100\times5+(5)^2$

$=10000+1000+25$

$=11025$

Rule 7 : $1^2+2^2+3^2+....+n^2=\dfrac{n(n+1)(2n+1)}{6}$

Example : Find the sum of $1^2+2^2+....+20^2$.

Solution : Here, $n=20$

$\therefore$ Required sum $=\dfrac{20(20+1)(2\times20+1)}{6}$

$=\dfrac{20\times21\times41}{6}$

$=2870$

Rule 8 : $(A+B+C)^2=A^2+B^2+C^2+2(AB+BC+CA)$

Example : Find the square of 15.

Solution : $(15)^2=(10+3+2)^2$

$=(10)^2+3^2+2^2+2(10\times3+3\times2+2\times10)$

$=100+9+4+2(30+6+20)$

$=113+2\times56$

$=113+112$

$=225$

Important Questions

1. Which of the following is not a perfect square?

(A) 19881 (B) 18971

(C) 17951 (D) 16831

2. $\sqrt{0.0625}=?$ **(NCERT)**

(A) 0.25 (B) 0.0025

(C) 0.025 (D) 0.00025

3. $\sqrt{900}+\sqrt{0.09}+\sqrt{0.000009}=?$

(A) 303.300 (B) 3030.3

(C) 3.0303 (D) 30.303

4. $\sqrt[3]{\dfrac{72.9}{0.4096}}$ equals to :

(A) 0.5626 (B) 5.625

(C) 182 (D) 13.6

5. $\dfrac{\sqrt[3]{8}}{\sqrt{16}}\div\sqrt{\dfrac{100}{49}}\times\sqrt[3]{125}$ equals to :

(A) 7 (B) $1\dfrac{3}{4}$

(C) $\dfrac{7}{100}$ (D) $\dfrac{4}{7}$

6. $\sqrt[3]{10648}\times\sqrt[3]{5832}=?$ **(NCERT)**

(A) 396 (B) 216

(C) 432 (D) 576

7. $\sqrt[3]{1-\dfrac{91}{216}}$ equals to :

(A) $\dfrac{1}{6}$ (B) $\dfrac{5}{6}$

(C) $\dfrac{7}{6}$ (D) $\dfrac{11}{6}$

8. $\sqrt{\sqrt{2500}+\sqrt{961}}=(?)^2$

(A) 81 (B) 3

(C) 6561 (D) 9

9. $\sqrt{5+\sqrt{11+\sqrt{19+\sqrt{29+\sqrt{49}}}}}=?$

(A) 3 (B) 2

(C) 4 (D) 6

10. $\dfrac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}}=?$ **(NCERT)**

(A) 2 (B) 4

(C) 8 (D) $\sqrt{2}$

11. $\sqrt{\dfrac{0.009\times0.036\times0.016\times0.08}{0.002\times0.0008\times0.0002}}=?$

(A) 34 (B) 36

(C) 38 (D) 39

12. $\sqrt{6+\sqrt{6+\sqrt{6+.....\infty}}}$

(A) 3 (B) 4

(C) 5 (D) 6

13. If $\sqrt{3}=1.732$, then $\sqrt{\dfrac{3}{4}}=?$

(A) 0.557 (B) 0.866

(C) 0.433 (D) 0.44

14. If $\dfrac{\sqrt{7}-2}{\sqrt{7}+2}=a\sqrt{7}+b$, then $a=?$

(A) $\dfrac{11}{3}$ (B) $\dfrac{-4}{3}$

(C) $\dfrac{4}{3}$ (D) $\dfrac{-4\sqrt{7}}{3}$

15. $\dfrac{1}{\sqrt{9}-\sqrt{8}}-\dfrac{1}{\sqrt{8}-\sqrt{7}}+\dfrac{1}{\sqrt{7}-\sqrt{6}}-\dfrac{1}{\sqrt{6}-\sqrt{5}}+\dfrac{1}{\sqrt{5}-\sqrt{4}}=?$

(A) 0 (B) 1

(C) 5 (D) $\dfrac{1}{3}$

16. If $x=7-4\sqrt{3}$, then $\left(x+\dfrac{1}{x}\right)=?$

(A) $3\sqrt{3}$ (B) $8\sqrt{3}$

(C) $14+8\sqrt{3}$ (D) 14

17. If $a=\dfrac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b=\dfrac{\sqrt{5}-1}{\sqrt{5}+1}$, then $\dfrac{a^2+ab+b^2}{a^2-ab+b^2}=?$ **(NCERT)**

(A) 0.75 (B) 1.25

(C) 0.6 (D) 1.33

18. $\sqrt[3]{(13.608)^2 - (13.392)^2} = ?$

(A) 0.6 (B) 0.06
(C) 1.8 (D) 2.6

19. $\sqrt[3]{\sqrt{0.000729}} + \sqrt[3]{0.008} = ?$ **(NCERT)**

(A) 0.1 (B) 0.5
(C) 0.06 (D) 0.8

20. Square root of

$\frac{(0.75)^3}{1-0.75} + \left[0.75 + (0.75)^2 + 1\right]$ is :

(A) 4 (B) 3
(C) 2 (D) 1

21. If $1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225$, then $2^3 + 4^3 + 6^3 + 8^3 + 10^3$ equals to :

(A) 450 (B) 900
(C) 1800 (D) $(225)^2$

22. $\sqrt[3]{333 + \sqrt[3]{987 + \sqrt[3]{2197}}}$ equals to :

(A) 21 (B) 18
(C) 7 (D) 3

23. $\frac{1}{\sqrt{9} - \sqrt{8}} = ?$

(A) $3 + 2\sqrt{2}$ (B) $3 - \sqrt[3]{2}$
(C) $\frac{1}{3 + \sqrt[3]{2}}$ (D) $\frac{1}{2}(3 - \sqrt[3]{2})$

24. Square root of

$\frac{0.324 \times 0.081 \times 4.624}{1.5625 \times 0.0289 \times 72.9 \times 64}$ will be :

(A) 24 (B) 2.40
(C) 0.024 (D) None of these

25. $(\sqrt{10} \times \sqrt{15})$ equals to :

(A) $5\sqrt{6}$ (B) $\sqrt[4]{5}$
(C) $\sqrt{30}$ (D) $\sqrt{25}$

26. The value of $\frac{112}{\sqrt{196}} \times \frac{\sqrt{576}}{12} \times \frac{\sqrt{256}}{8}$ will be :

(A) 8 (B) 32
(C) 12 (D) 16

27. $\sqrt{3.6 - 0.36} = ?$

(A) 1.8 (B) 3.24
(C) 10.4976 (D) None of these

28. $\left(\sqrt{65025}\right)^2 = (?)^2$

(A) 65025 (B) 32512.5
(C) 255 (D) 510

29. $\sqrt{?} + 7 = \sqrt{576}$

(A) 16 (B) 289
(C) 19 (D) 510

30. $\sqrt{529} + \sqrt{2304} = ?$

(A) 23 (B) 60
(C) 71 (D) None of these

31. $\sqrt{\frac{?}{196}} = \frac{72}{56}$ equals to :

(A) 18 (B) 14
(C) 324 (D) 212

32. $\sqrt[3]{(64)^{-4} \div (125)^{-2}}$ equals to :

(A) $\frac{25}{256}$ (B) 3200
(C) $\frac{1}{3200}$ (D) $\frac{1}{1000}$

33. $\sqrt[3]{1 + \sqrt[3]{343}} = ?$

(A) 2 (B) 3
(C) 4 (D) 7

34. $\sqrt[3]{4\frac{12}{125}}$ equals to :

(A) $1\frac{2}{5}$ (B) $1\frac{3}{5}$
(C) $1\frac{4}{5}$ (D) $2\frac{2}{5}$

35. The value of $\sqrt{5 + 2\sqrt{6}} - \frac{1}{\sqrt{5 + 2\sqrt{6}}}$ will be :

(A) $2\sqrt{2}$ (B) $2\sqrt{3}$
(C) $1 + \sqrt{5}$ (D) $\sqrt{5} - 1$

36. $1 - \frac{1}{1+\sqrt{2}} + \frac{1}{1-\sqrt{2}}$ equals to :

(A) $2\sqrt{2-1}$ (B) $1 - 2\sqrt{2}$
(C) $1 - \sqrt{2}$ (D) $-2\sqrt{2}$

37. The value of

$\left(2 + \sqrt{2} + \frac{1}{2+\sqrt{2}} + \frac{1}{\sqrt{2}-2}\right)$ will be :

(A) 2 (B) $2\sqrt{2}$
(C) $2 - \sqrt{2}$ (D) $2 + \sqrt{2}$

38. $\sqrt{0.02 + \sqrt{0.0049}}$ equals to :

(A) 0.03 (B) $\sqrt{0.27}$
(C) $\sqrt{.72}$ (D) 0.3

39. $\sqrt{15612 + \sqrt{154 + \sqrt{225}}}$ equals to :

(A) 13 (B) 15
(C) 25 (D) 125

40. $\sqrt{\frac{9}{16}} + (0.008)^{-2/3} - \left(\frac{7}{10}\right)^0$ equals to :

(A) $11\frac{2}{3}$ (B) $24\frac{3}{4}$
(C) $18\frac{1}{3}$ (D) $15\frac{2}{3}$

41. $\frac{\sqrt{98} - \sqrt{72} + \sqrt{50}}{\sqrt{18}} = ?$

(A) 6 (B) $\frac{\sqrt{38}}{3}$
(C) $-\frac{4}{3}$ (D) 2

42. $\sqrt{\frac{4}{3}} - \sqrt{\frac{3}{4}} = ?$

(A) $\frac{1}{2\sqrt{3}}$ (B) $\frac{-1}{2\sqrt{3}}$
(C) 1 (D) $\frac{5\sqrt{3}}{6}$

43. Simplify :

$\left[\sqrt{21 + \sqrt{11 + \sqrt{25}}}\right] \times [(3 \times 8 - 4) \div 4]^{-1}$

(A) 0 (B) 1
(C) 5 (D) 6

SOLUTIONS

1. (A) In a perfect sqaure, last two digits can not be 31, 51, 71. So, 19881 is a perfect square.

2. (A) $\sqrt{0.0625} = \sqrt{\frac{625}{10000}}$

$= \sqrt{\frac{5 \times 5 \times 5 \times 5}{10 \times 10 \times 10 \times 10}}$

$= \frac{25}{100} = 0.25$

3. (D) $\sqrt{900} + \sqrt{0.09} + \sqrt{0.000009}$

$= 30 + 0.3 + 0.003$

$= 30.303$

4. (B) $\sqrt[3]{\frac{72.9}{0.4096}} = \sqrt[3]{\frac{729000}{4096}}$

$= \sqrt[3]{\frac{9^3 \times 10^3}{2^{12}}}$

$= \frac{9 \times 10}{2^4} = \frac{90}{16}$

$= 5.625$

5. (B) $\frac{\sqrt[3]{8}}{\sqrt{16}} \div \frac{\sqrt{100}}{\sqrt{49}} \times \sqrt[3]{125}$

$= \frac{2}{4} \div \frac{10}{7} \times 5$

$= \frac{1}{2} \times \frac{7}{10} \times 5$

$= \frac{7}{4} = 1\frac{3}{4}$

6. (A) $\sqrt[3]{10648} \times \sqrt[3]{5832}$

$= \sqrt[3]{2^3 \times 11^3} \times \sqrt[3]{2^3 \times 9^3}$

$= 2 \times 11 \times 2 \times 9$

$= 396$

7. (B) $\sqrt[3]{1 - \frac{91}{216}} = \sqrt[3]{\frac{125}{216}}$

$= \sqrt[3]{\frac{5^3}{6^3}}$

$= \frac{5}{6}$

8. (B) $\sqrt{\sqrt{2500} + \sqrt{961}} = (?)^2$

$\Rightarrow \sqrt{50 + 31} = (?)^2$

$\Rightarrow \sqrt{81} = (?)^2$

$\Rightarrow 9 = (?)^2$

$\sqrt{9} = ?$

$? = 3$

9. (A) $\sqrt{5 + \sqrt{11 + \sqrt{19 + \sqrt{29 + \sqrt{49}}}}} = ?$

$\sqrt{5 + \sqrt{11 + \sqrt{19 + \sqrt{29 + 7}}}} = ?$

$\sqrt{5 + \sqrt{11 + \sqrt{19 + \sqrt{36}}}} = ?$

$\Rightarrow \sqrt{5 + \sqrt{11 + \sqrt{19 + 6}}} = ?$

$\Rightarrow \sqrt{5 + \sqrt{11 + \sqrt{25}}} = ?$

$\Rightarrow \sqrt{5 + \sqrt{11 + 5}} = ?$

$\Rightarrow \sqrt{5 + \sqrt{16}} = ?$

$\Rightarrow \sqrt{5 + 4} = ?$

$\Rightarrow \sqrt{9} = ?$

$\Rightarrow 3 = ?$

10. (A) $\frac{\sqrt{32} + \sqrt{48}}{\sqrt{8} + \sqrt{12}} = ?$

$= \frac{\sqrt{16 \times 2} + \sqrt{16 \times 3}}{\sqrt{4 \times 2} + \sqrt{4 \times 3}}$

$= \frac{4\sqrt{2} + 4\sqrt{3}}{2\sqrt{2} + 2\sqrt{3}}$

$= \frac{4(\sqrt{2} + \sqrt{3})}{2(\sqrt{2} + \sqrt{3})} = 2$

11. (B) $\sqrt{\frac{0.009 \times 0.036 \times 0.016 \times 0.08}{0.002 \times 0.0008 \times 0.0002}}$

Here, in numerator and denominator, digits after decimal point is equal. So, we have

$= \sqrt{\frac{9 \times 36 \times 16 \times 8}{2 \times 8 \times 2}}$

$= \frac{3 \times 6 \times 4}{2} = 36$

12. (A) $\sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots\ldots\ldots\infty}}} = ?$

$= \frac{\sqrt{1 + 4x} + 1}{2}$

(Put $x = 6$)

$= \frac{\sqrt{1 + 4 \times 6} + 1}{2}$

$= \frac{5 + 1}{2} = 3$

13. (B) $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$= \frac{1.732}{2} = 0.866$

14. (B) $\frac{\sqrt{7} - 2}{\sqrt{7} + 2} = a\sqrt{7} + b$

$\Rightarrow a\sqrt{7} + b = \frac{\sqrt{7} - 2}{\sqrt{7} + 2} \times \frac{\sqrt{7} - 2}{\sqrt{7} - 2}$

$= \frac{(\sqrt{7} - 2)^2}{7 - 4}$

$= \frac{7 - 4\sqrt{7} + 4}{3}$

$a\sqrt{7} + b = \frac{-4}{3}\sqrt{7} + \frac{11}{3}$

$\Rightarrow a = \frac{-4}{3}$

15. (C) $\frac{1}{\sqrt{9} - \sqrt{8}} - \frac{1}{\sqrt{8} - \sqrt{7}} + \frac{1}{\sqrt{7} - \sqrt{6}} - \frac{1}{\sqrt{6} - \sqrt{5}} + \frac{1}{\sqrt{5} - \sqrt{4}}$

$= (\sqrt{9} + \sqrt{8}) - (\sqrt{8} + \sqrt{7}) + (\sqrt{7} + \sqrt{6}) - (\sqrt{6} + \sqrt{5}) + (\sqrt{5} + \sqrt{4})$

$= \sqrt{9} + \sqrt{4}$

$= 3 + 2 = 5$

16. (D) $x = 7 - 4\sqrt{3}$

$x + \frac{1}{x} = 7 - 4\sqrt{3} + \frac{1}{7 - 4\sqrt{3}}$

$= 7 - 4\sqrt{3} + 7 + 4\sqrt{3}$

$= 14$

17. (D) $a = \frac{\sqrt{5} + 1}{\sqrt{5} - 1} \times \frac{\sqrt{5} + 1}{\sqrt{5} + 1}$

$= \frac{5 + 1 + 2\sqrt{5}}{5 - 1} = \frac{6 + 2\sqrt{5}}{4}$

$= \frac{3 + \sqrt{5}}{2}$

and $b = \frac{\sqrt{5} - 1}{\sqrt{5} + 1} \times \frac{\sqrt{5} - 1}{\sqrt{5} - 1}$

$= \frac{5 + 1 - 2\sqrt{5}}{5 - 1}$

$= \frac{6 - 2\sqrt{5}}{4} = \frac{3 - \sqrt{5}}{2}$

$ab =$

$\frac{\sqrt{5} + 1}{\sqrt{5} - 1} \times \frac{\sqrt{5} - 1}{\sqrt{5} + 1} = 1$

Here, $\frac{a^2 + ab + b^2}{a^2 - ab + b^2}$

$= \frac{\left(\frac{3 + \sqrt{5}}{2}\right)^2 + 1 + \left(\frac{3 - \sqrt{5}}{2}\right)^2}{\left(\frac{3 + \sqrt{5}}{2}\right)^2 - 1 + \left(\frac{3 - \sqrt{5}}{2}\right)^2}$

$= \frac{9 + 6\sqrt{5} + 5 + 4 + 9 - 6\sqrt{5} + 5}{9 + 6\sqrt{5} + 5 - 4 + 9 + 5 - 6\sqrt{5}}$

$= \frac{18 + 10 + 4}{18 + 10 - 4}$

$= \frac{32}{24} = \frac{4}{3} = 1.33$

18. (C) $\sqrt[3]{(13.608)^2 - (13.392)^2}$

$= \sqrt[3]{(13.608 + 13.392)(13.608 - 13.392)}$

$= \sqrt[3]{27 \times 0.216} = 3 \times 0.6 = 1.8$

19. (C) $\sqrt[3]{\sqrt{0.000729}} + \sqrt[3]{0.008}$

$= \sqrt[3]{\sqrt{\frac{729}{10^6}}} \times \sqrt[3]{\frac{8}{1000}}$

$= \sqrt[3]{\frac{27}{10^3}} \times \frac{2}{10}$

$= \frac{3}{10} \times \frac{2}{10}$

$= \frac{3}{10} \times \frac{1}{5}$

$= \frac{3}{50} = 0.06$

20. (A) $\dfrac{(0.75)^3}{1-0.75}+\left[0.75+(0.75)^2+1\right]$

$$=\frac{(0.75)^3+(1-0.75)\left[\begin{matrix}(0.75)^2\\+0.75+1^2\end{matrix}\right]}{(1-0.75)}$$

$$=\frac{(0.75)^3+1^3-(0.75)^3}{(1-0.75)}$$

$[\because (a^3-b^3)=(a-b)(a^2+ab+b)]$

$$=\frac{1}{0.25}=4$$

21. (C) $1^3+2^3+3^3+4^3+5^3=225$ ...(1)

$2^3+4^3+6^3+8^3+10^3$

$=2^3+(2\times2)^3+(2\times3)^3+(2\times4)^3+(2\times5)^3$

$=2^3(1^3+2^3+3^3+4^3+5^3)$

$=2^3\times225$ [From eq.(1)]

$=8\times225=1800$

22. (C) $\sqrt[3]{333+\sqrt[3]{987+\sqrt[3]{2197}}}$

$=\sqrt[3]{333+\sqrt[3]{987+13}}$

$=\sqrt[3]{333+\sqrt[3]{1000}}$

$=\sqrt[3]{333+10}$

$=\sqrt[3]{343}=7$

23. (A) $\dfrac{1}{\sqrt{9}-\sqrt{8}}=\dfrac{1}{\sqrt{9}-\sqrt{8}}\times\dfrac{\sqrt{9}+\sqrt{8}}{\sqrt{9}+\sqrt{8}}$

$$=\frac{\sqrt{9}+\sqrt{8}}{9-8}=\sqrt{9}+\sqrt{8}=3+2\sqrt{2}$$

24. (C) $\sqrt{\dfrac{324\times81\times4624}{15625\times289\times729\times64}}=0.024$

25. (A) $(\sqrt{10}\times\sqrt{15})=\sqrt{150}=5\sqrt{6}$

26. (B) $\dfrac{112}{14}\times\dfrac{24}{12}\times\dfrac{16}{8}=32$

27. (A) $\sqrt{3.6-0.36}=\sqrt{3.24}=1.8$

28. (C) $\left(\sqrt{65025}\right)^2=(?)^2$

or, $65025^{\frac{1}{2}\times2}=(?)^2$

or, $65025=?^2$

$\therefore ?=255$

29. (B) $\sqrt{?}=\sqrt{576}-7=24-7=17$

$\therefore ?=(17)^2=289$

30. (C) $?=\sqrt{529}+\sqrt{2304}$

$=23+48=71$

31. (C) $\sqrt{\dfrac{?}{196}}=\dfrac{72}{56}$

or, $\dfrac{\sqrt{?}}{14}=\dfrac{72}{56}$

or, $\sqrt{?}=\dfrac{72\times14}{56}=18$

$\therefore ?=(18)^2=324$

32. (A) $\sqrt[3]{(64)^{-4}\div(125)^{-2}}$

$=[(4^3)^{-4}\div(5^3)^{-2}]^{1/3}=[4^{-12}\div5^{-6}]^{1/3}$

$$=\left[4^{-12}\times\frac{1}{5^{-6}}\right]^{1/3}=\left(\frac{5^6}{4^{12}}\right)^{1/3}$$

$$=\frac{5^2}{4^4}=\frac{25}{256}$$

33. (A) $\sqrt[3]{1+\sqrt[3]{343}}=\sqrt[3]{1+\sqrt[3]{(7)^3}}$

$=\sqrt[3]{1+7}=\sqrt[3]{(2)^3}=2$

34. (B) $\sqrt[3]{4\dfrac{12}{125}}=\sqrt[3]{\dfrac{512}{125}}=\left(\dfrac{512}{125}\right)^{1/3}$

$=\dfrac{8}{5}=1\dfrac{3}{5}$

35. (A) $\sqrt{5+2\sqrt{6}}-\dfrac{1}{\sqrt{5+2\sqrt{6}}}$

$$=\frac{5+2\sqrt{6}-1}{\sqrt{5+2\sqrt{6}}}=\frac{4+2\sqrt{6}}{\sqrt{5+2\sqrt{6}}}$$

Let, $x=\dfrac{4+2\sqrt{6}}{\sqrt{5+2\sqrt{6}}}$

On squaring both sides,

$$x^2=\frac{16+24+16\sqrt{6}}{(5+2\sqrt{6})}=\frac{8(5+2\sqrt{6})}{(5+2\sqrt{6})}$$

or, $x^2=8$ or, $x=2\sqrt{2}$

36. (B) $1-\dfrac{1}{1+\sqrt{2}}+\dfrac{1}{1-\sqrt{2}}$

$$=1-\frac{1-\sqrt{2}}{1-2}+\frac{1+\sqrt{2}}{1-2}$$

$=1+1-\sqrt{2}-1-\sqrt{2}=1-2\sqrt{2}$

37. (A) $(2+\sqrt{2})+\dfrac{2-\sqrt{2}}{2^2-(\sqrt{2})^2}+\dfrac{\sqrt{2}+2}{(\sqrt{2})^2-2^2}$

$$=2+\sqrt{2}+\frac{2-\sqrt{2}}{2}+\frac{\sqrt{2}+2}{-2}$$

$$=\frac{2(2+\sqrt{2})+2-\sqrt{2}-\sqrt{2}-2}{2}$$

$$=\frac{4}{2}=2$$

38. (D) $\sqrt{0.02+\sqrt{0.0049}}=\sqrt{0.02+0.07}$

$=\sqrt{0.09}=0.3$

39. (D) $\sqrt{15612+\sqrt{154+\sqrt{225}}}$

$=\sqrt{15612+\sqrt{154+15}}$

$=\sqrt{15612+13}=\sqrt{15625}=125$

40. (B) $\sqrt{\dfrac{9}{16}}+(0.008)^{-2/3}-\left(\dfrac{7}{10}\right)^0$

$$=\frac{3}{4}+\frac{1}{(0.2)^2}-1=\frac{3}{4}+25-1$$

$=24\dfrac{3}{4}$

41. (D) $\dfrac{\sqrt{98}-\sqrt{72}+\sqrt{50}}{\sqrt{18}}$

$$=\frac{7\sqrt{2}-6\sqrt{2}+5\sqrt{2}}{3\sqrt{2}}=\frac{6\sqrt{2}}{3\sqrt{2}}=2$$

42. (A) $\sqrt{\dfrac{4}{3}}-\sqrt{\dfrac{3}{4}}=\dfrac{2}{\sqrt{3}}-\dfrac{\sqrt{3}}{2}=\dfrac{1}{2\sqrt{3}}$

43. (B) $\left[\sqrt{21+\sqrt{11+\sqrt{25}}}\right]\times[(3\times8-4)\div4]^{-1}$

$=\left[\sqrt{21+\sqrt{11+5}}\right]\times[(24-4)\div4]^{-1}$

$=\left[\sqrt{21+4}\right]\times\dfrac{1}{5}=5\times\dfrac{1}{5}=1$

Chapter 3 Indices and Surds

1. Indices

The base a raised to the power of n is equal to the multiplication of a, n times $a = a \times a \times ... \times a$ (n times). In a^n, a is the base and n is the indices.

1.1 Rules of Indices

S. No.	Laws or Rules	Description	Indices form
1.	Multiplication Rule	$a^m \times a^n = a^{m+n}$, where $m, n \in$ (+) ve integer and $a \neq 0$	$a^m \times a^n = a^{m+n}$
2.	Division Rule	$a^m \div a^n = a^{m-n}$, where $m, n \in$ (+) ve integer and $a \neq 0$	$a^m \div a^n = a^{m-n}$
3.	Power Rule	$(a^m)^n$, where $m, n \in$ (+) ve integer and $a \neq 0$	$(a^m)^n = a^{mn}$
4.	Power rules of product	$(a \times b \times c)^n$, where $n \in$ (+) ve integer and $a, b, c \neq 0$	$(a \times b \times c)^n = a^n b^n c^n$
5.	Inverse law	$a^m = \frac{1}{a^{-m}}$, where $m \in$ (+) ve integer and $a \neq 0$	$a^{-m} = \frac{1}{a^m}$
6.	Zero law of Index	a^0, where $a \neq 0$	$a^0 = 1$
7.	Even Indices law	$(-a)^m$, where $m \in$ (+) ve even integer and $a \neq 0$	$(-a)^m = a^m$
8.	Odd Indices law	$(-a)^m$, where $m \in$ (+) ve odd integer and $a \neq 0$	$(-a)^m = -a^m$
9.	**Indices law in Numerator and Denominator**	$\left(\frac{a}{b}\right)^m$, where $m \in$ (+) ve integer and $a, b \neq 0$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

1.2 Remember

(i) $a^* \neq 0$

(ii) $\left[\left(a^x\right)^y\right]^z \neq a^{x^{y+z}}$

(iii) $a^m \times a^n \neq a^{m \times n}$

(iv) $a^m + a^n \neq a^{m+n}$

(v) $\left(a^m\right)^n \neq a^{m^n}$

2. Surds

In mathematical language, to find out the root of any number is called surd. It is simply represented by $\sqrt[n]{x}$ or $x^{\frac{1}{n}}$. Where x is a positive rational number and n is a positive integer.

$\Rightarrow$ $\boxed{\sqrt[n]{x} = x^{\frac{1}{n}} = n^{\text{th}} \text{ surd}}$

2.1 Some Special type of Surds

S. No.	Surd's Name		Notation	Notation Example
1.	Square root	$\sqrt{}$ or $\sqrt[2]{}$	$\sqrt{x} = x^{1/2}$	$\sqrt{16} = (2^4)^{1/2} = 2^2 = 4$
2.	Cube root	$\sqrt[3]{}$	$\sqrt[3]{x} = x^{1/3}$	$\sqrt[3]{8} = 2$
3.	Fourth root	$\sqrt[4]{}$	$\sqrt[4]{x} = x^{1/4}$	$\sqrt[4]{16} = (2^4)^{1/4} = 2$

S.No.	Surd's Name	Symbols	Notation	Example
4.	Fifth root	$\sqrt[5]{\ }$	$\sqrt[5]{x} = x^{1/5}$	$\sqrt[5]{32} = (2^5)^{1/5} = 2$
	. . . .	. . . .	. . . .	. . . .
n.	n^{th} root	$\sqrt[n]{\ }$	$\sqrt[n]{x} = x^{1/n}$	$\sqrt[n]{2^n} = (2^n)^{\frac{1}{n}} = 2$

2.2 Classification of Surds

I. Like surds—The surds or radicals with same number of powers and radical numbers are called *Like surds*.

Examples : • $\sqrt{x}, 2\sqrt{x}, 5\sqrt{x}$

• $\sqrt[4]{xy}, 2\sqrt[4]{xy}, 5\sqrt[4]{xy}$

II. Unlike surds—The surds or radicals with different number of powers and radical numbers are called *Unlike surds*.

Examples : • $\sqrt{x}, \sqrt[3]{x}, \sqrt[5]{x}$

• $\sqrt[3]{x}, \sqrt[3]{y}, \sqrt[3]{z}$

• $\sqrt{x}, \sqrt[3]{y}, \sqrt[5]{xy}$

III. Conjugate surds—$a+\sqrt{b}$ and $a-\sqrt{b}$ are said to be *conjugate surds*.

2.3 Rationalisation

To change the denominator of a radical number into a rational number, multiply by rationalize factor to its numerator and denominator.

Example :
$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} \times \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}$$
$$= \frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}$$
$$= \frac{a+b+2\sqrt{ab}}{a-b}.$$

2.4 Important Rules on Surds

(i) $\sqrt[n]{a^n} = \left(a^n\right)^{\frac{1}{n}} = a$

(ii) $\sqrt[n]{ab} = (ab)^{1/n} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

(iii) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

(iv) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

(v) $\sqrt{ab^2} = b\sqrt{a}$

(vi) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(vii) $\sqrt[n]{a} = a^{1/n}$

(viii) $\sqrt{a} \times \sqrt{a} = a$

(ix) $\left(\sqrt{a}+\sqrt{b}\right)^2 = a+b+2\sqrt{ab}$

(Where, a and b be the positive rational numbers)

(x) $\left(\sqrt{a}-\sqrt{b}\right)^2 = a+b-2\sqrt{ab}$

(xi) $a-b = \left(\sqrt{a}-\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)$

2.5 Remember

(i) $\sqrt{a}+\sqrt{a^{-1}} \neq \sqrt{2a}$

(ii) $\left(\sqrt{\ }+\sqrt{\ }\right) = 2\sqrt{\ }$

(iii) $\left(\sqrt{2a}-\sqrt{a}\right) \neq \sqrt{a}$

(iv) $2\sqrt{a}-\sqrt{a} = \sqrt{a}$

(v) $\sqrt[2]{a}+\sqrt[3]{a} \neq \sqrt[5]{a}$

(vi) $-\sqrt[3]{a}-\sqrt[2]{a} \neq \sqrt{a}$

(vii) $\sqrt{2} = 1.414$

(viii) $\sqrt{3} = 1.732$

(ix) $\sqrt{5} = 2.238$

2.6 Type of Questions

Type 1 : If $a = m^{x+y}n^z, b = m^{y+z}n^x$ and $c = m^{z+x}n^y, a^{x-y}b^{y-z}c^{z-x} = 1$.

Example : $x = a^{m+n}b^l$, $y = a^{n+l}b^m$ and $z = a^{l+m}b^n$, $x^{m-n}y^{n-l}z^{l-m}$.

Sol : $x^{m-n}y^{n-l}z^{l-m} = 1$, since, m, n and l are cyclic.

Type 2 : $x^{a-b}x^{b-c}x^{c-a} = 1$

Example : Find $5^{3-2}5^{2-1}5^{1-3}$.

Sol : $5^{3-2}5^{2-1}5^{1-3} = 1$

Type 3 : $x^{a(b-c)} \times x^{b(c-a)} \times x^{c(a-b)} = 1$

Example : $10^{5(10-15)} \times 10^{10(15-5)} \times 10^{15(5-10)} = ?$

Sol : $10^{5(10-15)} \times 10^{10(15-5)} \times 10^{15(5-10)} = 1$

Type 4 : If $\left(\sqrt[3]{a}\right)^{x-1} = a, x = 4$.

Example : $\left[\sqrt[3]{64}\right]^{x-1} = 64$. Find x.

Sol. :

$$\left[\sqrt[3]{64}\right]^{x-1} = 64$$

We know that $\left(\sqrt[3]{a}\right)^{x-1} = a \Rightarrow x = 4$

Type 5 : $a \times a^2 \times a^3 \times \ldots\ldots\ldots \times a^n = a^{\frac{n(n+1)}{2}}$

Example : Find the value of $3 \times 3^2 \times 3^3 \times \ldots\ldots\ldots \times 3^{10}$.

Sol : $3 \times 3^2 \times 3^3 \times \ldots\ldots\ldots \times 3^{10} = a^{\frac{n(n+1)}{2}}$

$$= 3^{\frac{10(10+1)}{2}}$$

$$= 3^{5 \times 11} = 3^{55}$$

Type 6 : $a^{1^2} \times a^{2^2} \times a^{3^2} \times \ldots\ldots \times a^{n^2} = a^{\frac{n(n+1)(2n+1)}{6}}$

Example : Find the value of $3 \times 3^{2^2} \times 3^{3^2} \times 3^{4^2} \times 3^{5^2}$

Sol :

$$3 \times 3^{2^2} \times 3^{3^2} \times 3^{4^2} \times 3^{5^2} = 3^{1+2^2+3^2+4^2+5^2}$$

$$= 3^{\frac{5(5+1)(2\times5+1)}{6}}$$

$$= 3^{\frac{5\times6\times11}{6}} = 3^{55}$$

Type 7 : $a^{1^3} \times a^{2^3} \times a^{3^3} \times \ldots\ldots\ldots \times = a^{\left[\frac{n(n+1)}{2}\right]^2}$.

Example : Find the value of $10^{1^3} \times 10^{2^3} \times 10^{3^3} \times \ldots\ldots \times 10^{10^3}$.

Sol : $10^{1^3} \times 10^{2^3} \times 10^{3^3} \times \ldots\ldots\ldots \times 10^{10^3}$

$$= 10^{\left[\frac{10(10+1)}{2}\right]^2}$$

$$= 10^{(5\times11)^2}$$

$$= 10^{(55)^2}$$

$$= 10^{3025}$$

Type 8 : If $abc = 1$, then $\frac{1}{1+a+b^{-1}} + \frac{1}{1+a+b^{-1}} + \frac{1}{1+c+a^{-1}} = 1$.

Example : Find the solution of

$$\frac{1}{1+5+\left(\frac{1}{10}\right)^{-1}} + \frac{1}{1+\frac{1}{10}+2^{-1}} + \frac{1}{1+2+(5)^{-1}}$$

Sol : Here, $a = 5, b = \frac{1}{10}$ and $c = 2$ (Let)

$$abc = 5 \times \frac{1}{10} \times 2 = 1$$

We have,

$$\frac{1}{1+5+\left(\frac{1}{10}\right)^{-1}} + \frac{1}{1+\frac{1}{10}+2^{-1}} + \frac{1}{1+2+5^{-1}} = 1$$

Type 9: $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \times \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \times \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = 1$

Example : Find the value of $\left(\frac{a^x}{a^y}\right)^{\frac{1}{xy}} \times \left(\frac{a^y}{a^z}\right)^{\frac{1}{yz}} \times \left(\frac{a^z}{a^x}\right)^{\frac{1}{zx}}$.

Sol : $\left(\frac{a^x}{a^y}\right)^{\frac{1}{xy}} \times \left(\frac{a^y}{a^z}\right)^{\frac{1}{yz}} \times \left(\frac{a^z}{a^x}\right)^{\frac{1}{zx}} = 1$

[Since, x, y, z are in cyclic order]

Type 10 : $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} = 1$

Example : Solve : $\frac{1}{1+10^{y-x}+10^{z-x}} + \frac{1}{1+10^{x-y}+10^{z-y}} + \frac{1}{1+10^{y-z}+10^{x-z}}$.

Sol : According to $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} = 1$,

$$\frac{1}{1+10^{y-x}+10^{z-x}} + \frac{1}{1+10^{x-y}+10^{z-y}} + \frac{1}{1+10^{y-z}+10^{x-z}} = 1$$

Type 11: $\left(\frac{x^a}{x^b}\right)^{(a+b-c)} \times \left(\frac{x^b}{x^c}\right)^{(b+c-a)} \times \left(\frac{x^c}{x^a}\right)^{(c+a-b)} = 1$

Example : Find the solution of .

Sol : Here, $\left(\frac{5^2}{5^3}\right)^{(2+3-4)} \times \left(\frac{5^3}{5^4}\right)^{(3+4-2)} \times \left(\frac{5^4}{5^2}\right)^{(4+2-3)}$

$$\frac{1}{5} \times \left(\frac{1}{5}\right)^5 \times \left(\frac{1}{5^2}\right)^3 = 1$$

Type 12 : $\dfrac{a^{n-1}+a^n+a^{n+1}}{a^{n-1}-a^n-a^{n+1}} = \dfrac{a^n+a^{n+1}+a^{n+2}}{a^n-a^{n+1}-a^{n+2}} = \dfrac{1+a+a^2}{1-a-a^2}$

Example : Solve : $\dfrac{5^{n-1}+5^n+5^{n+1}}{5^{n-1}-5^n-5^{n+1}}$

Sol : $\dfrac{a^{n-1}+a^n+a^{n+1}}{a^{n-1}-a^n-a^{n+1}} = \dfrac{1+a+a^2}{1-a-a^2}$

Put $a = 5$,

$$\frac{1+5+5^2}{1-5-5^2} = \frac{31}{-29}$$

Type 13: If $a + b = 0$, then $\dfrac{\left(x+\frac{1}{y}\right)^a\left(x-\frac{1}{y}\right)^b}{\left(y+\frac{1}{x}\right)^a\left(y-\frac{1}{x}\right)^b} = 1$

Example : Solve : $\dfrac{\left(x+\frac{1}{y}\right)^2\left(x-\frac{1}{y}\right)^{-2}}{\left(y+\frac{1}{x}\right)^2\left(y-\frac{1}{x}\right)^{-2}}$.

Sol : Using Type 13, we have,

$a + b = 2 + (-2) = 0$

$$\Rightarrow \frac{\left(x+\frac{1}{y}\right)^2\left(x-\frac{1}{y}\right)^{-2}}{\left(y+\frac{1}{x}\right)^2\left(y-\frac{1}{x}\right)^{-2}} = 1$$

Type 14 : Divide a^{2n} by $(a + 1)$, the remainder will be 1.

Example : Find the remainder when 30^{50} is divided by 31.

Sol : Here, the power $2n = 50$ (even) and the base $a = 30$, So, the divisor $a + 1 = 30 + 1 = 31$ and the remainder will be 1.

Type 15 : Divide a^{2n+1} by $(a + 1)$, the remainder will be 1.

Example : Find the remainder when 30^{51} is divided by 31.

Sol : Here, the power $2n + 1 = 51$ (odd) and the base $a = 30$, So, the divisor $a + 1 = 30 + 1 = 31$ and the remainder will be 1.

Type 16 : Divide a^n by $(a - 1)$, the remainder will be 1.

Example : Find the remainder when 20^{21} is divided by 19.

Sol : Here, $a = 20$

and divisor $a - 1 = 20 - 1 = 19$

So, the remainder = 1

Type 17 : $[x^{(b-c)}]^{b+c} \times [x^{(c-a)}]^{c+a} \times [x^{(a-b)}]^{a+b} = 1$

Example : Find $(x^2)^8 \times (x^{-1})^7 \times (x^{-1})^9$.

Sol : Here, $(x^{5-3})^{(5+3)} \times (x^{3-4})^{(3+4)} \times (x^{4-5})^{(4+5)} = 1$

Type 18 : If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

Example : Simplify: $(x-y)^3 + (y-z)^3 + (z-x)^3$.

Sol :

Here, $A + B + C = (x-y) + (y-z) + (z-x) = 0$

$\Rightarrow A^3 + B^3 + C^3 = 3ABC$

$= 3(x-y)(y-z)(z-x)$

Type 19 : $\sqrt{a\sqrt{a\sqrt{a.....n}}} = a^{\frac{2^n-1}{2^n}}$.

Example : Solve : $\sqrt{16\sqrt{16\sqrt{16}}}$.

Sol :

$$\sqrt{a\sqrt{a\sqrt{a\ .....\ n}}} = a^{\frac{2^n-1}{2^n}}$$

$$\sqrt{16\sqrt{16\sqrt{16}}} = 16^{\frac{2^3-1}{2^3}}$$

$$= 16^{\frac{7}{8}}$$

$$= \left(2^4\right)^{\frac{7}{8}}$$

$$= 8\sqrt{2}$$

Type 20 : $\sqrt{a\sqrt{a\sqrt{a.....¥}}} = a$

Example : Solve: $\sqrt{5\sqrt{5\sqrt{5.....¥}}}$

Sol : $\sqrt{5\sqrt{5\sqrt{5.....¥}}} = a = 5$.

Type 21 : If $a_1 < a_2 < a_3 < ..... < a_n$, and $a_n - a_{n-1} = ...$ $= a_4 - a_3 = a_3 - a_2 = a_2 - a_1$, then

$$\left(\sqrt{a_n}-\sqrt{a_{n-1}}\right) < \left(\sqrt{a_{n-1}}-\sqrt{a_{n-2}}\right) < ..... < \left(\sqrt{a_2}-\sqrt{a_1}\right).$$

Example : Which one of the following is the largest?

$$\left(\sqrt{10}-\sqrt{9}\right), \left(\sqrt{9}-\sqrt{8}\right), \left(\sqrt{8}-\sqrt{7}\right), \left(\sqrt{7}-\sqrt{6}\right)$$

Sol : Here, $10 - 9 = 9 - 8 = 8 - 7 = 7 - 6 = 1$.

$$\left(\sqrt{10}-\sqrt{9}\right) < \left(\sqrt{9}-\sqrt{8}\right) < \left(\sqrt{8}-\sqrt{7}\right) < \left(\sqrt{7}-\sqrt{6}\right)$$

So, $\left(\sqrt{7}-\sqrt{6}\right)$ is the largest.

Type 22 : $\sqrt{\dfrac{a+\sqrt{b}}{a-\sqrt{b}}} = \dfrac{a+\sqrt{b}}{\sqrt{a^2-b}}$.

Example : Simplify : $\sqrt{\dfrac{5+2\sqrt{6}}{5-2\sqrt{6}}}$.

Sol :

$$\sqrt{\frac{a+\sqrt{b}}{a-\sqrt{b}}} = \frac{a+\sqrt{b}}{\sqrt{a^2-b}} \text{ से,}$$

$$\sqrt{\frac{5+2\sqrt{6}}{5-2\sqrt{6}}} = \sqrt{\frac{5+\sqrt{24}}{5-\sqrt{24}}}$$

$$= \frac{5+\sqrt{24}}{\sqrt{25-24}}$$

$$= 5+\sqrt{24}$$

$$= 5+2\sqrt{6}$$

Type 23 : $\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \frac{1}{\sqrt{a_3}+\sqrt{a_4}} \ldots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} = \frac{1}{k}\left(\sqrt{a_n}-\sqrt{a_1}\right)$

Where, $a_1 < a_2 < a_3 \ldots\ldots\ldots\ldots < a_n$

and $a_n - a_{n-1} = a_{n-1} - a_{n-2} = \ldots\ldots\ldots\ldots a_2 - a_1 = k$

Example : Simplify : $\frac{1}{\sqrt{11}+\sqrt{12}} + \frac{1}{\sqrt{12}+\sqrt{13}} + \frac{1}{\sqrt{13}+\sqrt{14}} + \frac{1}{\sqrt{14}+\sqrt{15}}$

Sol : $\frac{1}{\sqrt{11}+\sqrt{12}} + \frac{1}{\sqrt{12}+\sqrt{13}} + \frac{1}{\sqrt{13}+\sqrt{14}} + \frac{1}{\sqrt{14}+\sqrt{15}}$

Here, $a_1 < a_2 < \ldots\ldots\ldots\ldots a_n$

and $15-14 = 14-13 = 13-12 = 12-11 = k = 1$

Hence, required Solution is

$$= \frac{1}{k}\left(\sqrt{a_n}-\sqrt{a_1}\right) = \frac{1}{1}\left(\sqrt{15}-\sqrt{11}\right)$$

$$= \sqrt{15}-\sqrt{11}$$

Type 24 : $\frac{1}{\sqrt{a_1}-\sqrt{a_2}} - \frac{1}{\sqrt{a_2}-\sqrt{a_3}} - \frac{1}{\sqrt{a_3}-\sqrt{a_4}} - \ldots\ldots\ldots\ldots - \frac{1}{\sqrt{a_{n-1}}-\sqrt{a_n}}$

$$= \frac{1}{k}\left[\sqrt{a_1}-2\sqrt{a_3}-2\sqrt{a_4}\ldots\ldots\ldots\ldots-\sqrt{a_n}\right]$$

Where, $a_1 > a_2 > a_3 \ldots\ldots\ldots\ldots > a_n$

and $a_1 - a_2 = a_2 - a_3 = \ldots\ldots\ldots\ldots a_{n-1} - a_n = k$.

Example : Simplify : $\frac{1}{\sqrt{10}-\sqrt{9}} - \frac{1}{\sqrt{9}-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} - \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}}$.

Here, $10-9 = 9-8 = 8-7 = 7-6 = 6-5 = 1 = k$

Sol : Required result :

$$\frac{1}{k}\left[\sqrt{a_1}-2\sqrt{a_3}-2\sqrt{a_4}\ldots\ldots\ldots\ldots-\sqrt{a_n}\right]$$

$$= \frac{1}{1}\left[\sqrt{10}-2\sqrt{8}-2\sqrt{7}-2\sqrt{6}-\sqrt{5}\right]$$

$$= \sqrt{10}-2\sqrt{8}-2\sqrt{7}-2\sqrt{6}-\sqrt{5}$$

Type 25 : $\frac{1}{\sqrt{a_1}+\sqrt{a_2}} - \frac{1}{\sqrt{a_2}+\sqrt{a_3}} - \frac{1}{\sqrt{a_3}+\sqrt{a_4}} - \ldots\ldots\ldots\ldots - \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} = \frac{1}{k}\left[\sqrt{a_1}-2\sqrt{a_2}+\sqrt{a_n}\right]$

Where, $a_1 < a_2 < \ldots\ldots\ldots\ldots < a_n$

and $a_n - a_{n-1} = a_{n-1} - a_{n-2} = \ldots = a_2 - a_1 = k$ (constant)

Example : $\frac{1}{\sqrt{3}+\sqrt{4}} - \frac{1}{\sqrt{4}+\sqrt{5}} - \frac{1}{\sqrt{5}+\sqrt{6}} = ?$

Sol : $? = \sqrt{a_1}-2\sqrt{a_2}+\sqrt{a_n}$

$$= \sqrt{3}-2\sqrt{4}+\sqrt{6}$$

Type 26 : $\frac{1}{\sqrt{a_1}-\sqrt{a_2}} + \frac{1}{\sqrt{a_2}-\sqrt{a_3}} + \frac{1}{\sqrt{a_3}-\sqrt{a_4}} + \ldots\ldots\ldots\ldots + \frac{1}{\sqrt{a_{n-1}}-\sqrt{a_n}}$

$$= \frac{1}{k}\left[\sqrt{a_1}+2\left(\sqrt{a_2}+\sqrt{a_3}\ldots\ldots\ldots\ldots\sqrt{a_{n-1}}\right)+\sqrt{a_n}\right]$$

Where, $a_1 > a_2 > \ldots\ldots\ldots\ldots > a_n$

and $a_1 - a_2 = a_2 - a_3 = \ldots\ldots\ldots = a_{n-1} - a_n = k$ (constant)

Example : $\frac{1}{\sqrt{5}-\sqrt{4}} + \frac{1}{\sqrt{4}-\sqrt{3}} + \frac{1}{\sqrt{3}-\sqrt{2}} + \frac{1}{\sqrt{2}-\sqrt{1}} = ?$

Sol : $? = \sqrt{5}+2\left(\sqrt{4}+\sqrt{3}+\sqrt{2}\right)+\sqrt{1}$

Type 27 : $\frac{\sqrt{a}\pm\sqrt{b}}{\sqrt{a}\ m\sqrt{b}} = \frac{a+b\pm2\sqrt{ab}}{a-b}$

Example : $\frac{\sqrt{5+2\sqrt{6}}+\sqrt{5-2\sqrt{6}}}{\sqrt{5+2\sqrt{6}}-\sqrt{5-2\sqrt{6}}} = ?$

Sol : फ $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{a+b+2\sqrt{ab}}{a-b}$

$$= \frac{5+2\sqrt{6}+5-2\sqrt{6}+2\sqrt{5+2\sqrt{6}}\sqrt{5-2\sqrt{6}}}{5+2\sqrt{6}-\left(5-2\sqrt{6}\right)}$$

$$= \frac{10+2\sqrt{(5)^2-(2\sqrt{6})^2}}{4\sqrt{6}} = \frac{10+2\sqrt{25-24}}{4\sqrt{6}}$$

$$= \frac{12}{4\sqrt{6}} = \frac{3}{\sqrt{6}} = \frac{(\sqrt{3})^2}{\sqrt{2\times 3}} = \sqrt{\frac{3}{2}}$$

Type 28 : Method for arranging the radicals into ascending order

(i) First, make the powers of all the radicals equal.

(ii) Then, The largest number of radicals within the radical is the largest radical.

(iii) Finally, we can arrange the radicals in ascending order.

Example : Arrange the radicals $\sqrt[4]{10}, \sqrt[2]{5}, \sqrt[3]{15}$ in ascending order.

Sol : Here, the radicals have the powers 4, 2 and 3 respectively.

LCM of the powers $= 2 \times 2 \times 3 = 12$

(i) $\sqrt[4]{10} = 10^{\frac{1}{4}} = 10^{\frac{1}{4}\times\frac{3}{3}}$

$= \left(10^3\right)^{\frac{1}{12}} = \sqrt[12]{1000}$

(ii) $\sqrt[2]{5} = 5^{\frac{1}{2}} = 5^{\frac{1}{2}\times\frac{6}{6}} = \left(5^6\right)^{\frac{1}{12}} = \sqrt[12]{15625}$

(iii) $\sqrt[3]{15} = (15)^{\frac{1}{3}} = 15^{\frac{1}{3}\times\frac{4}{4}} = \left[(15)^4\right]^{\frac{1}{12}} = \sqrt[12]{15^4}$

$= \sqrt[12]{50625}$

Ascending order $\rightarrow \sqrt[4]{10}, \sqrt[2]{5}, \sqrt[3]{15}$.

Type 29 : $\sqrt{a+b\pm 2\sqrt{ab}} = \sqrt{a} \pm \sqrt{b}$

Example : Find the value of $\sqrt{8+2\sqrt{15}}$.

Sol : $\sqrt{8+2\sqrt{15}} = \sqrt{5+3+2\sqrt{5\times 3}}$

$= \sqrt{5}+\sqrt{3}$

Type 30 : If $x+\frac{1}{x} = a$, $\sqrt{x}+\frac{1}{\sqrt{x}} = \sqrt{\quad+2}$.

Example : If $x+\frac{1}{x} = 23$, find $\sqrt{x}+\frac{1}{\sqrt{x}}$.

Sol : $\sqrt{x}+\frac{1}{\sqrt{x}} = \sqrt{a+2}$

$= \sqrt{23+2} = 5$

Type 31 : If $x-\frac{1}{x} = a$, $\sqrt{x}-\frac{1}{\sqrt{x}} = \sqrt{a-2}$.

Example : If $x-\frac{1}{x} = 11$, then find $\sqrt{x}-\frac{1}{\sqrt{x}}$.

Sol : $\sqrt{x}-\frac{1}{\sqrt{x}} = \sqrt{a-2}$

$= \sqrt{11-2} = 3$

Type 32 : If $\sqrt{x}+\frac{1}{\sqrt{x}} = a$, $x+\frac{1}{x} = a^2-2$.

Example : If $\sqrt{x}+\frac{1}{\sqrt{x}} = 25$, find the value of $x+\frac{1}{x}$.

Sol : From, $x+\frac{1}{x} = a^2-2$,

$= 625-2$

$= 623$

Type 33 : If $\sqrt{x}-\frac{1}{\sqrt{x}} = a$, then $x+\frac{1}{x} = a^2+2$.

Example : If $\sqrt{x}-\frac{1}{\sqrt{x}} = 5$, then find the value of $x+\frac{1}{x}$.

Sol : From, $x+\frac{1}{x}$

$= a^2+2$

$= 5^2+2$

$= 27$

Type 35 : If $x^{1/3}+\frac{1}{x^{1/3}} = a$, then $x+\frac{1}{x} = a^3-3a$

Example : If $\sqrt[3]{x}+\frac{1}{\sqrt[3]{x}} = 5$, then find $x+\frac{1}{x}$.

Sol : From, $x+\frac{1}{x} = a^3-3a$

$= 5^3-3\times 5$

$= 125-15$

$= 110$

Type 36 : If $x^{1/3}-\frac{1}{x^{1/3}} = a$, then $x-\frac{1}{x} = a^3+3a$.

Example : If $\sqrt[3]{x}-\frac{1}{\sqrt[3]{x}} = 5$, then find $x-\frac{1}{x}$.

Sol : $x-\frac{1}{x} = a^3+3a$

$= 5^3+3\times 5$

$= 125+15 = 140$

Important Questions

1. Find the value of (?) in the following questions :

$(23)^{2.8} \times (23)^{7.2} \times (23)^{3.6} = (23)^{?}$

(A) 13.6 (B) 12.6
(C) 12.8 (D) 13.8

2. $(31)^{31} \times (31)^{-27} = ?$

(A) $(961)^2$ (B) $(31)^2$
(C) 29791 (D) 4

3. $1000^{12} \div 10^{30} = ?$

(A) $(1000)^2$ (B) 10
(C) $(100)^2$ (D) 100

4. $(3^?)^? = 19683$

(A) 3 (B) 9
(C) 4 (D) 8

5. If $m^n = 25$, $n^m = ?$

(A) 4 (B) 10
(C) 25 (D) 32

6. $\sqrt{(24)^4} + 224 = ? \times (20)^2$

(A) 20 (B) 4
(C) 2 (D) 16

7. Find the value of x in the following question :

$4^x \times 32^x \times 16^x = 2^{11}$

(A) 1 (B) 2
(C) 3 (D) 4

8. If $2^{2x-1} = \frac{1}{8^{x-3}}$, then $x = ?$

(A) – 2 (B) – 1
(C) 2 (D) 3

9. If $a^{2x+2} = 1$, where a is a (+ve) real number and $a \neq 1$, find x.

(A) – 2 (B) – 1
(C) 0 (D) 1

10. If $8^{x+1} = 64$, then x equals to :

(A) 1 (B) 9
(C) 27 (D) 81

11. If $(64)^{2x-5} = 4 \times (8)^{x-4}$, then the value of x :

(A) 2 (B) 11
(C) $\frac{10}{7}$ (D) $\frac{20}{9}$

12. If $\left(\frac{9}{4}\right)^x \cdot \left(\frac{8}{27}\right)^{x-1} = \frac{2}{3}$, then the value of x :

(A) 1 (B) 2
(C) 3 (D) 4

13. If $x = 7 - 4\sqrt{3}$, then find $x + \frac{1}{x}$.

(A) $3\sqrt{3}$ (B) $8\sqrt{3}$
(C) $14 + 8\sqrt{3}$ (D) 14

14. Find the largest number among the following : $\sqrt[3]{2}$, $\sqrt{3}$, $\sqrt[3]{5}$, 1.5

(A) 1.5 (B) $\sqrt{3}$
(C) $\sqrt[3]{2}$ (D) $\sqrt[3]{5}$

15. Find the largest number among the following : $2^{\frac{1}{3}}$, $3^{\frac{1}{3}}$, $8^{\frac{1}{8}}$, $9^{\frac{1}{9}}$

(A) $2^{\frac{1}{3}}$ (B) $3^{\frac{1}{3}}$
(C) $8^{\frac{1}{8}}$ (D) $9^{\frac{1}{9}}$

16. $\sqrt{0.01} + \sqrt{0.81} + \sqrt{1.21} + \sqrt{0.0009}$ equal to :

(A) 2.1 (B) 2.13
(C) 2.03 (D) 2.11

17. $2\sqrt[3]{32} - 2\sqrt[3]{4} + \sqrt[3]{500}$ equals to :

(A) $2\sqrt[3]{6}$ (B) $3\sqrt[3]{24}$
(C) $7\sqrt[3]{4}$ (D) 916

18. $\sqrt{\frac{0.49}{0.25}} + \sqrt{\frac{0.81}{0.36}}$ equals :

(A) $7\frac{9}{10}$ (B) $\frac{9}{10}$
(C) $2\frac{9}{10}$ (D) $4\frac{9}{10}$

19. If $\sqrt{2} = 1.4142$, then find $\frac{7}{3+\sqrt{2}}$.

(A) 1.5858 (B) 4.4142
(C) 3.4852 (D) 3.5858

20. Simplify: $\left[\sqrt[3]{\sqrt[6]{5^9}}\right]^4 \left[\sqrt[6]{\sqrt[3]{5^9}}\right]^4$

(A) 5^2 (B) 5^8
(C) 5^4 (D) 5^{12}

21. $\sqrt{8 - 2\sqrt{15}}$ equals to :

(A) $\sqrt{5} + \sqrt{3}$ (B) $5 - \sqrt{3}$
(C) $\sqrt{5} - \sqrt{3}$ (D) $3 - \sqrt{5}$

22. $\left[\left\{\left(-\frac{1}{2}\right)^2\right\}^2\right]^{-1}$ equals to :

(A) $\frac{1}{16}$ (B) 16
(C) $-\frac{1}{16}$ (D) –16

23. $\sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{7 + 4\sqrt{3}}}}$ equals to :

(A) 1 (B) 2
(C) 3 (D) 8

24. If $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $y = \frac{\sqrt{3}-1}{\sqrt{3}+1}$, then what will be the value of $x^2 + y^2$.

(A) 14 (B) 13
(C) 15 (D) 10

25. In $\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7} + b$, find a.

(A) $\frac{11}{3}$ (B) $-\frac{4}{3}$
(C) $\frac{4}{3}$ (D) $-\frac{4\sqrt{7}}{4}$

26. $\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{100}+\sqrt{99}} = ?$

(A) 1 (B) 9
(C) $\sqrt{99}$ (D) $\sqrt{99} - 1$

27. $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}}$ equals to :

(A) $2^{\frac{9}{2}}$ (B) $2^{\frac{11}{2}}$
(C) $2^{\frac{31}{32}}$ (D) $2^{\frac{39}{31}}$

28. $\frac{2^x + 2^{x-1}}{2^{x+1} - 2^x}$ equals to :

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$
(C) $2^{\frac{x-1}{x+1}}$ (D) 1

29. $\left[x^{b+c}\right]^{b-c} \times \left[x^{c+a}\right]^{c-a} \times \left[x^{a+b}\right]^{a-b}$ equals to :

(A) 0 (B) 1

(C) x (D) $\vec{a}\vec{r}^{\,2} + \vec{s}^{\,2} + \vec{s}^{\,2}$

30. If $x = \sqrt{\dfrac{\sqrt{5}+1}{\sqrt{5}-1}}$, then $x^2 - x - 1$.

(A) 0 (B) 1

(C) 2 (D) 5

31. $2^{32} - (2+1)(2^2+1)(2^4+1)(2^8+1) \times (2^{16}+1)$ equals to :

(A) 0 (B) 1

(C) 809436 (D) 809438

32. If a, b, c are real numbers, then $\sqrt{a^{-1}b}\,\sqrt{b^{-1}c}\,\sqrt{c^{-1}a} = ?$

(A) abc (B) $\dfrac{1}{abc}$

(C) $\sqrt{abc}$ (D) 1

33. $\left(\dfrac{a^x}{a^y}\right)^{\frac{1}{xy}} \left(\dfrac{a^y}{a^z}\right)^{\frac{1}{yz}} \times \left(\dfrac{a^z}{a^x}\right)^{\frac{1}{xz}} = ?$

(A) 1 (B) $x^{\frac{1}{abc}}$

(C) $x^{\frac{1}{ab+bc+ca}}$ (D) 0

34. $\dfrac{1}{1+a^{y-z}+a^{z-x}} + \dfrac{1}{1+a^{x-y}+a^{y-z}} + \dfrac{1}{1+a^{y-z}+a^{z-x}} = ?$

(A) 0 (B) 1

(C) a^{x-y-z} (D) a^{x+y+z}

35. $\dfrac{\sqrt{7}}{\sqrt{16+6\sqrt{7}} - \sqrt{16-6\sqrt{7}}}$ equals to :

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{3}$

(C) $\dfrac{1}{4}$ (D) $\dfrac{1}{5}$

36. $\dfrac{1}{\sqrt{100}-\sqrt{99}} - \dfrac{1}{\sqrt{99}-\sqrt{98}} + \dfrac{1}{\sqrt{98}-\sqrt{97}} \text{ ..... } - \dfrac{1}{\sqrt{3}-\sqrt{2}} + \dfrac{1}{\sqrt{2}-\sqrt{1}}$ equals to :

(A) 0 (B) 9

(C) 10 (D) 11

37. If $xyz = 1$, then $\dfrac{1}{1+x+y^{-1}} + \dfrac{1}{1+y+z^{-1}} + \dfrac{1}{1+z+x^{-1}}$ equals to :

(A) 0 (B) 1

(C) ab (D) $\dfrac{1}{ab}$

38. If $P^x = r^y = m$ and $r^w = p^z = n$, then which of the following statement is correct?

(A) $x\,w = y\,z$

(B) $x\,z = y\,w$

(C) $x + y = w + z$

(D) $x - y = w - z$

39. If $2^x = 3^y = 12^z$, then which of the following statement is correct?

(A) $xy = z(x + 2y)$

(B) $xy = 3z = (x + y)$

(C) $xyz = 3x + y$

(D) $xy = z(3x + y)$

40. $\dfrac{\sqrt[r]{9^{\left(r+\frac{1}{4}\right)}\sqrt{3.3^{-r}}}}{\sqrt{3^{2-r}}} = ?$

(A) 1 (B) $\dfrac{1}{3}$

(C) 3 (D) 9

41. $\dfrac{(243)^{\frac{n}{5}} \times 3^{2n+1}}{9^n.3^{n-1}} = ?$

(A) 1 (B) 9

(C) 3 (D) 3^n

42. $\dfrac{1}{3-\sqrt{8}} - \dfrac{1}{\sqrt{8}-\sqrt{7}} + \dfrac{1}{\sqrt{7}-\sqrt{6}} - \dfrac{1}{\sqrt{6}-\sqrt{5}}$ equals to :

(A) More than 3 (B) Between 2 and 3

(C) 0 (D) Between 0 and 1

Solutions

1. (A) $23^{2.8+7.2+3.6} = 2^{13.6}$

2. (A) $(31)^{31-27} = 31^4$

$= (31 \times 31)^2$

$= (961)^2$

3. (A) $(10^3)^{12} \div 10^{30} = 10^{36-30}$

$= 10^6 = (10^3)^2$

$= (1000)^2$

4. (A) $(3^?)^? = 19683$

3	19683
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

$= 3^9$

$(3^?)^? = (3^3)^3$

So, $? = 3$

5. (D) $m^n = 25 \quad m^n = 5^2$

$\Rightarrow n = 2$ and $m = 5$

$n^m = 2^5 = 32$

6. (C) $\sqrt{(24)^4} + 224 = ? \times (20)^2$

$\Rightarrow \dfrac{(24)^2 + 224}{(20)^2} = ?$

$\Rightarrow ? = \dfrac{576 + 224}{400}$

$\Rightarrow ? = \dfrac{800}{400}$

$? = 2$

7. (A) $4^x \times 32^x \times 16^x = 2^{11}$

$\Rightarrow (2^2)^x \times (2^5)^x \times (2^4)^x = 2^{11}$

$\Rightarrow 2^{2x+5x+4x} = 2^{11}$

$\Rightarrow 2^{11x} = 2^{11}$

$\Rightarrow 11x = 11$

$\Rightarrow x = 1$

8. (C) $2^{2x-1} = \dfrac{1}{8^{x-3}}$

$\Rightarrow 2^{2x-1} = \dfrac{1}{(2^3)^{2-3}}$

$\Rightarrow 2^{2x-1} = 2^{-3x+9}$

[Equating the powers of both sides]

$2x - 1 = -3x + 9$

$\Rightarrow 2x + 3x = 9 + 1$

$\Rightarrow 5x = 10$

$\Rightarrow x = 2$

9. (B) $a^{2x+2} = 1$

$\Rightarrow a^{2x+2} = 0$

$\Rightarrow 2x + 2 = a^0$

$\Rightarrow 2x = -2$

$\Rightarrow x = -1$

10. (A) $8^{x+1} = 64$

$8^{x+1} = 8^2$

$x + 1 = 2$

$x = 1$

11. (D) $(64)^{2x-5} = 4 \times (8)^{x-4}$

$(8^2)^{2x-5} = 4 \times (8)^{x-4}$

$\Rightarrow 8^{4x-10} \div 8^{x-4} = 4$

$\Rightarrow 8^{4x-10-x+4} = 4$

$\Rightarrow 8^{3x-6} = 4$

$\Rightarrow (2^3)^{3x-6} = 2^2$

$\Rightarrow 2^{9x-18} = 2^2$

Equating the powers of both sides,

$\Rightarrow 9x - 18 = 2$

$9x = 20$

$x = \frac{20}{9}$

12. (D) $\left(\frac{3^2}{2^2}\right)^x \times \left(\frac{2^3}{3^3}\right)^{x-1} = \frac{2}{3}$

$\Rightarrow \left(\frac{2}{3}\right)^{-2x} \times \left(\frac{2}{3}\right)^{3x-3} = \frac{2}{3}$

$\Rightarrow \left(\frac{2}{3}\right)^{-2x+3x-3} = \left(\frac{2}{3}\right)^1$

$\Rightarrow x - 3 = 1$

$\Rightarrow x = 4$

13. (D) $x = 7 - 4\sqrt{3}$

$\frac{1}{x} = 7 + 4\sqrt{3}$

Here, $x + \frac{1}{x}$

$= 7 - 4\sqrt{3} + 7 + 4\sqrt{3}$

$= 14$

14. (B) $\sqrt[3]{2}, \sqrt{3}, \sqrt[3]{5}, 1.5$

$2^{\frac{1}{3}}, 3^{\frac{1}{2}}, 5^{\frac{1}{3}}, 1.5$

L.C.M $= 2 \times 3 = 6$

(i) $2^{\frac{1}{3}} = \left(2^2\right)^{\frac{1}{6}} = (4)^{\frac{1}{6}}$

(ii) $3^{\frac{1}{3}} = \left(3^3\right)^{\frac{1}{6}} = (27)^{\frac{1}{6}}$

(iii) $5^{\frac{1}{3}} = \left(5^2\right)^{\frac{1}{6}} = (25)^{\frac{1}{6}}$

(iv) $1.5 = \frac{3}{2} = \left[\left(\frac{3}{2}\right)^6\right]^{\frac{1}{6}} = \left(\frac{729}{64}\right)^{\frac{1}{6}}$

The largest number $= 27^{\frac{1}{6}} = \sqrt{3}$

15. (B) L.C.M $= 9 \times 8 = 72$

(i) $2^{\frac{1}{3}} = \left(2^{24}\right)^{\frac{1}{72}}$

(ii) $3^{\frac{1}{3}} = \left(3^{24}\right)^{\frac{1}{72}}$

3	3, 8, 9
3	1, 8, 3
8	1, 8, 1
	1, 1, 1

(iii) $8^{\frac{1}{8}} = \left(2^3\right)^{\frac{1}{8}} = 2^{\frac{3}{8}} = \left(2^{27}\right)^{\frac{1}{72}}$

(iv) $9^{\frac{1}{9}} = \left(3^2\right)^{\frac{1}{9}} = 3^{\frac{2}{9}}$

$= \left(3^8\right)^{\frac{2}{72}} = \left(3^{16}\right)^{\frac{1}{72}}$

It is clear that $3^{\frac{1}{3}}$ is the largest number.

16. (B) $\sqrt{0.01} + \sqrt{0.81} + \sqrt{1.21} + \sqrt{0.0009}$

$= 0.1 + 0.9 + 1.1 + 0.03$

$= 2.13$

17. (C) $2\sqrt[3]{32} - 2\sqrt[3]{4} + \sqrt[3]{500}$

$= 2\sqrt[3]{8 \times 4} - 2\sqrt[3]{4} + \sqrt[3]{125 \times 4}$

$= 2 \times 2\sqrt[3]{4} - 2\sqrt[3]{4} + 5\sqrt[3]{4}$

$= 7\sqrt[3]{4}$

18. (C) $\sqrt{\frac{0.49}{0.25}} + \sqrt{\frac{0.81}{0.36}} = \sqrt{\frac{49}{25}} + \sqrt{\frac{81}{36}}$

$= \frac{7}{5} + \frac{9}{6}$

$= \frac{7}{5} + \frac{3}{2}$

$= \frac{14 + 15}{10}$

$= \frac{29}{10} = 2\frac{9}{10}$

19. (A) $\sqrt{2} = 1.4142$

$= \frac{7}{3 + \sqrt{2}} \times \frac{3 - \sqrt{2}}{3 - \sqrt{2}}$

$= \frac{7\left(3 - \sqrt{2}\right)}{9 - 2}$

$= 3 - \sqrt{2}$

$= 3 - 1.4142 = 1.5858$

20. (C) $\left[\sqrt[3]{\left(5^9\right)^{\frac{1}{6}}}\right]^4 \left[\sqrt[6]{\left(5^9\right)^{\frac{1}{3}}}\right]^4$

$= \left(\sqrt[3]{5^{\frac{3}{2}}}\right)^4 \left[\left(5^3\right)^{\frac{1}{6}}\right]^4$

$= \left[\left(\sqrt[3]{5^{\frac{3}{2}}}\right) \times \left(5^{\frac{1}{2}}\right)\right]^4$

$= \left[\left(5^{\frac{3}{2}}\right)^{\frac{1}{3}} \times \left(5^{\frac{1}{2}}\right)\right]^4$

$= (5^{\frac{1}{2} + \frac{1}{2}})^4 = 5^4$

21. (C) $\sqrt{8 - 2\sqrt{15}}$

$= \sqrt{5 + 3 - 2\sqrt{5 \times 3}}$

$= \sqrt{\left(\sqrt{5}\right)^2 + \left(\sqrt{3}\right)^2 - 2\sqrt{5}\sqrt{3}}$

$= \sqrt{\left(\sqrt{5} - \sqrt{3}\right)^2}$

$= \sqrt{5} - \sqrt{3}$

22. (B) $\left[\left\{\left(\frac{-1}{2}\right)^2\right\}^2\right]^{-1} = \left[\left(\frac{1}{4}\right)^2\right]^{-1}$

$= \left(\frac{1}{16}\right)^{-1} = 16$

23. (B) $\sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{7 + 4\sqrt{3}}}}$

$= \sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{\left(4 + 3 + 2\sqrt{4 \times 3}\right)}}}$

$= \sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{\left[\left(\sqrt{4} + \sqrt{3}\right)^2\right]}}}$

$= \sqrt{-\sqrt{3} + \sqrt{3 + 8\left(\sqrt{4} + \sqrt{3}\right)}}$

$(\because \sqrt{4} = 2)$

$= \sqrt{-\sqrt{3} + \sqrt{3 + 16 + 8\sqrt{3}}}$

$= \sqrt{-\sqrt{3} + \sqrt{\left(\sqrt{3}\right)^2 + (4)^2 + 2 \times 4\sqrt{3}}}$

$= \sqrt{-\sqrt{3} + \sqrt{\left(\sqrt{3} + 4\right)^2}}$

$= \sqrt{-\sqrt{3} + \sqrt{3} + 4}$

$= \sqrt{4} = 2$

24. (A) $x = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$

$= \frac{3 + 1 + 2\sqrt{3}}{2}$

$= \frac{4 + 2\sqrt{3}}{2}$

$= \frac{2\left(2 + \sqrt{3}\right)}{2}$

$= 2 + \sqrt{3}$

Similarly,

$$y = 2 - \sqrt{3}$$

$$x^2 + y^2 = \left(2+\sqrt{3}\right)^2 + \left(2-\sqrt{3}\right)^2$$

$$= 4 + 3 + 4\sqrt{3} + 4 + 3 - 4\sqrt{3}$$

$$= 14$$

25. (B) $a\sqrt{7} + b = \dfrac{\sqrt{7}-2}{\sqrt{7}+2} \times \dfrac{\sqrt{7}-2}{\sqrt{7}-2}$

$$= \frac{\left(\sqrt{7}-2\right)^2}{\left(\sqrt{7}\right)^2 - (2)^2}$$

$$= \frac{7+4-4\sqrt{7}}{7-4}$$

$$= \frac{11-4\sqrt{7}}{3}$$

$$b + a\sqrt{7} = \frac{11}{3} - \frac{4}{3}\sqrt{7}$$

[Equating the coefficients of a]

$$a = -\frac{4}{3}$$

26. (D) $\dfrac{1}{\sqrt{2}+1} + \dfrac{1}{\sqrt{3}+\sqrt{2}} + \ldots\ldots\ldots\ldots$

$$\frac{1}{\sqrt{100}+\sqrt{99}}$$

Here, $2 - 1 = 3 - 2$ .....

$= 100 - 99 = 1 = k$

Required solution: $\dfrac{1}{k}\left[\sqrt{a_n} - \sqrt{a_1}\right]$

$$= \frac{1}{1}\left[\sqrt{99} - 1\right]$$

$$= \sqrt{99} - 1$$

27. (C) $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}} = a^{\frac{2^n-1}{2^n}}$

$$= 2^{\frac{2^5-1}{2^5}}$$

$$= 2^{\frac{32-1}{32}}$$

$$= 2^{\frac{31}{32}}$$

28. (B) $\dfrac{2^x\left(1+2^{-1}\right)}{2^x\left(2^1-1\right)} = \dfrac{1+\frac{1}{2}}{1}$

$$= \frac{3}{2}$$

29. (B) $\left[x^{b+c}\right]^{b-c} + \left[x^{c+a}\right]^{c-a} \times \left(x^{a+b}\right)^{a-b} = 1$

30. (A) $x = \sqrt{\dfrac{\sqrt{5}+1}{\sqrt{5}-1}}$

$$= \frac{\sqrt{5}+\sqrt{1}}{\sqrt{(\sqrt{5})^2 - 1}}$$

$$= \frac{\sqrt{5}+1}{2}$$

$x^2 - x - 1$

$$= \left(\frac{\sqrt{5}+1}{2}\right)^2$$

$$- \frac{\sqrt{5}+1}{2} - 1$$

$$= \frac{5+1+2\sqrt{5}-2\sqrt{5}-2-4}{4}$$

$$= 0$$

31. (B) $2^{32} - (2+1)(2^2+1)(2^4+1)(2^8+1) \times (2^{16}+1)$

$$= 2^{32} - 1 \times (2+1)(2^2+1)(2^4+1) \times (2^8+1)(2^{16}+1)$$

$$= 2^{32} - (2-1)(2+1)(2^2+1)(2^4+1)(2^8+1)(2^{16}+1)$$

$$= 2^{32} - (2^2-1)(2^2+1)(2^4+1)(2^8+1)(2^{16}+1)$$

$$= 2^{32} - (2^4-1)(2^4+1)(2^8+1)(2^{16}+1)$$

$$= 2^{32} - (2^8-1)(2^8+1)(2^{16}+1)$$

$$= 2^{32} - (2^{16}-1)(2^{16}+1)$$

$$= 2^{32} - (2^{32}-1) = 1$$

32. (D) $\sqrt{a^{-1}b}.\sqrt{b^{-1}c}.\sqrt{c^{-1}a}$

$$= \sqrt{\frac{b}{a}} \times \sqrt{\frac{c}{b}} \times \sqrt{\frac{a}{c}} = 1$$

33. (A) Here x, y, z are in cyclic order.

So, Solution = 1

34. (B) Here x, y, z are in cyclic order.

So, Solution = 1

35. (A) $\dfrac{\sqrt{7}}{\sqrt{16+6\sqrt{7}} - \sqrt{16-6\sqrt{7}}}$

$$= \frac{\sqrt{7}}{\sqrt{9+7+2\sqrt{3^2\times 7}} - \sqrt{9+7-2\sqrt{3^2\times 7}}}$$

$$= \frac{\sqrt{7}}{\sqrt{(\sqrt{9})^2 + (\sqrt{7})^2 + 2\sqrt{9}\sqrt{7}} - \sqrt{(\sqrt{9})^2 + (\sqrt{7})^2 - 2\sqrt{9}\sqrt{7}}}$$

$$= \frac{\sqrt{7}}{\sqrt{(\sqrt{9}+\sqrt{7})^2} - \sqrt{(\sqrt{9}-\sqrt{7})^2}}$$

$$= \frac{\sqrt{7}}{\sqrt{9}+\sqrt{7}-(\sqrt{9}-\sqrt{7})}$$

$$= \frac{\sqrt{7}}{2\sqrt{7}} = \frac{1}{2}$$

36. (B) $\dfrac{1}{\sqrt{100}-\sqrt{99}} - \dfrac{1}{\sqrt{99}-\sqrt{98}} + \dfrac{1}{\sqrt{98}-\sqrt{97}} \ldots\ldots\ldots\ldots + \dfrac{1}{\sqrt{2}-\sqrt{1}}$.

Here, $100 - 99 = 99 - 98$ .....

$= 2 - 1 = 1$

Required solution

$\left(\sqrt{100} - 1\right) = 10 - 1 = 9$

37. (B) Here x, y, z are in cyclic order.

So, Solution = 1

38. (A) $p^x = m$ and $p^z = n$

$$\Rightarrow \quad p = m^{\frac{1}{x}} = n^{\frac{1}{z}}$$

$$\Rightarrow \quad m^{\frac{z}{x}} = n \qquad \ldots(1)$$

Similarly, $r = m^{\frac{1}{y}} = n^{\frac{1}{w}}$

$$\Rightarrow \quad m^{\frac{w}{y}} = n \qquad \ldots(2)$$

From eq. (1) and (2),

$$m^{\frac{z}{x}} = m^{\frac{w}{y}}$$

$$\Rightarrow \quad \frac{z}{x} = \frac{w}{y}$$

$$\Rightarrow \quad zy = xw$$

39. (A) Let, $2^x = 3^y = 12^z = k$

$$2 = k^{\frac{1}{x}}$$

$$3 = k^{\frac{1}{y}}$$

$$12 = k^{\frac{1}{z}}$$

$$(2 \times 2 \times 3) = k^{\frac{1}{z}}$$

$$k^{\frac{1}{x}} \times k^{\frac{1}{x}} \times k^{\frac{1}{y}} = k^{\frac{1}{z}}$$

$$k^{\left(\frac{2}{x}+\frac{1}{y}\right)} = k^{\frac{1}{z}}$$

$$\Rightarrow \quad \frac{2}{x} + \frac{1}{y} = \frac{1}{z}$$

$$\Rightarrow \quad 2yz + xz = xy$$

$$\Rightarrow \quad xy = z(x + 2y)$$

40. (D) $\dfrac{\sqrt[r]{9^{\left(r+\frac{1}{4}\right)}\sqrt{3.3^{-r}}}}{\sqrt{3^{2-r}}}$

$$= \frac{\sqrt[r]{3^{2r+\frac{1}{2}} \times 3^{\frac{1-r}{2}}}}{3^{\frac{2-r}{2}}}$$

$$= \sqrt[r]{3^{2r+\frac{1}{2}+\frac{1}{2}-\frac{r}{2}-1+\frac{r}{2}}}$$

$$= \sqrt[r]{3^{2r}} = \left(3^{2r}\right)^{\frac{1}{r}}$$

$$= 3^2 = 9$$

41. (B) $\dfrac{\left(3^5\right)^{\frac{n}{5}} \times 3^{2n+1}}{\left(3^2\right)^n .3^{n-1}}$

$$= 3^{n+2n+1-2n-n+1}$$

$$= 3^{n+2-n}$$

$$= 3^2 = 9$$

42. (D) $\dfrac{1}{\sqrt{9}-\sqrt{8}} - \dfrac{1}{\sqrt{8}-\sqrt{7}} + \dfrac{1}{\sqrt{7}-\sqrt{6}} - \dfrac{1}{\sqrt{6}-\sqrt{5}}$

Here $9-8 = 8-7$

$= 7-6 = 6-5 = 1$

$= \sqrt{9} - \sqrt{5}$

$= 3 - 2.236$

$= 0.764$

So, the value will be in between 0 and 1.

Chapter 4

Factors

1. Introduction

Arithmetic is a branch of mathematics for the study of numbers. For the study of images and figures, we use geometry term in mathematics. If we use variables or alphabet in place of numbers, its called *Algebra*.

In algebra, we use letters x, y, z, a, b, c ... etc which are called variables. With the help of letters, we can write the general form of formulas and rules. We can find the unknown values by using the variables in place of them and make ease of various problems of daily life.

On other hands, constant is a fixed value. For example, 5, 45, -7, -12, etc. The combination of variable and constant is called algebraic expression. $4x + 7$, $9x^2 + 4x + 5$, $2x - 1$ etc. are the algebraic expression. In the expression $4x + 7$, x is a variable with power 1, 4 is a coefficent of x and 7 is a constant value.

1.1 Types of Expressions

On the basis of number of terms, the expressions are classified into the follwoing ways—

I. **One-term expression**—All expressions with one term is called one-term expression.

Ex. : $4x$, $2xy$, $7xyz$, $9x^2y$, etc. Addition and subtraction are unavailable in these expressions.

II. **Two-term expression**—To form expressions, the terms are added.

Ex. : to form the expression $(2x + 3)$, $2x$ and 3 are added. Similarly, to form $(4x^2 - 9xy)$, $4x^2$ and $(-9xy)$ are added. So, all the expressions with two terms are called two-term expressions.

III. **Three-term expression**—All the expressions with three terms are called three-term expressions.

Ex. : In the expression $(4x^2 + 4x + 1)$, $4x^2$, $4x$ and 1 are the three terms by addition of them the expression is formed.

IV. **Like terms**—Like terms are terms that contain the same variables raised to the same power. Only the numerical coefficients are different.

Ex. : In the expression $4x^2 + 5x + 2xy - 9 - 2x + 4xy$, the terms $5x$ and $-2x$ are the like terms. The terms $2xy$ and $4xy$ are also the like terms. All those terms that are not like terms are called unlike terms. In the above expression, $4x^2$ and $2x$ are the unlike terms.

Additional Examples :

Ex. 1 : Find the variables and constants in the expression $5x^2y + 3x + 7$.

Sol. : Variables $\rightarrow$ x and y

Constants $\rightarrow$ 5, 3 and 7

Ex. 2 : Write the coefficients of x^2, y^5 and x^2y^5 in the expression $9x^2y^5$.

Sol. : In the expression $9x^2y^5$

(i) Coefficient of $x^2 = 9y^5 \times x^2$, *i.e.* $9y^5$.

(ii) Coefficient of $y^5 = 9x^2 \times y^5$, *i.e.* $9x^2$.

(iii) Coefficient of $x^2y^5 = 9 \times (x^2y^5)$, *i.e.* 9.

Ex. 3 : Which are the polynomials in the following algebraic expressions?

(i) $x + \frac{1}{x}$ (ii) $x^2 + \frac{1}{5}x + \sqrt{3}$

(iii) $3x^2 + \sqrt{5}x + \sqrt{x}$ (iv) $7 - x - x^2 - x^3$

Sol. : Option (ii) and (iv) are the polynomials.

In option (i), the 2nd term $\frac{1}{x}$ or x^{-1} is of negative power. Hence, it is not a polynomial.

In option (iii), the 3rd term $\sqrt{x}$ or $x^{\frac{1}{2}}$ is of fractional power. Hence, it is not a polynomial.

1.2 Power of Expressions

Degree of a term is the sum of powers of their variables. Example: The power of $5xy^2$ is 3 because the sum of the variables x and y $(1 + 2)$, *i.e.* 3.

Ex. : The powers of terms in the expression $x^3 + 5x^2 + 7x^2y + 4xy^3 + y^2$ are 3, 2, 3, 4 and 2 respectively. Out of which 4 is the maximum. Hence, the power of the expression is 4.

Note: If the coefficient of all the terms of a polynomial are zero, then it is called "zero polynomial". The power of zero polynomial is not defined.

2. Operations on Algebraic Expression

1. **Sum of Expression**—It can be shown as—

Ex. : If $x^3 = 1 + 7$, $3^3 = 1 + 7 + y$, and $4^3 = 1 + 7 + 4z$, then find $x + y + z$.

(A) 35 (B) 110

(C) 58 (D) 75

Sol. : (A) $x^3 = 1 + 7 \Rightarrow x = \sqrt[3]{8} = 2$

$3^3 = 1 + 7 + y \Rightarrow y = 19$

$4^3 = 1 + 7 + 4z \Rightarrow z = 14$

Hence, $x + y + z = 2 + 19 + 14 = 35$

2. **Subtraction of Expressions**—It can be shown as—

Ex. : Simplify- $(5x^2 + 12x - 84) - (3x^2 + 9x - 36)$.

(A) $5x^2 - 3x - 7$ (B) $2x^2 + 3x - 48$

(C) $-2x^2 - 3x + 48$ (D) $2x^2 - 3x + 36$

Sol. : (B) $(5x^2 + 12x - 84) - (3x^2 + 9x - 36)$

$= 5x^2 - 3x^2 + 12x - 9x - 84 + 36$

$= 2x^2 + 3x - 48$

3. **Multiplication of Expressions**—It can be shown as—

Ex. : Find the value of $(x - 3)$ $(x + 7)$.

(A) $x^2 - 21$ (B) $x^2 - 4x + 10$

(C) $x^2 + 4x - 21$ (D) $x^2 - 4x + 21$

Sol. : (C) $(x-3)\ (x+7) = x^2 - 3x + 7x - 21$

$= x^2 + 4x - 21$

4. Division of Expressions—It can be shown as—

Ex. 1. : Solve $(x^2 + 5x + 6) \div (x + 3)$.

(A) $x - 2$ (B) $x - 4$

(C) $x + 2$ (D) $x - 3$

Sol. : (C) $\frac{x^2+5x+6}{(x+3)} = \frac{(x+3)(x+2)}{(x+3)} = x + 2$

Ex. 2. : Solve $(x^3 + y^3) \div (x^2 - xy + y^2)$.

(A) $x - y$ (B) $x^2 - y^2$

(C) $x^2 + y^2$ (D) $x + y$

Sol. : (D) $\frac{\left(x^3+y^3\right)}{\left(x^2-xy+y^2\right)} = \frac{(x+y)\left(x^2-xy+y^2\right)}{\left(x^2-xy+y^2\right)} = x + y$

3. Factors of a Polynomial

Expressing a polynomial as a product of two or more monolithic polynomials is called a factor. For example, $x^2 - 4 = (x - 2)\ (x + 2)$, where $(x - 2)$ and $(x + 2)$ are the factors of the polynomial $x^2 - 4$.

3.1 Methods to Find Factors

I. $(A^2 - B^2)$ form—If any two polynomial are in the form of a and b, then factors of $a^2 - b^2 = (a - b)\ (a + b)$

Ex. : $4x^2 - 9y^2 = (2x)^2 - (3y)^2$

$= (2x - 3y)\ (2x + 3y)$

II. Factors of quadratic polynomial—If $ax^2 + bx + c$ is a quadratic polynomial, then at first find $a \times c = ac$ and then the factors of ac become $a + c = b$ or $a - c = b$.

Ex. : $x^2 + 5x + 6 = x^2 + 3x + 2x + 6$

$[\because 6 \times 1 = 6 = 3 \times 2$ and $3 + 2 = 5]$

$= x\ (x + 3) + 2(x + 3)$

$= (x + 3)\ (x + 2)$

III. Factors by making complete squares

Let, we have to find the factors of the expression $x^2 + 24x + 12$.

(i) At first, express x terms into $2x$ ( ) terms.

$= x^2 + 2(x)\ (12) + 12$

(ii) Add and subtract the square of ( ) terms in the $2x$ ( )

$= x^2 + 2x\ (12) + (12)^2 - (12)^2 + 12$

$= (x + 12)^2 - 132 \quad [\because (a + b)^2 = a^2 + 2ab + b^2]$

$= (x+12)^2 - \left(\sqrt{132}\right)^2$

$= (x+12)^2 - \left(\sqrt{132}\right)^2$

$= (x+12-\sqrt{132})\left(x+12+\sqrt{132}\right)$

$[\because a^2 - b^2 = (a - b)\ (a + b)]$

IV. Factors using remainder theorem—If a polynomial is of degree 3 or more than 3, then their factors are derived by remainder theorem.

Let we have to find the factors of the polynomial $x^3 + x^2 - x - 1$.

(i) At first, put $x = 1$,

$(1)^3 + (1)^2 - (1) - 1 = 0$

Hence, $(x - 1)$ is a factor.

(ii) Divide the polynomial by $(x - 1)$.

$x^2 + 2x + 1$ [$\because$ By theorem]

$x-1\overline{)\ x^3 + x^2 - x - 1}$

$x^3 - x^2$

$- \quad +$

$2x^2 - x - 1$

$2x^2 - 2x$

$- \quad +$

$x - 1$

$x - 1$

$- \quad +$

$\times \quad \times$

(iii) Quotient $= x^2 + 2x + 1$

$= (x + 1)^2$

Hence, $x^3 + x^2 - x - 1 = (x - 1)\ (x + 1)^2$

V. Cyclic Factors—If in a polynomial or algebraic expression, if we change the order of three variables a, b and c, the expression remains unchanged, then the expression is called cyclic expression.

Method to find out the cyclic factors—

(i) Write the terms of expression in the ascending or descending order of one variable.

(ii) Find the common factor after grouping.

(iii) Write the degrees of another variable of the remaining expression in the ascending or descending order.

(iv) Again, find the common factor from the remaining expression.

(v) Repeat the process until find all the factors of given expression.

Ex. : Factorize : $a^2b + b^2c + c^2a + a^2c + bc^2 + b^2a + 2abc$.

Sol. : Arrange the powers of a in descending order in the polynomial :

$a^2b + a^2c + ac^2 + ab^2 + 2abc + b^2c + bc^2$

$\Rightarrow a^2\ (b + c) + a\ (c^2 + b^2 + 2bc) + bc\ (b + c)$

$\Rightarrow a^2\ (b + c) + a\ (b + c)^2 + bc\ (b + c)$

$\Rightarrow (b + c)\ [a^2 + a\ (b + c) + bc]$

$\Rightarrow (b + c)\ (a^2 + ab + ac + bc)$

$\Rightarrow (b + c)\ [a\ (a + b) + c\ (a + b)]$

$\Rightarrow (b + c)\ (a + b)\ (a + c)$

IV. Factors of polynomials obtained from addition and subtraction of two polynomials :

(i) $a^3 - b^3 = (a - b)\ (a^2 + ab + b^2)$

(ii) $a^3 + b^3 = (a + b)\ (a^2 - ab + b^2)$

Ex. : Factorize : $27x^3 + 64y^3$.

Sol. : $27x^3 + 64y^3 = (3x)^3 + (4y)^3$

$= (3x + 4y)\ [(3x)^2 - 3x \times 4y + (4y)^2]$

$= (3x + 4y)\ [9x^2 - 12xy + 16y^2]$

Important Types

TYPE 1: Factors of polynomial $x^2 + bx + c$ will be :

$$x^2 + bx + c = (x + p)(x + q)$$

where, $p + q = b$ and $p \times q = c$

Example : Factorize: $x^2 + x - 30$.

Sol. : Let, $x^2 + x - 30 = (x + p)(x + q)$ ...(1)

where $p + q = 1$ ...(2)

$pq = -30$ ...(3)

$$p - q = \sqrt{(p + q)^2 - 4pq}$$

$$= \sqrt{1^2 - 4(-30)} = 11 \quad \text{... (4)}$$

On adding eq (2) and (4),

$$2p = 12 \Rightarrow p = 6$$

Put the value of p in eq (2),

$$q = 1 - 6 = -5$$

Put the value of p and q in eq (1),

$$x^2 + x - 30 = (x + 6)(x - 5)$$

TYPE 2: Factors of polynomial $ax^2 + bx + c$ will be :

$$ax^2 + bx + c = a\left[x - \left(\frac{-b + \sqrt{D}}{2a}\right)\right]\left[x - \left(\frac{-b - \sqrt{D}}{2a}\right)\right]$$

where, $D = b^2 - 4ac$

Example : Factorize $2x^2 - x - 1$.

Sol. : Here, $a = 2$, $b = -1$, and $c = -1$

$$\text{Required factors} = a\left[x - \left(\frac{-b + \sqrt{D}}{2a}\right)\right]\left[x - \left(\frac{-b - \sqrt{D}}{2a}\right)\right]$$

$$= 2\left[\text{अ.}\left\{\frac{.(.1). \sqrt{(.1)^2 . 4\ 2(.1)}}{2\ 2}\right\}\right]$$

$$\left[x - \left\{\frac{-(-1) - \sqrt{(-1)^2 - 4 \times 2(-1)}}{2 \times 2}\right\}\right]$$

$$= 2\left[x - \left\{\frac{1 + 3}{4}\right\}\right]\left[x - \left\{\frac{1 - 3}{4}\right\}\right]$$

$$= 2(x - 1)\left(x + \frac{1}{2}\right)$$

$$= 2\left[\frac{(x - 1)(2x + 1)}{2}\right]$$

$$= (x - 1)(2x + 1)$$

TYPE 3: Factors of polynomial $(a - b)$ will be :

$$a - b = \left(\sqrt{a}\right)^2 - \left(\sqrt{b}\right)^2$$

$$a - b = \left(\sqrt{a} - \sqrt{b}\right)\left(\sqrt{a} + \sqrt{b}\right)$$

Example : Factorize $2x - 3y$.

Sol. : $2x - 3y = \left(\sqrt{2x}\right)^2 - \left(\sqrt{3y}\right)^2$

$$= \left(\sqrt{2x} - \sqrt{3y}\right)\left(\sqrt{2x} + \sqrt{3y}\right)$$

TYPE 4: Factors of polynomial $(a - b)^3 + (b - c)^3 + (c - a)^3$:

$$(a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a)$$

Example : Factorize : $(x - y)^3 + (y - z)^3 + (z - x)^3$.

Sol. : Here, $x - y + y - z + z - x = 0$

So, if $A + B + C = 0$, then

$$A^3 + B^3 + C^3 = 3ABC$$

$$(x - y)^3 + (y - z)^3 + (z - x)^3 = 3(x - y)(y - z)(z - x)$$

TYPE 5: Factors of polynomial $a^4 - b^4$:

$$a^4 - b^4 = (a - b)(a + b)(a^2 + b^2)$$

Example : Factorize: $x^4 - 16y^4$.

Sol. : $x^4 - 16y^4 = (x)^4 - (2y)^4$

$$= (x - 2y)(x + 2y)[x^2 + (2y)^2]$$

$$= (x - 2y)(x + 2y)(x^2 + 4y^2)$$

TYPE 6: Factors of polynomial $a^6 - b^6$:

$$a^6 - b^6 = (a - b)(a + b)(a^2 - ab + b^2)(a^2 + ab + b^2)$$

Example : Factorize: $x^6 - \frac{1}{x^6}$.

Sol. : Since,

$$a^6 - b^6 = (a - b)(a + b)(a^2 - ab + b^2)(a^2 + ab + b^2)$$

$$x^6 - \frac{1}{x^6} = \left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)$$

$$\left(x^2 - x \times \frac{1}{x} + \frac{1}{x^2}\right)\left(x^2 + x \times \frac{1}{x} + \frac{1}{x^2}\right)$$

$$= \left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)\left(x^2 - 1 + \frac{1}{x^2}\right)\left(x^2 + 1 + \frac{1}{x^2}\right)$$

TYPE 7: Factors of polynomial $a^8 - b^8$:

$$a^8 - b^8 = (a - b)(a + b)(a^2 + b^2)(a^4 + b^4)$$

Example : Factorize $6561x^8 - 256y^8$.

Sol. : $6561x^8 - 256y^8 = (3x)^8 - (2y)^8$

$$= (3x - 2y)(3x + 2y)[(3x)^2 + (2y)^2][(3x)^4 + (2y)^4]$$

$$= (3x - 2y)(3x + 2y)(9x^2 + 4y^2)(81x^4 + 16y^4)$$

TYPE 8: Factors of polynomial $a^n - b^n$:

$$a^n - b^n = (a - b)(a + b)(a^2 + b^2)(a^4 + b^4) \ldots\ldots \left(a^{\frac{n}{2}} - b^{\frac{n}{2}}\right)$$

where, $n = 4, 8, 16$ .....etc

Example : Factorize $z^{16} - \frac{1}{z^{16}}$.

Sol. : Since, $x^n - y^n = (a - b)(a + b)(a^2 + b^2) \ldots\ldots \left(a^{\frac{n}{2}} - b^{\frac{n}{2}}\right)$

$$z^{16} - \frac{1}{z^{16}} = \left(z - \frac{1}{z}\right)\left(z + \frac{1}{z}\right)\left(z^2 + \frac{1}{z^2}\right)$$

$$\left(z^4 + \frac{1}{z^4}\right)\left(z^8 + \frac{1}{z^8}\right)$$

TYPE 9: Factorize $[a(b-c)]^3 + [b(c-a)]^3 + [c(a-b)]^3$:

$$[a(b-c)]^3 + [b(c-a)]^3 + [c(a-b)]^3$$
$$= 3abc\ (a-b)(b-c)(c-a)$$

Example : Factorize $[x(y-z)]^3 + [y(z-x)]^3 + [z(x-y)]^3$.

Sol. : Let, $A = x(y-z)$

$B = y(z-x)$

$C = z(x-y)$

$A + B + C = x(y-z) + y(z-x) + z(x-y)$

$= xy - xz + yz - yx + zx - zy = 0$

$\therefore\ A^3 + B^3 + C^3 = 3ABC$

Required factors $= 3[x(y-z)] \times [y(z-x)] \times [z(x-y)]$

$= 3xyz\,(x-y)(y-z)(z-x)$

TYPE 10 : Factors of polynomial $(a+b+c)^2 - (a-b-c)^2$:

$$(a+b+c)^2 - (a-b-c)^2 = 4a(b+c)$$

Example : Factorize $(2x + 3y + 4z)^2 - (2x - 3y + 4z)^2$.

Sol : Here, let $a = 2x$, $b = 3y$ and $c = 4z$

$(a+b+c)^2 - (a-b-c)^2 = 4a(b+c)$

$= 4 \times 2x\,(3y + 4z)$

$= 8x\,(3y + 4z)$

TYPE 11: Factors of polynomial $a^4 + \frac{1}{a^4} + 1$:

$$a^4 + \frac{1}{a^4} + 1 = \left(a + \frac{1}{a} \text{ n } 1\right)\left(a + \frac{1}{a} + 1\right)\left(a^2 + \frac{1}{a^2} - 1\right)$$

Example : Factorize $81x^4 + \frac{1}{81x^4} + 1$

Sol. : $81x^4 + \frac{1}{81x^4} + 1 = (3x)^4 + \frac{1}{(3x)^4} + 1$

$= \left(3x + \frac{1}{3x} + 1\right)\left(3x + \frac{1}{3x} - 1\right)\left[(3x)^2 + \left(\frac{1}{3x}\right)^2 \quad 1\right]$

$= \left(3x + \frac{1}{3x} + 1\right)\left(3x + \frac{1}{3x} - 1\right)\left(9x^2 + \frac{1}{9x^2} - 1\right)$

TYPE 12: Factors of polynomial $a^6 + \frac{1}{a^6}$:

$$a^6 + \frac{1}{a^6} = \left(a^2 + \frac{1}{a^2}\right)\left(a^4 + \frac{1}{a^4} - 1\right)$$

Example : Factorize $64x^6 + \frac{1}{64x^6}$

Sol. : $64x^6 + \frac{1}{64x^6} = (2x)^6 \cdot \frac{1}{(2x)^6}$

$= \left[(2x)^2 + \frac{1}{(2x)^2}\right]\left[(2x)^4 + \frac{1}{(2x)^4} - 1\right]$

$= \left(4x^2 + \frac{1}{4x^2}\right)\left(16x^4 + \frac{1}{16x^4} - 1\right)$

TYPE 13: Factors of polynomial $a^3 + b^3 + c^3 - 3abc$:

$$a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
$$= \frac{1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

Example : Factorize $x^3 - \frac{y^3}{8} - \frac{z^3}{27} - \frac{xyz}{2}$

Sol.: $= (x)^3 + \left(\frac{-y}{2}\right)^3 + \left(\frac{z}{3}\right)^3 - 3(x)\left(\frac{-y}{2}\right)\left(\frac{z}{3}\right)$

here, $a = x$, $b = -\frac{y}{2}$ and $c = -\frac{z}{3}$,

$= \left(x - \frac{y}{2} + \frac{z}{3}\right)$

$\left[x^2 + \left(\frac{-y}{2}\right)^2 + \left(\frac{z}{3}\right)^2 - x\left(\frac{-y}{2}\right) - \left(\frac{-y}{2}\right)\left(\frac{z}{3}\right) - \left(\frac{z}{3}\right)(x)\right]$

$= \left(x - \frac{y}{2} + \frac{z}{3}\right)\left(x^2 + \frac{y^2}{4} + \frac{z^2}{9} + \frac{xy}{2} + \frac{yz}{6} - \frac{zx}{3}\right)$

TYPE 14: Factors of polynomial $a^9 + b^9$:

$$a^9 + b^9 = (a+b)(a^2 - ab + b^2)(a^6 - a^3b^3 + b^6)$$

Example : Factorize $12x^9 + y^9$.

Sol. : $512x^9 + y^9 = (2x)^9 + y^9$

$= (2x + y)[(2x)^2 - 2xy + y^2][(2x)^6 - (2x)^3y^3 + y^6]$

$= (2x + y)(4x^2 - 2xy + y^2)(64x^6 - 8x^3y^3 + y^6)$

TYPE 15: Factors of polynomial $a^4 + b^4$:

$$a^4 + b^4 = \left(a^2 + b^2 - \sqrt{2}ab\right)\left(a^2 + b^2 + \sqrt{2}ab\right)$$

Example : Factorize $16x^4 + 625y^4$.

Sol. $16x^4 + 625y^4 = (2x)^4 + (5y)^4$

$= \left[(2x)^2 + (5y)^2 - \sqrt{2}\,(2x)(5y)\right]$

$\left[(2x)^2 + (5y)^2 + \sqrt{2}\,(2x)(5y)\right]$

$= \left(4x^2 + 25y^2 - 10\sqrt{2}xy\right)$

$\left(4x^2 + 25y^2 + 10\sqrt{2}xy\right)$

TYPE 16: Factors of polynomial $a^9 - b^9$:

$$a^9 - b^9 = (a-b)(a^2 + ab + b^2)(a^6 + a^3b^3 + b^6)$$

Example : Factorize $\frac{1}{64x^9} - y^9$

Sol : $\frac{1}{64x^9} - y^9 = \frac{1}{(2x)^9} - (y^3)^3$

$= \left(\frac{1}{2x} - y\right)\left[\left(\frac{1}{2x}\right)^2 + \frac{1}{2x}y + (y)^2\right]$

$\left[\left(\frac{1}{2x}\right)^6 + \left(\frac{1}{2x}\right)^3 y^3 + y^6\right]$

$= \left(\frac{1}{2x} - y\right)\left[\frac{1}{4x^2} + \frac{y}{2x} + y^2\right]$

$\left[\frac{1}{64x^6} + \frac{y^3}{8x^3} + y^6\right]$

TYPE 17: Factor the polynomial $a^{12} - b^{12}$:

$$a^{12} - b^{12} = (a + b)(a - b)(a^2 + b^2)(a^2 + ab + b^2)(a^2 - ab + b^2)(a^4 + a^2b^2 + b^4)$$

Example : Factorize $64x^{12} - 729y^{12}$.

Sol. : $64x^{12} - 729y^{12} = (\sqrt{2}x)^{12} - (\sqrt{3}y)^{12}$

$$= (\sqrt{2}x + \sqrt{3}y)(\sqrt{2}x - \sqrt{3}y)\left[(\sqrt{2}x)^2 + (\sqrt{3}y)^2\right]\left[(\sqrt{2}x)^2 + (\sqrt{2}x)(\sqrt{3}y) + (\sqrt{3}y)^2\right] \times \left[(\sqrt{2}x)^2 - (\sqrt{2}x)(\sqrt{3}y) + (\sqrt{3}y)^2\right]\left[(\sqrt{2}x)^4 + (\sqrt{2}x)^2(\sqrt{3}y)^2 + (\sqrt{3}y)^4\right]$$

$$= (\sqrt{2}x + \sqrt{3}y)(\sqrt{2}x - \sqrt{3}y)(2x^2 + 3y^2)\left(2x^2 + \sqrt{6}xy + 3y^2\right)\left(2x^2 - \sqrt{6}xy + 3y^2\right) \times \left(4x^4 + 6x^2y^2 + 9y^4\right)$$

TYPE 18: Factors of polynomial $a^4 + a^2b^2 + b^2$:

$$a^4 + a^2b^2 + b^4 = (a^2 + b^2 - ab)(a^2 + b^2 + ab)$$

Example : Factorize $16x^4 + 36x^2y^2 + 81y^4$.

Sol. : $16x^4 + 36x^2y^2 + 81y^4$

$$= (2x)^4 + (2x)^2(3y)^2 + (3y)^4$$

$$= [(2x)^2 + (3y)^2 - (2x)(3y)][(2x)^2 + (3y)^2 + (2x)(3y)]$$

$$= (4x^2 + 9y^2 - 6xy)(4x^2 + 9y^2 + 6xy)$$

TYPE 19: Factors of polynomial $a^4 - a^2b^2 + b^4$:

$$a^4 - a^2b^2 + b^4 = \left(a^2 + b^2 - \sqrt{3}ab\right)\left(a^2 + b^2 + \sqrt{3}ab\right)$$

Example : Factorize $16x^4 - 36x^2y^2 + 81y^4$.

Sol. : $16x^4 - 36x^2y^2 + 81y^4 = (2x)^4 - (2x)^2(3y)^2 + (3y)^4$

$$= \left[(2x)^2 + (3y)^2 - \sqrt{3}(2x)(3y)\right]\left[(2x)^2 + (3y)^2 + \sqrt{3}(2x)(3y)\right]$$

$$= \left[4x^2 + 9y^2 - 6\sqrt{3}xy\right]\left[4x^2 + 9y^2 + 6\sqrt{3}xy\right]$$

TYPE 20: Factor the polynomial $a^n - 1$:

$$a^n - 1 = (a - 1)(a^{n-1} + a^{n-2} + \ldots\ldots\ldots + a^2 + a + 1)$$

Example : Factorize $x^5 - 1$.

Sol. : $x^5 - 1 = (x - 1)(x^4 + x^3 + x^2 + x + 1)$

Important Formulas :

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)^3 = a^3 + 3ab(a + b) + b^3$
- $(a - b)^3 = a^3 - 3ab(a - b) - b^3$
- $a^2 - b^2 = (a - b)(a + b)$
- $a^3 - b^3 = (a - b)(a^2 + ab + b^2) = (a - b)^3 + 3ab(a - b)$
- $a^3 + b^3 = (a + b)(a^2 - ab + b^2) = (a + b)^3 - 3ab(a + b)$
- $(a + b)^2 = (a - b)^2 + 4ab$
- $(a - b)^2 = (a + b)^2 - 4ab$
- $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$
- $\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$
- $\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$
- $\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$
- $\left(x + \frac{1}{x}\right)^2 = \left(x - \frac{1}{x}\right)^2 + 4$
- $\left(x - \frac{1}{x}\right)^2 = \left(x + \frac{1}{x}\right)^2 - 4$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Important Questions

1. If $a + b + 1 = 0$, then what will be the value of $a^3 + b^3 + 1 - 3ab$? **(NCERT)**

(A) -1 (B) 1

(C) 3 (D) 0

2. If $a - b = 3$, $b - c = 5$ and $c - a = 1$, then what will be the value of $\frac{a^3 + b^3 + c^3 - 3abc}{a + b + c}$?

(A) 10.5 (B) 15.5

(C) 17.5 (D) 20.5

3. If $x + \frac{1}{x} = 3$, then what will be the value of $x^5 + \frac{1}{x^5}$?

(A) 113 (B) 129

(C) 123 (D) 126

4. If $x + y = a$ and $xy = b^2$, then find the value of $x^3 - x^2y - xy^2 + y^3$ in the form of a and b :

(A) $(a^3 + 4b^2)a$ (B) $a^3 - 3b^2$

(C) $a^3 - 4b^2a$ (D) $a^3 + 3b^2$

5. If $x + \frac{1}{x} = 5$, then find $\frac{2x}{3x^2 - 5x + 3}$. **(NCERT)**

(A) 5 (B) $\frac{1}{5}$

(C) 4 (D) $\frac{1}{4}$

6. If $a^2 + b^2 + c^2 = 2(a - b - c) - 3$, then find $2a - 3b + 4c$.

(A) 3 (B) 1

(C) 2 (D) 4

7. If $x^2 + y^2 - 4x - 4y + 8 = 0$, then find the value of $x - y$.

(A) 4 (B) -4

(C) 0 (D) 8

8. If $(a-1)^2 + (b+2)^2 + (c+1)^2 = 0$, then the value of $2a - 3b + 7c$ will be :

(A) 12 (B) 3

(C) -11 (D) 1

9. If $x + \frac{1}{4x} = \frac{3}{2}$, then the value of $8x^3 + \frac{1}{8x^3}$ will be **(NCERT)**

(A) 18 (B) 36

(C) 24 (D) 16

10. If $(3a+1)^2 + (b-1)^2 + (2c-3)^2 = 0$, then the value of $(3a + b + 2c)$ is :

(A) 3 (B) -1

(C) 2 (D) 5

11. If $\frac{a}{3} = \frac{b}{2}$, then what will be the value of $\frac{2a+3b}{3a-2b}$?

(A) $\frac{12}{5}$ (B) $\frac{5}{12}$

(C) 1 (D) $\frac{12}{7}$

12. If $4x = 18y$, find $\left(\frac{x}{y} - 1\right)$.

(A) $\frac{1}{3}$ (B) $\frac{7}{2}$

(C) $\frac{2}{3}$ (D) $\frac{3}{2}$

13. If $x = 12$ and $y = 4$, then find $(x+y)^{\frac{x}{y}}$: **(NCERT)**

(A) 4096 (B) 3066

(C) 3616 (D) 4226

14. If $\text{ग}^{\text{ल}}$. $\frac{1}{\text{ग}^{\text{ल}}} = 727$, then find $\text{ग}^3 \quad \frac{1}{\text{ग}^3}$:

(A) 125 (B) 140

(C) 155 (D) 170

15. If $a^3 - b^3 - c^3 - 3abc = 0$, then

(A) $a = b = c$ (B) $a + b + c = 0$

(C) $a + c = b$ (D) $a = b + c$

16. If $x + y + z = 0$, then the expression $\frac{xyz}{(x+y)(y+z)(z+x)}$ equals to : **(NCERT)**

(A) -1 (B) 1

(C) $xy + yz + zx$ (D) None of these

17. If a, b and c be the real numbers and $a + b + c = 0$, then find $a^3 + b^3 + c^3$.

(A) 1 (B) 0

(C) $3abc$ (D) $a^2 + b^2 + c^2$

18. If $x = \frac{4ab}{a+b}(a \neq b)$, then find the value of $\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b}$?

(A) a (B) $2ab$

(C) b (D) 2

19. If $x + \frac{1}{x} = 2$ and x is real, then what will be the value of $x^{17} + \frac{1}{x^{19}}$?

(A) 1 (B) 0

(C) 2 (D) -2

20. If $x + y + z = 0$, then find $\frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy}$.

(A) $(xyz)^2$ (B) $x^2 + y^2 + z^2$

(C) 9 (D) 3

21. If $x = b + c - 2a$, $y = c + a - 2b$, $z = a + b - 2c$, what will be the value of $x^2 + y^2 - z^2 + 2xy$? **(NCERT)**

(A) 0 (B) $a + b + c$

(C) $a - b + c$ (D) $a + b - c$

22. If $a + b + c = 0$, what will be the value of product of

$$\left(\frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}\right) \times \left(\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}\right) ?$$

(A) 8 (B) -3

(C) 9 (D) 0

23. If $a + b + c = 8$, then find the value of $(a-4)^3 + (b-3)^3 + (c-1)^3 - 3(a-4)(b-3)(c-1)$.

(A) 2 (B) 4

(C) 1 (D) 0

24. If $\left(x + \frac{1}{x}\right)^2 = 3$, then find the value of $x^{206} + x^{200} + x^{90} + x^{84} + x^{18} + x^{12} + x^6 + 1$.

(A) 0 (B) 1

(C) 84 (D) 206

25. If $\frac{3-5x}{x} + \frac{3-5y}{y} + \frac{3-5z}{z} = 0$, then what will be the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$?

(A) -5 (B) 5

(C) 2 (D) 3

26. If $3x + \frac{1}{2x} = 5$, then what will be the value of $8x^3 + \frac{1}{27x^3}$?

(A) $118\frac{1}{2}$ (B) $30\frac{10}{27}$

(C) 0 (D) 1

27. If $xy(x+y) = 1$, then find the value of $\frac{1}{x^3y^3} - x^3 - y^3$. **(NCERT)**

(A) 0 (B) 1

(C) 3 (D) -2

28. If $a^4 + b^4 = a^2b^2$, then find the value of $a^6 + b^6$.

(A) 0 (B) 1

(C) -1 (D) 2

29. If $x^4 + x^{-4} = 322$, then $(x - x^{-1})$ will be:

(A) 3 (B) 4

(C) 0 (D) -1

30. If $x \times y = 2$ and $x^2 + y^2 = 20$, then the value of $(x+y)^2$ will be : **(NCERT)**

(A) 24 (B) 42

(C) 12 (D) 21

31. If $\frac{x}{2x^2 + 5x + 2} = \frac{1}{6}$, find $\left(x + \frac{1}{x}\right)$.

(A) 2 (B) $\frac{1}{2}$

(C) $-\frac{1}{2}$ (D) -2

32. If $a + b + c = 0$, then find the value of $\frac{1}{(a+b)(b+c)} + \frac{1}{(a+c)(b+a)} + \frac{1}{(c+a)(c+b)}$. **(NCERT)**

(A) 1 (B) 0

(C) -1 (D) -2

33. The value of $\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} + \frac{(c-a)^2}{(a-b)(b-c)}$ is :

(A) 0 (B) 3

(C) $\frac{1}{3}$ (D) 2

34. If $\frac{2x-y}{x+2y} = \frac{1}{2}$, then find $\frac{3x-y}{3x+y}$. **(NCERT)**

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$

(C) $\frac{4}{5}$ (D) 1

35. If $\frac{p}{q}-\frac{q}{p}=4$, then find $\frac{p^3}{q^3}+\frac{q^3}{p^3}$.

(A) $43\sqrt{5}$ (B) $34\sqrt{5}$

(C) $4\sqrt{5}$ (D) $\sqrt{5}$

Direction (Q. No. 36 to 53)

Factorize the following expressions :

36. $ax + bx + ay + by$

(A) $(a + b)(x + y)$

(B) $(a - b)(x - y)$

(C) $(a + b)(x - y)$

(D) $(a - b)(x + y)$

37. $x^2 + xy + xz + yz$

(A) $(x + y)(x + z)$

(B) $(x - y)(x - z)$

(C) $(x - y)(x + z)$

(D) $(x + y)(x - z)$

38. $ab(x^2 + y^2) + xy(a^2 + b^2)$

(A) $(ax - by)(ax + by)$

(B) $(ax + by)(bx + ay)$

(C) $(a + b)(x + y)$

(D) $(a - b)(x - y)$

39. $\overline{x^2} + \left(\cdot + \frac{1}{\cdot}\right)x + 1$

(A) $(x + a)(x - a)$

(B) $(x + a)\left(x - \frac{1}{a}\right)$

(C) $(x + a)\left(x + \frac{1}{a}\right)$

(D) None of these

40. $x^2 + 4x + 4$

(A) $(x - 2)(x + 2)$

(B) $(x - 2)(x - 2)$

(C) $(x + 2)^2$

(D) $(x - a)^2$

41. $1 - 8x + 16x^2$ **(NCERT)**

(A) $(1 - 4x)(1 + 4x)$

(B) $(2 - x)(2 + x)$

(C) $(1 + 4x)^2$

(D) $(1 - 4x)^2$

42. $a^2 - 50ab + 625b^2$

(A) $(a - 25b)(a + 25b)$

(B) $(5a + b)(5a - b)$

(C) $(a + 25b)^2$

(D) $(a - 25b)^2$

43. $144x^2 + 264xy + 121y^2$

(A) $(12x + 11y)(x + y)$

(B) $(x - y)(11x + 12y)$

(C) $(12x - 11y)^2$

(D) $(12x + 11y)^2$

44. $25x^2 - 10x + 1 - 36y^2$ **(NCERT)**

(A) $(5x + 6y - 1)(5x - 6y - 1)$

(B) $(25x - 6y)(25 + 6y)$

(C) $(25x - 6y)^2$

(D) $(25x + 6y)^2$

45. $\left(x + \frac{1}{x}\right)^3 - 2x - \frac{2}{x}$

(A) $\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

(B) $\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right)$

(C) $\left(x - \frac{1}{x}\right)\left(x^2 - \frac{1}{x^2}\right)$

(D) None of these

46. $6\sqrt{3}x^2 + 47x + 5\sqrt{3}$

(A) $(2x - 7)\left(3\sqrt{3}x - 5\right)$

(B) $(2x + 5\sqrt{3})\left(3\sqrt{3}x + 1\right)$

(C) $(2x + 7)\left(3\sqrt{3}x + 5\right)$

(D) None of these

47. $\frac{25x^2}{18} - \frac{49y^2}{32}$ **(NCERT)**

(A) $\frac{1}{2}\left(\frac{5x}{9} - \frac{7y}{4}\right)\left(\frac{5x}{9} + \frac{7y}{4}\right)$

(B) $\frac{1}{2}\left(\frac{5x}{9} - \frac{7y}{4}\right)\left(\frac{5x}{9} - \frac{7y}{4}\right)$

(C) $\left(\frac{5x}{9} + \frac{7y}{4}\right)^2$

(D) None of these

48. $9x^2 - 30xy + 25y^2$

(A) $(3x - 5y)^2$

(B) $(3x + 5y)^2$

(C) $(3x - 5y)(3x + 5y)$

(D) None of these

49. $36 - 12k + k^2$ **(NCERT)**

(A) $(6 + k)^2$

(B) $(6 - k)^2$

(C) $(6 + k)(6 - k)$

(D) None of these

50. $12x^3 - 14x^2 - 10x$

(A) $2x(3x - 5)(2x + 1)$

(B) $(3x - 5)(2x + 1)$

(C) $2x(3x + 5)(2x + 1)$

(D) $2x(3x - 5)(2x - 5)$

51. $15x^4 + 3x^2 - 18$ **(NCERT)**

(A) $3(5x^2 + 6)(x + 1)(x - 1)$

(B) $3(5x^2 + 6)(x - 1)^2$

(C) $3(5x^2 + 6)(x + 1)^2$

(D) $(5x^2 + 6)(x - 1)$

52. $8x^3 + \frac{1}{8x^3} + 2x + \frac{1}{2x}$

(A) $\left(2x + \frac{1}{2x}\right)\left(2x - \frac{1}{2x}\right)$

(B) $\left(2x + \frac{1}{2x}\right)^2$

(C) $\left(2x + \frac{1}{2x}\right)\left(4x^2 + \frac{1}{4x^2}\right)$

(D) None of these

53. $27x^3 + y^3 + z^3 - 9xyz$

(A) $(3x - y - z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$

(B) $(3x + y - z)(9x^2 - y^2 - z^2 + 3xy + yz + 3xz)$

(C) $(3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$

(D) None of these

54. If $x - a$, is a factor of $x^3 - a^2x + x + 3$, then find the value of a.

(A) 2 (B) -2

(C) -3 (D) 3

55. If $(x - a)$ is a factor of $x^3 - mx^2 - 2nax + na^2$, then **(NCERT)**

(A) $a = m + n$ (B) $a = m - n$

(C) $a = mn$ (D) $a = m \div n$

56. Factorize $x^3 - 7x + 6$.

(A) $(x + 1)(x + 2)(x + 3)$

(B) $(x - 1)(x - 2)(x - 3)$

(C) $(x - 1)(x + 3)(x - 2)$

(D) $(x + 1)(x + 2)(x - 3)$

57. What number be added to or subtracted from $81x^2 + 4y^2$ for perfect square?

(A) $\pm 36xy$ (B) xy

(C) $9xy$ (D) $\pm 3xy$

58. Factorize the algebraic expression $m^4 - 3m^3 - 2m^2 - 3m + 1$. **(NCERT)**

(A) $(m^2 + m + 1)m$

(B) $(m^2 - 4m)(m^2 + 4m)$

(C) $(m^2 - 4m + 1)(m^2 + m + 1)$

(D) $(m + 1)(m - 1)$

59. $(x - y)$ equals to :

(A) $\left[\sqrt[3]{x} - \sqrt[3]{y}\right]\left[x^{\frac{2}{3}} + (xy)^{\frac{1}{3}} + y^{\frac{2}{3}}\right]$

(B) $\left[\sqrt[3]{x} + \sqrt[3]{y}\right]\left[x^{\frac{2}{3}} + (xy)^{\frac{1}{3}} + y^{\frac{2}{3}}\right]$

(C) $\left[\sqrt[3]{x} + \sqrt[3]{y}\right]\left[x^{\frac{2}{3}} + (xy)^{\frac{1}{3}} + y^{\frac{1}{3}}\right]$

(D) $\left[\sqrt[3]{x} + \sqrt[3]{y}\right]\left[x^{\frac{2}{3}} + y^{\frac{2}{3}}\right]$

60. $x^2 + \frac{1}{81} - \frac{2}{9}x$ has a factor:

(A) $\left(x - \frac{1}{9}\right)^2$ (B) $\left(x + \frac{1}{9}\right)$

(C) $\left(x + \frac{1}{3}\right)$ (D) $\left(x - \frac{1}{3}\right)$

61. A factor of $y^4 + 3y^2 - 28$ is:

(A) $y - 2$ (B) $y^2 + 2$

(C) $y^2 - 2$ (D) $y^2 - 7$

62. A factor of $4 - 2ab - (a^2 + b^2)$ is:

(A) $2 + a - b$ (B) $2 - a + b$

(C) $2 - a - b$ (D) $a + b - 2$

63. If expression $2x^3 - px^2 + x + q$ has two factors $(x + 1)$ and $(x + 2)$, then find p and q:

(A) $-5, -2$ (B) 7, 8

(C) 7, 10 (D) 15, 12

64. $(x + 1)$ is a factor of $x^4 + 9x^3 + 7x^2 + 9ax + 5a^2$, then find the value of a.

(A) $\frac{9 \pm \sqrt{1001}}{5}$ (B) $\frac{9 \pm \sqrt{11}}{10}$

(C) $\frac{9 \pm \sqrt{101}}{10}$ (D) $9 \pm \sqrt{21}$

65. If $(x + a)$, is a factor of the polynomials $x^2 + px + q$ and $x^2 + mx + n$, then:

(NCERT)

(A) $a = \frac{n - q}{m - p}$ (B) $a = \frac{m - p}{n - q}$

(C) $a = \frac{m + p}{n + q}$ (D) $a = \frac{n + q}{m + p}$

SOLUTIONS

1. (D) Here, $a + b + 1 = 0$ ...(1)

$a^3 + b^3 + 1 - 3ab$

$= a^3 + b^3 + 1^3 - 3ab$

$= 3ab - 3ab = 0$

2. (C) $a - b = 3$ ...(1)

$b - c = 5$ ...(2)

$c - a = 1$ ...(3)

On adding the squares of eq.(1), (2) and (3),

$(a - b)^2 + (b - c)^2 + (c - a)^2$

$= 3^2 + 5^2 + 1^2$

$\Rightarrow a^2 - 2ab + b^2 + b^2 - 2bc + c^2 + c^2 + a^2 - 2ac = 35$

$\Rightarrow 2(a^2 + b^2 + c^2 - ab - bc - ca) = 35$

$\Rightarrow a^2 + b^2 + c^2 - ab - bc - ca = \frac{35}{2}$ ...(4)

$\Rightarrow \frac{a^3 + b^3 + c^3 - 3abc}{a + b + c}$

$= \frac{(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)}{(a + b + c)}$

$= a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{35}{2}$ [$\because$ from eq.(4)]

$= 17.5$

3. (C) Here, $x + \frac{1}{x} = 3$

$x^5 + \frac{1}{x^5} = 3^5 - 5 \times 3^3 + 5 \times 3$

$= 243 - 135 + 15$

$= 243 - 120 = 123$

4. (C) $x + y = a$ ...(1)

$xy = b^2$ ...(2)

$(x - y)^2 = (x + y)^2 - 4xy$

$(x - y)^2 = a^2 - 4b^2$ ...(3)

$= x^3 - x^2y - xy^2 + y^3$

$= (x^3 + y^3) - xy(x + y)$

$= (x + y)(x^2 - xy + y^2) - xy(x + y)$

$= (x + y)(x^2 - xy + y^2 - xy)$

$= (x + y)(x - y)^2$

$= a(a^2 - 4b^2)$ [From eq. (1) & (3)]

$= a^3 - 4ab^2$

5. (B) $x + \frac{1}{x} = 5$

$x^2 + 1 = 5x$

$x^2 - 5x + 1 = 0$ [Multiplying by 3]

$3x^2 - 15x + 3 = 0$

$3x^2 + 3 = 15x$ ...(1)

Here,

$\frac{2x}{3x^2 - 5x + 3} = \frac{2x}{15x - 5x}$

$= \frac{2x}{10x} = \frac{1}{5}$

6. (B) $a^2 + b^2 + c^2 = 2(a - b - c) - 3$

$\Rightarrow a^2 + b^2 + c^2 = 2a - 2b - 2c - 1 - 1 - 1$

$\Rightarrow (a^2 - 2a + 1) + (b^2 + 2b + 1) + (c^2 + 2c + 1) = 0$

$\Rightarrow (a - 1)^2 + (b + 1)^2 + (c + 1)^2 = 0$

[On equating the coefficients]

$\Rightarrow a - 1 = 0, b + 1 = 0$ and $c + 1 = 0$

$a = 1, b = -1, c = -1$

$2a - 3b + 4c = 2 \times 1 - 3(-1) + 4(-1)$

$= 2 + 3 - 4 = 1$

7. (C) $x^2 + y^2 - 4x - 4y + 8 = 0$

$\Rightarrow (x^2 - 4x + 4) + (y^2 - 4y + 4)$

$\Rightarrow (x - 2)^2 + (y - 2)^2 = 0$

$\Rightarrow x - 2 = 0$ and $y - 2 = 0$

$x = 2, y = 2$

$x - y = 2 - 2 = 0$

8. (D) $(a - 1)^2 + (b + 2)^2 + (c + 1)^2 = 0$

$\Rightarrow a - 1 = 0, b + 2 = 0$ and $c + 1 = 0$

$a = 1, \ b = -2$ and $c = -1$

$2a - 3b + 7c$

$= 2(1) - 3(-2) + 7(-1)$

$= 2 + 6 - 7 = 1$

9. (A) $x + \frac{1}{4x} = \frac{3}{2}$ ...(1)

On multiplying by 2,

$2x + \frac{1}{2x} = 3$ ...(2)

On cubing on both sides,

$\left(2x + \frac{1}{2x}\right)^3 = (3)^3$

$8x^3 + 3 \times 2x \times \frac{1}{2x}\left(2x + \frac{1}{2x}\right) + \frac{1}{8x^3} = 27$

$8x^3 + \frac{1}{8x^3} + 3(3) = 27$

[From eq (2)]

$8x^3 + \frac{1}{8x^3} = 27 - 9$

$8x^3 + \frac{1}{8x^3} = 18$

10. (A) $(3a + 1)^2 + (b - 1)^2 + (2c - 3)^2 = 0$

$\Rightarrow 3a + 1 = 0, b - 1 = 0$ and $2c - 3 = 0$

$3a = -1 \qquad b = 1 \qquad 2c = 3$

$a = \frac{1}{3} \qquad c = \frac{3}{2}$

$$3a+b+2c=3\left(-\frac{1}{3}\right)+1+2\left(\frac{3}{2}\right)$$
$$=-1+1+3=3$$

11. (A) $\frac{a}{3}=\frac{b}{2}$

$\Rightarrow$ $\frac{a}{b}=\frac{3}{2}$ ...(1)

$$\frac{2a+3b}{3a-2b}=\frac{2\left(\frac{a}{b}\right)+3}{3\left(\frac{a}{b}\right)-2}$$
$$=\frac{2\left(\frac{3}{2}\right)+3}{3\left(\frac{3}{2}\right)-2}$$
$$=\frac{6}{\frac{9}{2}-2}$$
$$=\frac{6}{\frac{9-4}{2}}$$
$$=\frac{12}{5}$$

12. (B) $4x=18y$

$$\frac{x}{y}=\frac{18}{4}$$
$$=\frac{9}{2}$$
$$\frac{x}{y}-1=\frac{9}{2}-1$$
$$=\frac{9-2}{2}=\frac{7}{2}$$

13. (A) $x=12$ $y=4$

$$(\text{ग}\ \text{ल})^{\frac{\text{ग}}{\text{ल}}}=(12\ \ 4)^{\frac{12}{4}}$$
$$=(16)^3=4096$$

14. (B) $\text{ग}^4.\ \frac{1}{\text{ग}^4}=727$

$$\left(\text{ग}^2.\ \frac{1}{\text{ग}^2}\right)^2\ \ 2=727$$
$$\left(\text{ग}^2.\ \frac{1}{\text{ग}^2}\right)^2=727+2$$
$$\text{ग}^2.\ \frac{1}{\text{ग}^2}=\sqrt{727+2}$$
$$\text{ग}^2.\ \frac{1}{\text{ग}^2}=27 \quad ...(1)$$

$$\left(\text{ग}\ \ \frac{1}{\text{ग}}\right)^2.\ \ 2=27$$
$$\text{ग}\ \ \frac{1}{\text{ग}}=\sqrt{27\ .\ 2}$$
$$\text{ग}\ \ \frac{1}{\text{ग}}=5 \quad ...(2)$$
$$\text{ग}^3\ \ \frac{1}{\text{ग}^3}=\left(\text{ग}\ \ \frac{1}{\text{ग}}\right)\left(\text{ग}^2.\ \ \frac{1}{\text{ग}^2}.\ \ 1\right)$$
$$=5\ (27+1)$$
$$=28\times 5=140$$

15. (D) $a^3-b^3-c^3-3abc=0$ ...(1)

We know that if

$$a^3+b^3+c^3=3abc$$
$\Rightarrow$ $a+b+c=0$
$$a^3+(-b)^3+(-c^3)=3abc$$
$\Rightarrow$ $a-b-c=0$
$\Rightarrow$ $a=b+c$

16. (A) $x+y+z=0$ ...(1)

$$\frac{xyz}{(x+y)(y+z)(z+x)}$$
$$=\frac{xyz}{(-z)\times(-x)(-y)}=-1$$

17. (C) $3abc$

18. (D) $x=\frac{4ab}{a+b}$ ...(1)

$$\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}$$
$$=\frac{(x+2a)(x-2b)+(x-2a)(x+2b)}{(x-2a)(x-2b)}$$
$$\Rightarrow\ \frac{x^2+x(2a-2b)-4ab+x^2+x(2b-2a)-4ab}{x^2-x(2a+2b)+4ab}$$
$$=\frac{2x^2-8ab+x(2a-2b+2b-2a)}{x^2-x(2a+2b)+4ab}$$
$$=\frac{2x^2-8ab}{x^2-x(2a+2b)+x(a+b)}$$
$$=\frac{2(x^2-4ab)}{x^2-x(2a+2b-a-b)}$$
$$=\frac{2[x^2-x(a+b)]}{x^2-x(a+b)}=2$$

19. (C) Here, $x+\frac{1}{x}=2\Rightarrow x=1.$

$$x^{17}+\frac{1}{x^{19}}=(1)^{17}+\frac{1}{(1)^{19}}$$
$$=1+1=2$$

20. (D) $x+y+z=0$ ...(1)

$$\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}=\frac{x^3+y^3+z^3}{xyz}$$
$$=\frac{3xyz}{xyz}=3$$

21. (A) Here, $x=b+c-2a$ ...(1)

$y=c+a-2b$ ...(2)

$z=a+b-2c$ ...(3)

To find the sum

$$x+y+z=0$$
$$x^2+y^2-z^2+2xy$$
$$=x^2+2xy+y^2-z^2$$
$$=(x+y)^2-z^2$$
$$=(-z)^2-z^2=0$$

22. (C) $a+b+c=0$ ...(1)

$$=\left(\frac{a+b}{c}+\frac{b+c}{a}+\frac{c+a}{b}\right)\times$$
$$\left(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\right)$$
$$=\left[\left(\frac{-c}{c}\right)+\left(\frac{-a}{a}\right)+\left(\frac{-b}{b}\right)\right]$$
$$\left[\frac{a}{-a}+\frac{b}{-b}+\frac{c}{-c}\right]$$
$$=(-1-1-1)(-1-1-1)=9$$

23. (D) $a+b+c=8$

$$a+b+c=4+3+1$$
$(a-4)+(b-3)+(c-1)=0$ ...(1)

Therefore

$(a-4)^3+(b-3)^3+(c-1)^3-3$
$(a-4)\ (b-3)\ (c-1)$

24. (A) $\left(x+\frac{1}{x}\right)^2=3$

$$x+\frac{1}{x}=\sqrt{3} \quad ...(1)$$

On cubing the both sides,

$$\left(x+\frac{1}{x}\right)^3=(\sqrt{3})^3$$
$$x^3+\frac{1}{x^3}+3x\times\frac{1}{x}\left(x+\frac{1}{x}\right)$$
$$=3\sqrt{3}$$
$$x^3+\frac{1}{x^3}+3(\sqrt{3})=3\sqrt{3}$$
$$x^6+1=0 \quad ...(2)$$

$x^{206} + x^{200} + x^{90} + x^{84} + x^{18} + x^{12} + x^6 + 1$

$= x^{200}(x^6 + 1) + x^{84}(x^6 + 1) + x^{12}(x^6 + 1) + 1(x^6 + 1)$

$= x^{200}(0) + x^{84}(0) + x^{12}(0) + 1(0)$

$= 0$

25. (B) $\frac{3-5x}{x} + \frac{3-5y}{y} + \frac{3-5z}{z} = 0$

$\frac{3}{x} - \frac{5x}{x} + \frac{3}{y} - \frac{5y}{y} + \frac{3}{z} - \frac{5z}{z} = 0$

$\frac{3}{x} + \frac{3}{y} + \frac{3}{z} = 15$

$3\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 15$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 5$

26. (B) $3x + \frac{1}{2x} = 5$ ...(1)

$\left[\text{Multiplying by } \frac{2}{3} \text{ on both sides}\right]$

$\frac{2}{3} \times 3x + \frac{2}{3} \times \frac{1}{2x} = 5 \times \frac{2}{3}$

$2x + \frac{1}{3x} = \frac{10}{3}$ ...(2)

On cubing the both sides,

$\left(2x + \frac{1}{3x}\right)^3 = \left(\frac{10}{3}\right)^3$

$\Rightarrow 8x^3 + 3 \times 2x \times \frac{1}{3x}\left(2x + \frac{1}{3x}\right) + \frac{1}{27x^3} = \frac{1,000}{27}$

$\Rightarrow 8x^3 + 2\left(\frac{10}{3}\right) + \frac{1}{27x^3} = \frac{1,000}{27}$

[from eq. (1)]

$\Rightarrow 8x^3 + \frac{1}{27x^3} = \frac{1,000}{27} - \frac{20}{3}$

$= \frac{1,000 - 180}{27} = \frac{820}{27}$

$= 30\frac{10}{27}$

27. (C) $xy(x + y) = 1$ ...(1)

$xy = \frac{1}{x + y}$

$\frac{1}{x^3y^3} - x^3 - y^3$

$= (x + y)^3 - x^3 - y^3$

$= x^3 + y^3 + 3x^2y + 3y^2x - x^3 - y^3$

$= 3xy(x + y)$

$= 3 \times 1$ [From eq.(1)]

$= 3$

28. (A) $a^4 + b^4 = a^2b^2$

On adding $2a^2b^2$ on both sides,

$a^4 + 2a^2b^2 + b^4 = a^2b^2 + 2a^2b^2$

$(a^2 + b^2)^2 = 3a^2b^2$

$a^2 + b^2 = \sqrt{3}\,ab$ ...(1)

On cubing the both sides,

$(a^2 + b^2)^3 = \left(\sqrt{3}\,ab\right)^3$

$a^6 + b^6 + 3a^2b^2(a^2 + b^2) = 3\sqrt{3}\,a^3b^3$

$a^6 + b^6 + 3a^2b^2\left(\sqrt{3}ab\right) = 3\sqrt{3}\,a^3b^3$

[From eq.(1)]

$a^6 + b^6 = 0$

29. (B) $x^4 + x^{-4} = 322$ ...(1)

$x^4 + \frac{1}{x^4} = 322$

$\left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 322$

$x^2 + \frac{1}{x^2} = \sqrt{324}$

$\left(x - \frac{1}{x}\right)^2 + 2 = 18$

$x - \frac{1}{x} = \sqrt{16}$

$x - \frac{1}{x} = 4$

30. (A) $x \times y = 2$ ...(1)

$x^2 + y^2 = 20$ ...(2)

$(x + y)^2 = x^2 + y^2 + 2xy$

$= 20 + 2(2)$

$= 24$

31. (B) $\frac{x}{2x^2 + 5x + 2} = \frac{1}{6}$

$6x = 2x^2 + 5x + 2$

$\Rightarrow 2x^2 - x + 2 = 0$

$\Rightarrow 2x^2 + 2 = x$

Divide by 2x,

$x + \frac{1}{x} = \frac{1}{2}$

32. (B) $a + b + c = 0$ ...(1)

$\frac{1}{(a+b)(b+c)} + \frac{1}{(a+c)(b+a)} + \frac{1}{(c+a)(c+b)}$

$\Rightarrow = \frac{c + a + b + c + a + b}{(a+b)(b+c)(c+a)}$

$= \frac{2(a+b+c)}{(a+b)(b+c)(c+a)}$

$= \frac{2(0)}{(a+b)(b+c)(c+a)}$

$= 0$ [From eq.(1)]

33. (A) $\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} + \frac{(c-a)^2}{(a-b)(b-c)}$

$= \frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)}$

Here, let $A = a - b$

$B = b - c$

$C = c - a$

$A + B + C = a - b + b - c + c - a$

$= 0$

Hence, $A^3 + B^3 + C^3 = 0$

$= \frac{0}{(a-b)(b-c)(c-a)}$

$= 0$

34. (B) $\frac{2x - y}{x + 2y} = \frac{1}{2}$

$\Rightarrow 4x - 2y = x + 2y$

$3x = 4y$

$\frac{x}{y} = \frac{4}{3}$ ...(1)

$\frac{3x - y}{3x + y} = \frac{3\left(\frac{x}{y}\right) - 1}{3\left(\frac{x}{y}\right) + 1}$

$= \frac{3\left(\frac{4}{3}\right) - 1}{3\left(\frac{4}{3}\right) + 1}$

$= \frac{4 - 1}{4 + 1} = \frac{3}{5}$

35. (B) चक्र $(a+b)^2 = (a-b)^2 + 4ab$

$$\left(\frac{p}{q}+\frac{q}{p}\right)^2 = \left(\frac{p}{q}-\frac{q}{p}\right)^2 + 4\frac{p}{q}\times\frac{q}{p}$$

$$\left(\frac{p}{q}+\frac{q}{p}\right)^2 = (4)^2 + 4$$

$$\frac{p}{q}+\frac{q}{p} = \sqrt{20} = 2\sqrt{5} \quad \ldots(1)$$

On cubing on both sides,

$$\left(\frac{p}{q}+\frac{q}{p}\right)^3 = (2\sqrt{5})^3$$

$$\frac{p^3}{q^3}+\frac{q^3}{p^3}+3\frac{p}{q}\times\frac{q}{p}\left(\frac{p}{q}+\frac{q}{p}\right) = 8\times 5\sqrt{5}$$

$$\Rightarrow \frac{p^3}{q^3}+\frac{q^3}{p^3} = 40\sqrt{5} - 3(2\sqrt{5})$$

[From eq.(1)]

$$= 34\sqrt{5}$$

36. (A) $ax + bx + ay + by = x(a+b) + y(a+b)$

$$= (a+b)(x+y)$$

37. (A) $x^2 + xy + xz + yz$

$$= x(x+y) + z(x+y)$$

$$= (x+y)(x+z)$$

38. (B) $ab(x^2+y^2) + xy(a^2+b^2)$

$$= abx^2 + aby^2 + xya^2 + xyb^2$$

$$= ax(bx+ay) + by(ay+bx)$$

$$= (ax+by)(bx+ay)$$

39. (C) $x^2 + \left(a+\frac{1}{a}\right)x + 1$

$$= x^2 + ax + \frac{1}{a}x + 1$$

$$= x(x+a) + \frac{1}{a}(x+a)$$

$$= (x+a)\left(x+\frac{1}{a}\right)$$

40. (C) $x^2 + 4x + 4 = (x)^2 + 2(x)(2) + (2)^2$

$$= (x+2)^2$$

41. (D) $1 - 8x + 16x^2$

$$= (1)^2 - 2(1)(4x) + (4x)^2$$

$$= (1-4x)^2$$

42. (D) $a^2 - 50ab + 625b^2$

$$= (a)^2 - 2(a)(25b) + (25b)^2$$

$$= (a-25b)^2$$

43. (D) $144x^2 + 264xy + 121y^2$

$$= (12x)^2 + 2(12x)(11y) + (11y)^2$$

$$= (12x+11y)^2$$

44. (A) $25x^2 - 10x + 1 - 36y^2$

$$= (5x-1)^2 - (6y)^2$$

$$= (5x-1+6y)(5x-1-6y)$$

$$= (5x+6y-1)(5x-6y-1)$$

45. (B) $\left(1+\frac{1}{x}\right)^3 - 2\left(x+\frac{1}{x}\right)$

$$= \left(1+\frac{1}{x}\right)\left[\left(1+\frac{1}{x}\right)^2 - 2\right]$$

$$= \left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+2-2\right)$$

$$= \left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)$$

46. (B) $6\sqrt{3}x^2 + 47x + 5\sqrt{3}$

$$= 6\sqrt{3}x^2 + (45+2)x + 5\sqrt{3}$$

$$= 6\sqrt{3}x^2 + 45x + 2x + 5\sqrt{3}$$

$$= 3\sqrt{3}x(2x+5\sqrt{3}) + 1(2x+5\sqrt{3})$$

$$= (2x+5\sqrt{3})(3\sqrt{3}x+1)$$

47. (D) $\frac{1}{2}\left[\frac{25x^2}{9} - \frac{49y^2}{16}\right]$

$$= \frac{1}{2}\left[\left(\frac{5x}{3}\right)^2 - \left(\frac{7y}{4}\right)^2\right]$$

$$= \frac{1}{2}\left(\frac{5x}{3}-\frac{7y}{4}\right)\left(\frac{5x}{3}+\frac{7y}{4}\right)$$

48. (A) $9x^2 - 30xy + 25y^2$

$$= (3x)^2 - 2(3x)(5y) + (5y)^2$$

[चक्र $a^2 - 2ab + b^2 = (a-b)^2$]

$$= (3x)^2 - 2(3x)(5y) + (5y)^2$$

$$= (3x-5y)^2$$

49. (B) $36 - 12k + k^2$

$$= (6)^2 - 2(6)k + k^2$$

चक्र $a^2 - 2ab + b^2 = (a-b)^2$

$$= (6-k)^2$$

50. (A) $12x^3 - 14x^2 - 10x$

$$= 2x(6x^2 - 7x - 5)$$

$$= 2x[6x^2 - 10x + 3x - 5]$$

$$= 2x[2x(3x-5) + 1(3x-5)]$$

$$= 2x(3x-5)(2x+1)$$

51. (A) $15x^4 + 3x^2 - 18 = 3[5x^4 + x^2 - 6]$

Let $x^2 = y$

$$= 3[5y^2 + y - 6]$$

$$= 3[5y^2 + 6y - 5y - 6]$$

$$= 3[y(5y+6) - 1(5y+6)]$$

$$= 3(5y+6)(y-1)$$

Put the value of x,

$$= 3(5x^2+6)(x^2-1)$$

$$= 3(5x^2+6)(x+1)(x-1)$$

52. (C) $8x^3 + \frac{1}{8x^3} + 2x + \frac{1}{2x}$

$$= (2x)^3 + \left(\frac{1}{2x}\right)^3 + \left(2x+\frac{1}{2x}\right)$$

$$= \left(2x+\frac{1}{2x}\right)\times\left(4x^2 + \frac{1}{4x^2} - 2x\times\frac{1}{2x}\right) + \left(2x+\frac{1}{2x}\right)$$

[$\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)$]

$$= \left(2x+\frac{1}{2x}\right)\left(4x^2 + \frac{1}{4x^2} - 1 + 1\right)$$

$$= \left(2x+\frac{1}{2x}\right)\left(4x^2 + \frac{1}{4x^2}\right)$$

53. (C) $27x^3 + y^3 + z^3 - 9xyz$

$$= (3x)^3 + (y)^3 + (z)^3 - 3(3xyz)$$

$$= (3x)^3 + y^3 + z^3 - 3(3xyz)$$

$$= (3x+y+z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$$

54. (C) $x-a$, is a factor of $p(x)$.

$$\Rightarrow p(a) = 0$$

$$\Rightarrow a^3 - a^2\times a + a + 3 = 0$$

$$\Rightarrow a + 3 = 0$$

$$\Rightarrow a = -3$$

55. (A) $x^3 - mx^2 - 2nax + na^2$ and $(x-a)$ are factors of $p(x)$.

$$p(a) = 0$$

$$p(a) = a^3 - ma^2 - 2na\times a + na^2$$

$$0 = a^3 - ma^2 - 2na^2 + na^2$$

$$0 = a^3 - ma^2 - na^2$$

$$0 = a - m - n$$

$$a = m + n$$

56. (C) Factors of constant term 6 are ± 1, ± 2, ± 3.

So, put $x = 1$,

Remainder $= (1)^3 - 7\times(1) + 6$

$$= 1 - 7 + 6 = 0$$

$\because$ We get 0 as remainder when, $x = 1$.

So, $x-1$ is a factor of the expression.

$$x^3 - 7x + 6$$

$$= x^3 - x^2 + x^2 - x - 6x + 6$$

$$= x^2(x-1) + x(x-1) - 6(x-1)$$

$$= (x-1)(x^2 + x - 6)$$

$$= (x-1)[x^2 + 3x - 2x - 6]$$

$$= (x-1)[x(x+3) - 2(x+3)]$$

$$= (x-1)(x+3)(x-2)$$

57. (A) To make $81x^2 + 4y^2$ as perfect square,

$$= \pm 2\sqrt{81x^2\times 4y^2}$$

$$= \pm 2\times 9x\times 2y$$

$$= \pm 36xy$$

58. (C) $m^4 - 3m^3 - 2m^2 - 3m + 1$

$= m^4 - 3m^3 - 2m^2 - 3m + 1$

$= (m^4 + 1) - 3m^3 - 3m - 2m^2$

$= (m^4 + 1) - 3m(m^2 + 1) - 2m^2$ ...(1)

Let $m^2 + 1 = x$...(2)

On squaring both sides,

$(m^2 + 1)^2 = x^2$

$m^4 + 1 + 2m^2 = x^2$

$= m^4 + 1$

$= x^2 - 2m^2$ ...(3)

From eq.(1)

$= x^2 - 2m^2 - 3mx - 2m^2$

$= x^2 - 3mx - 4m^2$

$= x^2 - (4 - 1)mx - 4m^2$

$= x^2 - 4mx + mx - 4m^2$

$= x(x - 4m) + m(x - 4m)$

$= (x - 4m)(x + m)$

Put the value of x,

$= (m^2 + 1 - 4m)(m^2 + 1 + m)$

$= (m^2 - 4m + 1)(m^2 + m + 1)$

59. (A) $x - y = \left(\sqrt[3]{x}\right)^3 - \left(\sqrt[3]{y}\right)^3$

$[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

$= \left(\sqrt[3]{x} - \sqrt[3]{y}\right)\left[\left(x^{\frac{1}{3}}\right)^2 + x^{\frac{1}{3}} y^{\frac{1}{3}} + \left(y^{\frac{1}{3}}\right)^2\right]$

$= \left(\sqrt[3]{x} - \sqrt[3]{y}\right)\left[x^{\frac{2}{3}} + (xy)^{\frac{1}{3}} + y^{\frac{2}{3}}\right]$

60. (A) $x^2 + \frac{1}{81} - \frac{2}{9}x$

$= x^2 - \frac{2}{9}x + \frac{1}{81}$

$= x^2 - 2 \times \frac{1}{9}x + \left(\frac{1}{9}\right)^2$

$= \left(x - \frac{1}{9}\right)^2$

61. (A) $y^4 + 3y^2 - 28$

Let $y^2 = x$ ...(1)

$x^2 + 3x - 28 = x^2 + (7 - 4)x - 28$

$= x^2 + 7x - 4x - 28$

$= x(x + 7) - 4(x + 7)$

$= (x + 7)(x - 4)$

Put the value of x,

$(y^2 + 7)(y^2 - 4)$

$= (y^2 + 7)[(y^2) - (2)^2]$

$= (y^2 + 7)(y + 2)(y - 2)$

62. (C) $2^2 - (a^2 + b^2 + 2ab)$

$= 2^2 - (a + b)^2$

Formula, $A^2 - B^2 = (A - B)(A + B)$

$= [2 - (a + b)][2 + (a + b)]$

$= (2 + a + b)(2 - a - b)$

63. (A) $(x + 1)$ is a factor of $2x^3 - px^2 + x + q$. So, put $x = -1$,

$2(-1)^3 - p(-1)^2 + (-1) + q = 0$

$-2 - p - 1 + q = 0$

$q - p = 3$ ...(1)

Similarly, $x + 2 = 0 \Rightarrow$ put $x = -2$

$2(-2)^3 - p(-2)^2 + (-2) + q = 0$

$-16 - 4p - 2 + q = 0$

$q - 4p = 18$ ...(2)

Subtract eq. (2) from eq. (1),

$3p = -15 \Rightarrow p = -5$

Hence, $q - (-5) = 3$

$q = 3 - 5$

$q = -2$

64. (C) Put $x = -1$ in $x^4 + 9x^3 + 7x^2 + 9ax + 5a^2$

$(-1)^4 + 9(-1)^3 + 7(-1)^2 + 9a(-1) + 5a^2 = 0$

$1 - 9 + 7 - 9a + 5a^2 = 0$

$5a^2 - 9a - 1 = 0$

It is a quadratic equation in a varaible.

$a = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 5(-1)}}{2 \times 5}$

$= \frac{9 \pm \sqrt{81 + 20}}{10} = \frac{9 \pm \sqrt{101}}{10}$

65. (A) $x + a = 0 \Rightarrow x = -a$

$(-a)^2 + p(-a) + q = 0$

$a^2 - pa + q = 0$ ...(1)

Similarly,

$(-a)^2 + m(-a) + n = 0$

$a^2 - ma + n = 0$ ...(2)

Subtract eq. (2) from eq (1),

$-pa + ma + q - n = 0$

$a(m - p) = n - q$

$a = \frac{\text{द}\ .}{\text{उ}\quad \text{ब}}$

Chapter 5

L.C.M and H.C.F

1. Multiples

The number, multipled by an integer (not a fraction) is called multiples. Normally, the skip counting or "count by" numbers are most often called multiples.

Example :
- Multiples of 2 — 2, 4, 6, 8, 10..... etc.
- Multiples of 3 — 3, 6, 9, 12, 15..... etc.
- Multiples of 5 — 5, 10, 15, 20, 25..... etc.

Note : Each number completely divides all its multiples.

2. Factors

A factor of a number is an exact divisor of that number. Example, Ansh wants to find those numbers which exactly divide 6. He divides 6 by number less than or equal to 6, i.e. 1, 2, 3, 4, 5, 6. He found that the numbers 1, 2, 3 and 6 are exact divisors of 6. These numbers are called *factors* of 6.

Example :
- 32 is exactly divisible by 1, 2, 4, 8, 16, 32. So, these numbers are called the factors of 32.
- 35 is exactly divisible by 1, 5, 7, 35. So, these numbers are called the factors of 35.

3. Prime and Composite Numbers

Prime Numbers—The numbers other than 1 whose only factors are 1 and the number itself are called Prime numbers. For example, 2, 3, 5, 7, 11, 13 and so on. 2 is the smallest prime number which is even and odd otherwise.

Composite numbers—Numbers having more than two factors are called Composite numbers. For example, 4, 6, 8, 9, 10, 12 and so on.

Note – 1 is neither a prime nor a composite number.

4. Prime Factorization

In a factorization, if the only factors are prime numbers such as 2, 3, 5, 7, 11, etc. Such a factorization of a number is called a prime factorization.

Example : Find out the prime factors of 48.

Sol : $48 = 2 \times 2 \times 2 \times 2 \times 3$

2	48
2	24
2	12
2	6
3	3
	1

$\Rightarrow$ prime factors of 48

5. LCM (Least Common Multiple)

The Lowest Common Multiple of two or more given numbers is the lowest (or smallest or least) of their common multiples. There are various methods of finding out the LCM of the given numbers.

5.1 Methods of finding out the LCM

(i) **Division Method—**We can find the LCM of the given numbers by using following steps.

Step 1 : Write the given number in a series.

Step 2 : Divide by the smallest possible numbers that cannot be divided by that number are written as it is in the next line.

Step 3 : Continue the process till all numbers are divided completely.

Step 4 : The multiplication of divisors of each row is the required LCM of given numbers.

Example: Find out the LCM of 14, 18, 20.

Sol. :

2	14	18	20
2	7	9	10
3	7	9	5
3	7	3	5
5	7	1	5
7	7	1	1
	1	1	1

L.C.M. = $2 \times 2 \times 3 \times 3 \times 5 \times 7 = 1260$

(ii) **Prime Factorization Method—**We can find the LCM of the given numbers by using the following steps :

Step 1: First find the factors of each given number.

Step 2: Choose highest power of each factor among them.

Step 3: Find LCM by taking the product of these factors with highest power of each.

Example: Find out the LCM of 14, 18, 20.

Sol. : $14 = 2 \times 7$

$18 = 3 \times 3 \times 2$

$20 = 2 \times 2 \times 5$

Hence, the required LCM = $2 \times 2 \times 3 \times 3 \times 7 \times 5 = 1260$

6. HCF (Highest Common Factor)

The Highest Common Factor of two or more given numbers is the highest (or greatest) of their common factors. It is also called Greatest Common Divisor (GCD). There are various methods of finding out the HCF of the given numbers.

6.1 Methods to find out H.C.F

Example : Find H.C.F of 12, 18, 24.

(i) **Division method—** H.C.F of 12, 18, 24

$$12 \overline{)\ 18\ (\ 1}$$
$$\underline{12}$$
$$6 \overline{)\ 12\ (\ 2}$$
$$\underline{12}$$
$$\times$$

$$6\ )\ 24\ (\ 4$$
$$\underline{24}$$
$$\times$$

So, the H.C.F of 12, 18, 24 = 6

(ii) Factorization method—

$12 = 2 \times 2 \times 3$

$18 = 2 \times 3 \times 3$

$24 = 2 \times 2 \times 2 \times 3$

After factorization, factor with less power has been taken in H.C.F. So, the required H.C.F of the given numbers

$= 2 \times 3 = 6$

7. Type of Questions

(I) Questions based on LCM and HCF of fractions—In such type of questions, we have to find out LCM or HCF of a group of fractions. To find the LCM or HCF, follow the rules given below :

- $\text{LCM of fractions} = \dfrac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$
- $\text{HCF of fractions} = \dfrac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$

(II) Questions based on LCM and HCF of numbers—Such type of questions are based on product of numbers, LCM and HCF of the numbers.

If x and y be the two numbers and their LCM and HCF are a and b, then the relation among them will be :

$$x \times y = a \times b$$

First number × Second number = LCM × HCF

Important Questions

1. The LCM of two numbers is four times their HCF and their sum is 125. If one number is 100, what will be the other?

(A) 5 (B) 25

(C) 100 (D) 125

2. The HCF and LCM of two numbers are 8 and 1728, respectively. Accordingly, how many pairs of such digits are there?

(A) 2 (B) 3

(C) 4 (D) 5

3. The LCM of two numbers is 520 and their HCF is 4. If one of them is 52, Find the other?

(A) 40 (B) 42

(C) 50 (D) 52

4. The sum of the squares of the digits of the greatest prime number of two digits is: **(NCERT)**

(A) 148 (B) 130

(C) 97 (D) 118

5. Find the least number which if divided by 24 and 36, the remainder will be 14 and 26 respectively.

(A) 82 (B) 62

(C) 102 (D) 92

6. The possible maximum length (in cm) that can be used to accurately measure 495 cm, 900 cm, 1665 cm is:

(A) 25 (B) 35

(C) 45 (D) 15

7. What is the smallest number that is divided by 12, 24 36 and 45 and leaves the remainder 6 in each case? **(NCERT)**

(A) 366 (B) 354

(C) 360 (D) 372

8. Find the least number from the following which is divided by 12, 16, 24 and 36 and leaves the remainder 5?

(A) 243 (B) 139

(C) 149 (D) 245

9. The HCF and LCM of two numbers are 44 and 264, respectively. If the first number is divided by 2, the quotient is 44, which of the following will be the second number?

(A) 147 (B) 528

(C) 132 (D) 264

10. The difference between the largest 4-digit number and the smallest 4-digit number which starting from 3 and ending with 5 is : **(NCERT)**

(A) 900 (B) 909

(C) 999 (D) 990

11. The number of pairs of positive integers whose sum is 99 and their HCF is 9:

(A) 5 (B) 4

(C) 3 (D) 2

12. The sum of two numbers is 84 and their HCF is 12. The number of total pairs of such numbers will be:

(A) 2 (B) 3

(C) 4 (D) 5

13. The largest four-digit number, which is divisible by each of the numbers 12, 18, 21 and 28, is:

(A) 9576 (B) 9928

(C) 9828 (D) 9324

14. The sum of two numbers is 36 and their HCF and LCM are 3 and 105 respectively. The sum of the reciprocal of the two numbers will be:

(A) 13 (B) $\frac{9}{11}$

(C) $\frac{7}{35}$ (D) $\frac{4}{35}$

15. The smallest perfect square number, which is divided by each of 16, 20 and 24, is :

(A) 1600 (B) 3600

(C) 6400 (D) 14400

16. The two numbers are in the ratio 3 : 4 and their LCM is 48. The sum of those two numbers will be : **(NCERT)**

(A) 32 (B) 28

(C) 26 (D) 24

17. What is the largest number which divides 729 and 901 leaves the remainder 9 and 5 respectively? **(NCERT)**

(A) 15 (B) 16

(C) 19 (D) 20

18. If $P = 2^3.3^{10}.5$; $Q = 2^5.3.7$, then find the HCF of P and Q.

(A) 2.3.5.7 (B) 3.2^3

(C) $2.^2.3^7$ (D) $2^5.3^{10}.5.7$

19. A, B, C start running in the same direction from the same point in a circular stadium at the same time. A round is completed in 252 seconds, A in 308 seconds and C in 198 seconds. How long will they meet again at the starting point?

(A) 26 min 18 sec

(B) 42 min 36 sec

(C) 45 minutes

(D) 46 min 12 sec

20. A milk seller has 21 liters of cow milk, 42 liters of toned milk and 63 liters of double toned milk. If he wants to pack them in tin cans in such a way that each container contains the same amount of milk and does not want to mix any two types of milk in one box, then the required minimum number of boxes is: **(NCERT)**

(A) 3 (B) 6

(C) 9 (D) 12

21. Between 100 and 600, the number of integers divisible by both 4 and 6 is :

(A) 40 (B) 42

(C) 41 (D) 50

22. Which is the largest four digit number which if divided by 3, 5, 7, 9, the remainder remains 1, 3, 5, 7 respectively?

(A) 9763 (B) 9764

(C) 9766 (D) 9765

23. The LCM of two positive integers is twice the larger number. The difference between the smallest number and the HCF of the two integers is 4. Accordingly, what is the smaller number?

(A) 6 (B) 8

(C) 10 (D) 12

24. Which of the following fractions $\frac{2}{7}, \frac{1}{3}, \frac{1}{6}, \frac{3}{4}$ is the largest? **(NCERT)**

(A) $\frac{5}{6}$ (B) $\frac{1}{3}$

(C) $\frac{2}{7}$ (D) $\frac{3}{4}$

25. The product of two numbers is 2028 and their HCF is 13. Accordingly, find the number of such pairs.

(A) 1 (B) 2

(C) 3 (D) 4

26. Three bells ring together at 11 am. All the three, ring at intervals of 20 minutes, 30 minutes and 40 minutes respectively. Accordingly, what time will they ring together again?

(A) 2 pm (B) 1 pm

(C) 1.15 pm (D) 1.30 pm

27. The LCM of two numbers is 4284 and their HCF is 34. If one of them is 204, then what is the other number?

(A) 714 (B) 814

(C) 914 (D) 614

28. The LCM of two numbers is 120 and their HCF is 10. Accordingly, which of the following numbers can be the sum of those two numbers? **(NCERT)**

(A) 140 (B) 80

(C) 60 (D) 70

29. The four runners started their race from a single point on a circular path. He took 200 seconds, 300 seconds, 360 seconds and 450 seconds respectively to complete a round of that path. Accordingly, how much time will they be able to meet again at their initial point for the first time?

(A) 1800 sec (B) 3600 sec

(C) 2400 sec (D) 4800 sec

30. The HCF of two numbers is 11 and the LCM is 7700. If one of them is 275, then how much is the other?

(A) 279 (B) 283

(C) 308 (D) 318

31. The LCM of three different numbers is 120. Accordingly, which of the following numbers cannot be the HCF of those numbers?

(A) 8 (B) 12

(C) 24 (D) 35

32. The traffic-related lights at three different crossroads change color after 24 seconds, 36 seconds and 54 seconds respectively. If all three of them change color together at 10: 15: 00 am, what time will the three of them change color next time?

(A) 10 : 16 : 54 am

(B) 10 : 18 : 36 am

(C) 10 : 17 : 02 am

(D) 10 : 22 : 12 am

33. The gcd and lcm of two numbers are 13 and 1989, respectively. If one of them is 117, what is the other number?

(A) 121 (B) 143

(C) 217 (D) 221

34. If the HCF of any two numbers is 12 and the LCM is 924, then how many pairs of such numbers will there be? **(NCERT)**

(A) 0 (B) 1

(C) 2 (D) 3

35. Which of the following is the smallest number, which is divided by 5, 6, 7, 8 to get remainder 3, but is also divisible by 9?

(A) 1463 (B) 1573

(C) 1683 (D) 1793

36. If $x = 2^2 \times 3^3 \times 7^2$, and $y = 2^3 \times 3 \times 5$, then what is the HCF of x and y? **(NCERT)**

(A) $2^3 \times 3^3 \times 5 \times 7^2$

(B) $2^2 \times 3^2 \times 7^2$

(C) $2^2 \times 3$

(D) 5×7^2

37. What is the smallest number from which, if 675 is multiplied, the product becomes a perfect cube?

(A) 7 (B) 8

(C) 5 (D) 6

38. What is the smallest number that, if divided by 35, 45, 55, the remainder gets 18, 28, 38 respectively?

(A) 3448 (B) 3482

(C) 2468 (D) 3265

39. What is the maximum number that dividing 110 and 128 gives a uniform remainder of 2? **(NCERT)**

(A) 8 (B) 18

(C) 28 (D) 38

40. Find the least number which is divided by 12, 18, 36 and 45, leaves the remainders 8, 14, 32 and 41 respectively.

(A) 186 (B) 176

(C) 180 (D) 178

41. The ratio of two numbers is 3: 4 and their least common multiple is 180. What is the second number? **(NCERT)**

(A) 30 (B) 60

(C) 45 (D) 90

Solutions

1. (B) If LCM = L and HCF = H, then

According to question,

$$L = 4 \times H = 4H$$

and $L + H = 125$

$\Rightarrow 4H + H = 125$

$\Rightarrow 5H = 125$

$\therefore H = \frac{125}{5} = 25$

$\therefore L = 4H = 4 \times 25 = 100$

$\therefore 2^{nd}$ number $= \frac{\text{LCM} \times \text{HCF}}{\text{First Number}}$

$= \frac{100 \times 25}{100} = 25$

2. (A) Let the numbers be $8x$ and $8y$. Then,

LCM of $8x$ and $8y = 8xy$

$\Rightarrow 8xy = 1728$

$\therefore xy = \frac{1728}{8} = 216$

$\Rightarrow x$ and $y = (1, 216), (8, 27)$

So, only two pairs are poosible.

3. (A) 2^{nd} number $= \frac{\text{LCM} \times \text{HCF}}{\text{First Number}}$

$= \frac{520 \times 4}{52} = 40$

4. (B) ∵ The Largest two-digit prime number = 97

∴ Required Sum = $(9)^2 + (7)^2$

$= 81 + 49 = 130$

5. (B) ∵ $24 - 14 = 10$ and $36 - 26 = 10$

∴ LCM (24, 36) = 72

$24 = 2 \times 2 \times 2 \times 3$

$36 = 2 \times 2 \times 3 \times 3$

∴ Required number

$= 72 - 10 = 62$

6. (C) HCF (495, 900, 1665) = 45

$495 = \underline{3} \times \underline{3} \times \underline{5} \times 11$

$900 = \underline{3} \times \underline{3} \times 4 \times \underline{5} \times 5$

$1665 = \underline{3} \times \underline{3} \times \underline{5} \times 37$

∴ Required number $= 3 \times 3 \times 5 = 45$

7. (A)

2	12,	24,	36,	45
2	6,	12,	18,	45
3	3,	6,	9,	45
3	1,	2,	3,	15
	1,	2,	1,	5

∴ L.C.M $= 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$= 360$

∴ Required number $= 360 + 6 = 366$

8. (C)

2	12,	16,	24,	36
2	6,	8,	12,	18
2	3,	4,	6,	9
3	3,	2,	3,	9
	1,	2,	1,	3

∴ L.C.M $= 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$= 144$

∴ Required number $= 144 + 5 = 149$

9. (C) 1st number $= 44 \times 2 = 88$

1st number × 2nd number

= L.C.M × H.C.F

$\Rightarrow$ $88 \times$ 2nd number $= 44 \times 264$

$\therefore$ 2nd number $= \frac{44 \times 264}{88} = 132$

10. (D) According to question,

Largest 4-digit number = 3995,

Smallest 4-digit number = 3005

Required number = 3995 – 3005

= 990

11. (A) Given, HCF = 9

Sum of x and y = 99

If, $x = 9$, then $y = 90$; Pair = (9, 90)

If, $x = 18$ then $y = 81$; Pair = (18, 81)

If, $x = 27$ then $y = 72$; Pair = (27, 72)

If, $x = 36$ then $y = 63$; Pair = (36, 63)

If, $x = 45$ then $y = 54$; Pair = (45, 54)

So, total number of pairs = 5

12. (A) Let, the numbers be $12x$ and $12y$.

According to question,

$12x + 12y = 84$

$\Rightarrow$ $12(x + y) = 84$

$\Rightarrow$ $x + y = \frac{84}{12} = 7$

$\therefore$ $x = 3$ and $y = 4$,

$x = 5$ and $y = 2$

∴ Required number of pairs = 2

13. (C) L.C.M (12, 18, 21, 28) = 252

∴ Required number $= 252 \times 39 = 9828$

14. (D) Let the numbers by $3x$ and $3y$

According to question,

$3x + 3y = 36$

$\Rightarrow$ $3(x + y) = 36$

$\Rightarrow$ $x + y = 12$

and $3xy = 105$

$\Rightarrow$ $xy = 35$

$\therefore$ $x = 5$ and $y = 7$

Hence, the numbers = 15 and 21.

Sum of reciprocals of the numbers

$= \frac{1}{15} + \frac{1}{21}$

$= \frac{7+5}{105}$

$= \frac{12}{105}$

$= \frac{4}{35}$

15. (B) L.C.M (16, 20 and 24)

$=$

2	16,	20,	24
2	8,	10,	12
2	4,	5,	6
	2,	5,	3

$= 2 \times 2 \times 2 \times 2 \times 5 \times 3 = 240$

∴ Perfect square number

$= \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5} \times \underline{3 \times 3}$

$= 3600$

16. (B) Let $3x$ and $4x$ be the two numbers.

According to question,

$12x = 48$

$\therefore$ $x = \frac{48}{12} = 4$

∴ Required sum $= (3x + 4x)$

$= 7x = 7 \times 4$

$= 28$

17. (B) Required number

= HCF[(729 – 9 = 720), (901 – 5 = 896)]

720) 896 (1

720

176) 720 (4

704

16) 176 (11

16

16

×

18. (B) $P = 2^3.3^{10}.5$

$Q = 2^5.3.7$

$\therefore$ H.C.F $= 2^3.3$

19. (D) Required time = LCM(252, 308, 198)

= 2772 sec = 46 min 12 sec

2	252	308	198
2	126	154	99
3	63	77	99
3	21	77	33
7	7	77	11
11	1	11	11
	1	1	1

$\therefore$ LCM $= 2 \times 2 \times 3 \times 3 \times 7 \times 11$

$= 2772$

20. (B) Maximum quantity in each box = HCF (21, 42, 63) = 21

$21 = 3 \times 7$

$42 = 2 \times 3 \times 7$

$63 = 3 \times 3 \times 7$

$HCF = 3 \times 7 = 21$

Required least number of boxes

$= \frac{21}{21} + \frac{42}{21} + \frac{63}{21}$

$= 1 + 2 + 3 = 6$

21. (C) LCM (4 and 6) = 12

Numbers between 1 to 600 divisible by $12 = \frac{600}{12} - 1 = 50 - 1 = 49$

Numbers between 1 to 100 divisible by $12 = \frac{100}{12} = 8$

∴ Numbers between 100 to 600 divisible by 12 = 49 – 8 = 41

22. (A) Since, difference between divisor and respective remainder is equal.

LCM (3, 5, 7, 9) = 315

Largest 4-digit number = 9999

315) 9999 (31

945

549

315

234

∴ Number divisible by 945 = 9999 – 234

= 9765

Required number = 9765 – 2

= 9763

23. (B) Let, the larger number and smaller number be x and y.

Their LCM = $2x$

and $y - \text{HCF} = 4$ ...(i)

Using formula,

$$x \times y = \text{HCF} \times \text{LCM}$$

$$x \times y = \text{HCF} \times 2x$$

$$\text{HCF} = \frac{y}{2}$$

From eq. (1)

$$y - \frac{y}{2} = 4$$

$$\frac{2y - y}{2} = 4$$

$$y = 8$$

Hence, the smallest number = 8.

24. (D) LCM of $\frac{2}{7}, \frac{1}{3}, \frac{1}{6}, \frac{3}{4} = \frac{24, 28, 14, 63}{84}$

Hence, ascending order of the fractions $= \frac{1}{6} < \frac{2}{7} < \frac{1}{3} < \frac{3}{4}$

The largest fraction = $\frac{3}{4}$.

25. (B) Let the numbers be $13x$ and $13y$.

According to question,

$$13x \times 13y = 2028$$

$$\Rightarrow \quad 169xy = 2028$$

$$\therefore \quad xy = \frac{2028}{169} = 12$$

$$\therefore \quad x = 3 \text{ and } y = 4$$

or $x = 1$ and $y = 12$

Hence, Number of required pairs = 2

26. (B) Required time

= 11 : 00 : 00 + LCM(20, 30, 40)

= 11 : 00 : 00 + 120 min

= 11 : 00 : 00 + 02 : 00 : 00

= 13 : 00 : 00 or 1pm

2	20,	30,	40
2	10,	15,	20
5	5,	15,	10
	1,	3,	2

L.C.M. = $2 \times 2 \times 5 \times 3 \times 2 = 120$

27. (A) 2^{nd} number = $\frac{\text{LCM} \times \text{HCF}}{\text{First Number}}$

$$= \frac{4284 \times 34}{204} = 714$$

28. (D) Let, $10x$ and $10y$ be the two numbers.

LCM of $10x$ and $10y$

$$= 10 \times x \times y = 10xy$$

$$\Rightarrow \quad 120 = 10xy$$

$$\therefore \quad xy = \frac{120}{10} = 12$$

$\therefore$ If $x = 3$ and $y = 4$, then

$10x + 10y = 30 + 40 = 70$

29. (A) Required time = LCM(200, 300, 360, 450)

= 1800 sec

30. (C) 2^{nd} number = $\frac{\text{LCM} \times \text{HCF}}{\text{First Number}}$

$$= \frac{7700 \times 11}{275}$$

= 308

31. (D) Required HCF can not be 35.

32. (B) LCM (24, 36, 54) = 216 sec

= 3 min 36 sec

$\therefore$ Required time = 10 : 15 : 00 + 0 : 03 : 36

= 10 : 18 : 36 am

2	24,	36,	54
2	12,	18,	27
3	6,	9,	27
3	2,	3,	9
	2,	1,	3

$\therefore$ L.C.M. = $2 \times 2 \times 3 \times 3 \times 2 \times 3$

= 216

33. (D) 2^{nd} number = $\frac{13 \times 1989}{117} = 221$

34. (B) Let $12x$ and $12y$ be the two numbers.

Then,

$$12xy = 924$$

$$\therefore \quad xy = \frac{924}{12} = 77$$

$$\therefore \quad x = 11 \text{ and } y = 7$$

So, the number of possible pairs is only 1.

35. (C) LCM (5, 6, 7, 8) = 840

$\therefore$ Required number = $840 \times 2 + 3$

= 1680 + 3 = 1683,

which is divisible by 9.

2	5	6	7	8
	5	3	7	4

$\therefore$ LCM = $2 \times 5 \times 3 \times 7 \times 4 = 840$

36. (C) According to question,

$$x = 2^2 \times 3^3 \times 7^2$$

and $$y = 2^3 \times 3 \times 5$$

Hence, HCF= $2^2 \times 3$

37. (C) $675 = 5 \times 5 \times 3 \times 3 \times 3 = 3^3 \times 5^2$

$\therefore$ Required number = 5

38. (B) According to question,

$35 - 18 = 17$

$45 - 28 = 17$

$55 - 38 = 17$

LCM of 35, 45 and 55 = 3465

$\therefore$ Required number = 3465 + 17 = 3482

39. (B) Required number = HCF of (110 – 2) and (128 – 2)

= HCF (108, 126)

$$\begin{array}{r} 108 \overline{)126(1} \\ \underline{108} \\ 18 \overline{)108(6} \\ \underline{108} \\ \times \end{array}$$

40. (B) Here, Divisor – Remainder = 4

$\Rightarrow \quad 12 - 8 = 18 - 14$

$= 36 - 32 = 45 - 41$

$= 4$

$\therefore$ Required number

= LCM(12, 18, 36, 45) – 4

2	12,	18,	36,	45
2	6,	9,	18,	45
3	3,	9,	9,	45
3	1,	3,	3,	15
	1,	1,	1,	5

$\therefore \quad$ LCM = $2 \times 2 \times 3 \times 3 \times 5$

= 180

$\therefore$Required number = 180 – 4 = 176

41. (B) Let $3x$ and $4x$ be the two numbers.

Then,

LCM = $3 \times 4 \times x = 12x$

$\therefore \quad 12x = 180$

$$\Rightarrow \quad x = \frac{180}{12} = 15$$

$\therefore$ 2nd number = $4x = 4 \times 15$

= 60

Chapter 6 Linear Equation of One Variable

1. Introduction

You must have heard or seen many algebraic expressions and equations. Example : $4x, 2x+7, 4x+5y, x^2+y^2$ etc. are algebraic expressions and $2x-6=0; 4x=10, 5m+7=17$ etc. are equations. All such linear equations in which the maximum power of the variable being used is one, is called "linear equation with one variable". Example : $2y=0$; $4z=9$; $\frac{5}{7}z+6=0$ etc. are linear equation with one variable.

Variable Parity

$5\underset{\downarrow}{m}+7=\underset{\downarrow}{17}$ ← Equation

Here, the algebraic equation has a parity sign ('='). The expression on the left side of this parity is called the L.H.S and the expression on the right side is called the R.H.S.

Hence, moving a term from one side of the equation to another is called *substitute* process.

2. Methods to Solve the Equations

2.1 If one side of linear expression is variable and another side is a number

Example 1. Find the solution : $2x-6=8$

Sol:

Step 1. To place only variables in the left side, first add 6 on both sides

$2x-6+6=8+6$ or $2x=14$

Step 2. On dividing both sides by 2,

$\frac{2x}{2}=\frac{14}{2}$

$x=7$

Example 2. Find the solution : $\frac{Z}{4}=12$

Sol:

Step 1. On multiplying both sides by 4,

$\frac{Z}{4}\times 4=12\times 4$

$Z=48$

Example 3. Perimeter of a square is 16 cm. Find its side.

Sol: Let, side of a square is x cm.

Perimeter of square $=4\times$ side

$\therefore \quad 4\times x=16$ (given)

On dividing both sides by 4,

$x=\frac{16}{4}=4$

$\therefore$ side $=4$ cm

Example 4. Present age of rajsesh's father is three times of present age of Rajesh. After 5 years, the sum of their ages will be 70 years. Find their present ages.

Sol: Let, Rajesh's present age = x years

Then, Rajesh's father present age = $3x$ years

After 5 years,

Rajesh's age $=(x+5)$ years

Rajesh's father age $=(3x+5)$ years

According to question,

$(x+5)+(3x+5)=70$ (given)

$(x+3x)+(5+5)=70$

$4x+10=70$

Subtract 10 from both sides,

$4x+10-10=70-10$

or $4x=60$

On dividing both sides by 4,

$x=\frac{60}{4}=15$

Hence, Rajesh's present age = 15 years

and Rajesh's father present age = 45 years

Example 5. If sum of three consecutive multiples of 11 is 363, then find the numbers. **(NCERT)**

Sol: Let, x be a multiple of 11, then its next two multiples will be $(x+11)$ and $(x+11+11)$ or $(x+22)$.

According to question,

$x+(x+11)+(x+22)=363$

$3x+33=363$

On dividing both sides by 3,

$x+11=121$

Subtract 11 from both sides,

$x=121-11=110$

So, the three consecutive multiples– 110, 121, 132

2.2 If variable is present on both sides

Example 6. Solve : $4x+6=2x+18$

Sol: $4x+6=2x+18$

or, $4x=2x+18-6$

or, $4x=2x+12$

Subtract $2x$ from both sides,

$4x-2x=2x+12-2x$

$2x=12$

or, $x=6$

Here, subtract $2x$ means - to transfer $2x$ into left hand side.

Example 7. Solve : $2x + \frac{5}{3} = \frac{26}{3} - x$

Sol: $2x + \frac{5}{3} = \frac{26}{3} - x$

Separating the variables and constant terms,

$$2x + x = \frac{26}{3} - \frac{5}{3}$$

or, $$3x = \frac{26-5}{3}$$

or, $$3x = \frac{21}{3} = 7$$

or, $$x = \frac{7}{3}$$

Example 8. Solve : $\frac{6x+1}{3} + 1 = \frac{x-3}{6}$

Sol: $\frac{6x+1}{3} + 1 = \frac{x-3}{6}$

$\because$ LCM of denominators 3 and 6 = 6

Multiplying by 6 on both sides,

$$6 \times \frac{(6x+1)}{3} + 6 = 6 \times \frac{(x-3)}{6}$$

$$2(6x+1) + 6 = x - 3$$

$$12x + 2 + 6 = x - 3$$

$$12x + 8 = x - 3$$

$$11x = -11 \text{ or } x = -1$$

Important Questions

1. Solution of which of the following equations is neither a fraction nor an integer?

(A) $5x - 8 = x + 4$

(B) $3x + 2 = 7 + 4x$

(C) $4x - 18 = 2 + 4x$

(D) $5x + 4 = 3x + 4$

2. Moving a term from one side of the equation to another is called—

(A) Associative (B) Communicative

(C) Distributive (D) Substitute

3. If $\frac{5x}{3} - 4 = \frac{2x}{5}$, then the value of $2x - 7$ is –

(A) $\frac{19}{13}$ (B) $\frac{-13}{19}$

(C) 0 (D) $\frac{13}{19}$

4. For what value of x, the expressions $3x - 4$ and $2x + 1$ will be equal—

(A) 5 (B) 3

(C) 1 (D) 4

5. If $-\frac{2}{5}x = -\frac{5}{2}$, then value of x will be—

(A) 1 (B) $\frac{25}{4}$

(C) $\frac{4}{25}$ (D) –1

6. Let, the unit digit in a two-digit number is m and tens' digit is 5 more than unit digit, then find the number.

(A) $11m + 50$ (B) $10m + 50$

(C) $11m + 5$ (D) $10m + 5$

7. Puja's present age is three times the age of Priya. If 3 years ago, Priya's age was 21 years, find the present age of Puja?

(A) 8 yrs (B) 72 yrs

(C) 24 yrs (D) 36 yrs

8. Sum of three multiples of 5 is 360. Find the smallest multiple.

(A) 25×5 (B) 23×5

(C) 26×5 (D) 30×5

9. In expression $2y = 5y - \frac{18}{5}$, the value of y is—

(A) $\frac{3}{5}$ (B) $\frac{5}{3}$

(C) $\frac{6}{5}$ (D) $\frac{5}{6}$

10. When 9 is subtracted from the product of x and y, then 11 is obtained. Find x.

(A) 4 (B) 5

(C) 9 (D) 11

Directions (Q. No 11 to 15)

Solve the following equation :

11. $\frac{2x-3}{4x+5} = \frac{1}{3}$

(A) 4 (B) 5

(C) 6 (D) 7

12. $\frac{8}{x} = \frac{5}{x-1}$

(A) $\frac{8}{3}$ (B) $\frac{5}{3}$

(C) 5 (D) 3

13. $\frac{y-(4-3y)}{2y-(3+4y)} = \frac{1}{5}$

(A) $\frac{5}{22}$ (B) $\frac{17}{22}$

(C) $\frac{15}{22}$ (D) 1

14. $10m - 5 - 7m = 5m + 15 - 8$

(A) –7 (B) –5

(C) –6 (D) –4

15. $\frac{1}{2}(m+1) + \frac{1}{3}(m-1) = \frac{5}{12}(m-2)$

(A) $\frac{-12}{5}$ (B) $\frac{5}{12}$

(C) $\frac{-5}{12}$ (D) $\frac{12}{5}$

16. Split a number 540 into such two parts that a part is $\frac{7}{2}$ times of other.

(A) 120, 420 (B) 100, 440

(C) 190, 360 (D) 240, 340

17. Length of a ractangle is 7 m more than its breadth. If perimeter of the rectangle is 234 m, then its length will be–

(A) 55 m (B) 62 m

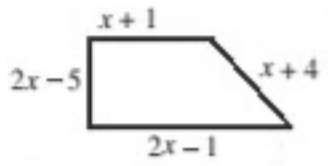
(C) 48 m (D) 42 m

18. If $\frac{1}{2}$ is subtracted from an amount and multiply the diffeence by 4, then 5 is obtained as result. Find the amount.

(A) $\frac{5}{2}$ (B) $\frac{3}{4}$

(C) $\frac{3}{5}$ (D) $\frac{7}{4}$

19. For what value of x, perimeter of the following figure is 41 m?



(A) 4 (B) 5

(C) 6 (D) 7

20. Rahul thinks a number. He doubled that number and added 20 to it. Then divide the resulting number by 25, resulting in 4. Find out what was the number thought by Rahul? **(NCERT)**

(A) 40 (B) 50

(C) 60 (D) 75

21. The sum of two numbers is 85. If one number is 7 more than the other, find the number. **(NCERT)**

(A) 39, 46 (B) 27, 58

(C) 50, 35 (D) 45, 40

22. The father's age is 20 years more than double his son's age. If the sum of their ages is 65 years, find the age of the father.

(A) 50 yrs (B) 40 yrs

(C) 20 yrs (D) 65 yrs

23. The length of a rectangle is twice its width. If the perimeter of the rectangle is 66 cm, find its length.

(A) 11 cm (B) 22 cm

(C) 33 cm (D) 20 cm

24. The number of boys in a class is 2/5 of the number of girls. If the number of boys in the class is 10, find the number of girls. **(NCERT)**

(A) 25 (B) 20

(C) 15 (D) 30

25. Rahim's father is three times the age of Rahim. If the sum of their ages is 56 years, then find the age of the father.

(A) 30 yrs (B) 42 yrs

(C) 45 yrs (D) 51 yrs

26. Rita has a 10 m cloth. He divided it into two parts in such a way that one piece is 4 meters longer than the other. What is the length of a small piece? **(NCERT)**

(A) 3 (B) 4

(C) 5 (D) 7

27. A total of 50000 are to be distributed as prizes to 200 persons. A prize is either 500 or 100. Find the number of prizes of the 100 note. **(NCERT)**

(A) 75 (B) 125

(C) 150 (D) 50

28. A wallet has 100 and 50 notes which have a total value of 25000. If the number of 100 notes is one more than the number of 50 notes, find the number of 50 notes.

(A) 150 (B) 166

(C) 200 (D) 125

29. Increasing 8 in a number gives 26. Find the number. **(NCERT)**

(A) 8 (B) 9

(C) 17 (D) 18

30. The present age of Reena and Meena is in the ratio of 4: 5. After 8 years, their age ratio will be 5: 6. Find the present age of Meena. **(NCERT)**

(A) 30 yrs (B) 32 yrs

(C) 40 yrs (D) 50 yrs

31. The denominator of a rational number is 8 greater than its numerator. If 1 is subtracted from the denominator and 17 is added to the numerator, the number becomes $\frac{3}{2}$. Find the Rational Number. **(NCERT)**

(A) $\frac{11}{2}$ (B) $\frac{3}{10}$

(C) $\frac{13}{21}$ (D) $\frac{25}{13}$

32. The sum of the digits of a two-digit number is 11. If the digits change their place, the newly created number is 27 less than the original number. Find the original number. **(NCERT)**

(A) 72 (B) 27

(C) 74 (D) 47

33. Three years ago, Atul's age was 4 times the age of Parul. After 5 years from today, Atul's age will be twice that of Parul's age. Find the present age of Parul. **(NCERT)**

(A) 19 yrs (B) 7 yrs

(C) 12 yrs (D) 11 yrs

34. The perimeter of a rectangular plot is 32 m. is. If the length is 2 m be increased and the width is reduced by 1 m, the area of the plot remains the same. Find the length of the plot. **(NCERT)**

(A) 6 m (B) 10 m

(C) 16 m (D) 14 m

Solutions

1. (C) In the expression $4x - 18 = 2 + 4x$, Coefficients of x is same in both sides. So, the solutions is not possible.

2. (D) Substitiue

3. (B) $\frac{5x}{3} - 4 = \frac{2x}{5}$

$\because$ LCM of 5 and 3 = 15

So, multiplying by 15 on both sides,

$$25x - 60 = 6x$$

Separating the variables and constant terms,

$$25x - 6x = 60$$

$$19x = 60 \text{ or } x = \frac{60}{19}$$

$$\therefore \quad 2x - 7 = \frac{120}{19} - 7$$

$$= \frac{120 - 133}{19} = \frac{-13}{19}$$

4. (A) According to question,

$$3x - 4 = 2x + 1$$

Separating the variables and constant terms,

$$3x - 2x = 1 + 4$$

$$x = 5$$

5. (C) $\frac{-2}{5}x = -\frac{5}{2}$

On cross multiplication,

$$x = \frac{-5}{2} \times \frac{5}{-2}$$

$$x = \frac{-25}{-4} \text{ or } \frac{25}{4}$$

6. (A) $\because$ Unit digit = m

$\therefore$ Tens digit = $m + 5$

According to question,

Number = 10 (tens digit) + unit digit

$$= 10(m + 5) + m$$

$$= 10m + 50 + m$$

$$= 11m + 50$$

7. (B) Let, Priya's present age = x yrs

Puja's present age = $3x$ yrs

According to question,

$$x - 3 = 21$$

$$x = 21 + 3 = 24 \text{ yrs}$$

$\therefore$ Puja's present age = $3x$

$$= 3 \times 24 = 72 \text{ yrs}$$

8. (B) Let, three consecutive multiples of 5are $5x$, $5(x + 1)$, and $5(x + 2)$.

According to question,

$$5x + 5(x + 1) + 5(x + 2) = 360$$

$$5x + 5x + 5 + 5x + 10 = 360$$

$$15x + 15 = 360$$

$$15x = 360 - 15$$

$$x = \frac{345}{15}$$

$$= 23$$

So, the smallest multiple $= 5x$

$= 23 \times 5$

9. (C) $2y = 5y - \frac{18}{5}$

Shift y varible to LHS,

$$2y - 5y = -\frac{18}{5}$$

$$-3y = \frac{-18}{5}$$

$$y = \frac{18}{5\times 3} = \frac{6}{5}$$

10. (B) According to question,

$4x - 9 = 11$

$4x - 9 + 9 = 11 + 9$ (On adding 9)

$4x = 20$

$$x = \frac{20}{4} = 5$$

11. (D) $\frac{2x-3}{4x+5} = \frac{1}{3}$

By cross multiplication,

$6x - 9 = 4x + 5$

Separate variable and constant terms,

$6x - 4x = 5 + 9$

$2x = 14$ or $x = 7$

12. (A) $\frac{8}{x} = \frac{5}{x-1}$

By cross multiplication,

$8(x - 1) = 5x$

$8x - 8 = 5x$

Separate variable and constant terms,

$8x - 5x = 8$

$3x = 8$ or $x = \frac{8}{3}$

13. (B) $\frac{y-(4-3y)}{2y-(3+4y)} = \frac{1}{5}$

$$\frac{y-4+3y}{2y-3-4y} = \frac{1}{5}$$

$$\frac{4y-4}{-2y-3} = \frac{1}{5}$$

By cross multiplication,

$5(4y - 4) = -2y - 3$

$20y - 20 = -2y - 3$

Separate variable and constant terms,

$20y + 2y = 20 - 3$

$22y = 17$

$$y = \frac{17}{22}$$

14. (C) $10m - 5 - 7m = 5m + 15 - 8$

$(10m - 7m) - 5 = 5m + 7$

$3m - 5 = 5m + 7$

Separate the variable and constant terms,

$3m - 5m = 7 + 5$

$-2m = 12$

$$m = \frac{12}{-2} = -6$$

15. (A) $\frac{1}{2}(m + 1) + \frac{1}{3}(m - 1) = \frac{5}{12}(m - 2)$

$\because$ LCM $(2, 3, 12) = 12$

Multiplying by 12 on both sides,

$6(m + 1) + 4(m - 1) = 5(m - 2)$

$6m + 6 + 4m - 4 = 5m - 10$

$10m + 2 = 5m - 10$

$10m - 5m = -10 - 2$

$$5m = -12 \text{ or } m = \frac{-12}{5}$$

16. (A) Let the first part $= x$

and other part $= \frac{7}{2}x$

According to question,

$$x + \frac{7}{2}x = 540$$

$$\frac{2x+7x}{2} = 540$$

$$\frac{9x}{2} = 540$$

$$x = \frac{540\times 2}{9}$$

$x = 120$

$\therefore$ Other part $= \frac{7}{2}x$

$$= \frac{7}{2}\times 120 = 420$$

17. (B) Let, width of rectangle $= x$ m

and its length $= (x + 7)$ m

$\therefore$Perimeter of rectangle $= 234$ m

$x + (x + 7) + x + (x + 7) = 234$

$4x + 14 = 234$

$4x = 220$

$x = 55$

$\therefore$ its length $= x + 7$

$= 62$ m

18. (D) Let, the amount $= ₹x$

According to question,

$$4\left[x - \frac{1}{2}\right] = 5$$

$$x - \frac{1}{2} = \frac{5}{4}$$

$$x = \frac{5}{4} + \frac{1}{2}$$

$$x = \frac{7}{4}$$

19. (D) $(x + 1) + (x + 4) + (2x - 1) + (2x - 5) = 41$

$(x + x + 2x + 2x) + (1 + 4 - 1 - 5) = 41$

$6x - 1 = 41$

or, $x = 7$ m

20. (A) Let, the number be x.

According to question,

$$\frac{2x+20}{25} = \frac{4}{1}$$

By cross multiplication,

$2x + 20 = 100$

$2x = 100 - 20$

$2x = 80$

or, $x = 40$

21. (A) Let, the numbers be x and $(x + 7)$.

According to question,

$x + x + 7 = 85$

$2x = 78$ or $x = 39$

Hence, the numbers are 39 and 46.

22. (A) Let, Son's age $= x$ *yrs*

Father's age $= (65 - x)$ yrs

According to question,

$65 - x = 2x + 20$

$3x = 45$ or $x = 15$

So, Father's age $= 65 - 15$

$= 50$ yrs

23. (B) Let, width of rectangle $= x$ cm

and length of the rectangle $= 2x$ cm

According to question,

$2(2x + x) = 66$

$3x = 33$ or $x = 11$

Hence, the length $= 22$ cm

24. (A) Girls $= 10 \times \frac{5}{2} = 25$

25. (B) Let, Rahim's age $= x$ *yrs*

His father's age $= 3x$ *yrs*

According to question,

$$3x + x = 56$$

$$4x = 56 \text{ or } x = 14$$

So, the father's age = 42 yr.

26. (A) Let,

Length of smaller part = x m

Length of larger part = $(x + 4)$ m

Accroding to question,

$$x + 4 + x = 10$$

$$2x = 6 \text{ or } x = 3 \text{ m}$$

27. (B) Let, x persons get 500 note and (200 - x) persons get 100 note.

According to question,

$$500x + 100(200 - x) = 50000$$

$$5x + 200 - x = 500$$

$$4x = 300 \text{ or } x = 75$$

$$\therefore \quad 200 - x = 200 - 75$$

$$= 125$$

28. (B) Let, note of 50 = x

notes of 100 = $x + 1$

According to question,

$$50x + 100(x + 1) = 25000$$

$$150x = 24900$$

$$x = \frac{2490}{15} = 166$$

29. (B) Required number $= \frac{26-8}{2}$

$$= \frac{18}{2} = 9$$

30. (C) Let, Reena's age = $4x$

Meena's age = $5x$

According to question,

$$\frac{4x+8}{5x+8} = \frac{5}{6}$$

$$25x + 40 = 24x + 48$$

$$x = 8$$

Meena's age = $5x = 40$ yrs

31. (C) Let, the rational number = $\frac{x}{x+8}$

According to question,

$$\frac{x+17}{(x+8)-1} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{x+17}{x+7} = \frac{3}{2}$$

$$\Rightarrow \quad 3x + 21 = 2x + 34$$

$$\Rightarrow \quad x = 13$$

$\therefore$ the rational number = $\frac{13}{21}$

32. (C) Let, the number = $10x + y$

So, $x + y = 11$ ...(1)

and $10x + y = 10y + x + 27$

$$9x - 9y = 27$$

or $x - y = 3$ ...(2)

On solving eq. (1) and eq. (2),

$x = 7, y = 4$

So, the munber = $7 \times 10 + 4$

$= 74$

33. (B) Let,

Parul's age = x *years*

Atul's age= y *years*

According to question,

$$4(x - 3) = y - 3$$

$\Rightarrow \quad 4x - y = 9$ ...(1)

and $2(x + 5) = (y + 5)$

$2x - y = -5$ ...(2)

On solving eq. (1) and (2),

$x = 7, y = 19$

So, Parul's age = 7 years

34. (B) Let, Plot's length = x *m*

its width = y *m*

According to question,

$$2(x + y) = 32$$

or, $x + y = 16$ ...(1)

and $(x + 2)(y - 1) = xy$

$\Rightarrow \quad xy - x + 2y - 2 = xy$

$\Rightarrow \quad x - 2y = -2$ ...(2)

On solving eq. (1) and (2),

$x = 10, y = 6$

So, Plot's length = 10 m

Chapter 7 Ratio and Proportion

1. Ratio

The rational number obtained by using the operation of division into two same quantites is called the ratio of these quantities. Suppose there are two quantities a and b where $b \neq 0$. The ratio of a and b is expressed by a/b or $a : b$. a is called a predecessor and b is called a successor.

Example– Ratio between 20 km and 100 km $= \frac{20}{100} = \frac{1}{5}$ or 1 : 5.

2. Types of Ratio

I. Duplicate Ratio— If $a : b$, then duplicate ratio of $a : b = a^2 : b^2$

Example: Duplicate ratio of $2 : 5 = (2)^2 : (5)^2 = 4 : 25$

II. Triplicate Ratio—If $a : b$, then triplicate ratio of $a : b = a^3 : b^3$

Example: Triplicate ratio of $1 : 3 = (1)^3 : (3)^3 = 1 : 27$

III. Sub-duplicate Ratio—If $a : b$, then sub-duplicate ratio of $a : b = \sqrt{a} : \sqrt{b}$

Example: Sub-duplicate ratio of $16 : 49 = \sqrt{16} : \sqrt{49} = 4 : 7$

IV. Sub-Triplicate Ratio—If $a : b$, then sub-triplicate ratio of $a : b = \sqrt[3]{a} : \sqrt[3]{b}$

Example: Sub-triplicate ratio of $125 : 8 = \sqrt[3]{125} : \sqrt[3]{8} = 5 : 2$

V. Reciprocal Ratio—If $a : b$, then reciprocal ratio of $a : b = \frac{1}{a} : \frac{1}{b}$ or $b : a$

Example: Reciprocal ratio of $4 : 5 = 5 : 4$

VI. If $a : b$ and $c : d$, then Compound ratio will be–

$a \times c : b \times d$ **or** $ac : bd$

Example: Compound ratio of 2 : 3 and 5 : 7

$= 2 \times 5 : 3 \times 7$

$= 10 : 21$

3. Proportion

When two ratios are kept equal, they are called proportions. If a, b, c, d are in proportion, then they are expressed as–

$$a : b : c : d$$

Here, a and d are known as external terms and c and d are known as internal terms. Hence, we have

Product of external terms = Product of internal terms

or, $$ad = bc$$

3.1 Properties of Proportion

I. If $a : b : : c : d$, then a- first term, b- 2nd term, c- 3rd term, and d- 4th term. Here, d is also called fourth proportional of a, b, and c.

$$d = \frac{bc}{a}$$

Example: Fourth proportional of 6, 4 and 3 $= \frac{4 \times 3}{6} = 2$.

II. If a, b and c are in proportion, then c is called third proportion of a and b.

$$a : b : : b : c \quad \textbf{or} \quad c = \frac{b^2}{a}$$

Example: Third proportion of 2 and 5 $= \frac{5 \times 5}{2} = 12.5$

III. If a, x and b are in proportion, then x is called mean proportion of a and b.

$$a : x : : x : b$$

$$\text{or } x = \sqrt{ab}$$

Example: Mean proportion of 4 and 16 $= \sqrt{4 \times 16}$

$= \sqrt{64} = 8$

IV. If A : B = $a : b$ and B : C = $c : d$, then

$$A : B : C = ac : bc : bd$$

$$A : C = ac : bd$$

Example: If $a : b = 2 : 3$ and $b : c = 4 : 5$, $a : c = ?$

$$a : b : c = 2 \times 4 : 3 \times 4 : 3 \times 5$$

$$= 8 : 12 : 15$$

$$a : c = 8 : 15$$

Important Questions

1. The ages of A and B are in the ratio of 6: 5 and the sum of their ages is 44 years. After 8 years the ratio of their ages will be— **(NCERT)**

(A) 7 : 6 (B) 6 : 5

(C) 4 : 3 (D) 8 : 7

2. The ratio of $\frac{2}{3}$ part of 12.60 to 0.6 part of 21 will be–

(A) $\frac{1}{2} : \frac{1}{3}$ (B) 2 : 3

(C) 3 : 2 (D) 3 : 5

3. The ratio of the interior angles of a triangle is 1 : 2 : 3. The value of the largest angle in radians will be– **(NCERT)**

(A) $\frac{\pi}{2}$ (B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{3}$ (D) $\frac{3\pi}{2}$

4. Sheela and Shilpa had some coins in the ratio 5: 3. If Sheela had 400 and Shilpa had only 5 rupees' coins, how many coins did Shilpa have?

(A) 96 (B) 56

(C) 48 (D) 24

5. 234 had to distribute into three persons A, B and C in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$, but

it was distributed in the ratio 2 : 3 : 4, mistakenly. Who benefited the most and how much? **(NCERT)**

(A) A, ₹ 52 (B) B, ₹ 35

(C) A, ₹ 56 (D) C, ₹ 50

6. The sum of three numbers is 58. The ratio of the first number to the second number is 4 : 9 and the ratio of the second number to the third number is 9: 16. The sum of the first and third numbers is–

(A) 40 (B) 29

(C) 28 (D) 50

7. The average marks obtained by all the students of class A and class B in an examination were 68.4 and 71.2 respectively. If the combined average of the marks obtained by the students of both the classes is 70, then the ratio of the number of students of class A and class B is–

(A) 15 : 6 (B) 3 : 7

(C) 4 : 3 (D) 3 : 4

8. Which of the following is incorrect–

(A) If quantites a, b, c, d are in proportion, then $a \times d = b \times c$

(B) If quantites a, b, c, d are in proportion, then $ac = bd$

(C) Inverse proportion of $a : b$ is $b : a$.

(D) If quantites a, b, c are in G.P., then $b^2 = ac$

9. If $a : b = 4 : 5, b : c = 6 : 9, c : d = 15 : 19$, then $a : b : c : d$ will be–

(A) 8 : 10 : 15 : 19

(B) 10 : 11 : 15 : 19

(C) 8 : 9 : 15 : 19

(D) 19 : 15 : 6 : 8

10. If half of a number equals to 0.07 of other number, then their ratio will be–

(A) 50 : 7 (B) 5 : 7

(C) 7 : 50 (D) 1 : 14

11. Ratio between X's income and Y's income is 5 : 4 and ratio between their expenses is 3 : 2. If each saves 1600 at end of the year, the find X's income.

(A) ₹ 3600 (B) ₹ 5000

(C) ₹ 4000 (D) ₹ 1600

12. There are 2500 students in a school. If the number of girls in the school is 1400, then find the ratio between the number of boys and girls in the school.

(A) 11 : 14 (B) 10 : 15

(C) 14 : 25 (D) 25 : 14

13. If X : Y = 3 : 4 and Y : Z = 8 : 9, then X : Y : Z will be– **(NCERT)**

(A) 9 : 7 : 6 (B) 6 : 8 : 9

(C) 6 : 9 : 7 (D) 8 : 9 : 6

14. If $x : y = 7 : 3$, then find $\frac{xy + y^2}{x^2 - y^2}$.

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$

(C) $\frac{3}{7}$ (D) $\frac{7}{3}$

15. If A : B : C = 2 : 3 : 5, then find the ratio $\frac{B+C}{A} : \frac{C+A}{B} : \frac{A+B}{C}$.

(A) 4 : 7 : 1 (B) 12 : 7 : 3

(C) 4 : 7 : 3 (D) 4 : 7 : 13

16. If $p : q = r : s = t : u = 2 : 3$, then find the value of $(mp + nr + ot) : (mq + ns + ou)$.

(A) 1 : 3 (B) 1 : 2

(C) 2 : 3 (D) 3 : 2

17. Which number should be added to each of the numbers 6, 7, 15 and 17 for making them proportional? **(NCERT)**

(A) 6 (B) 5

(C) 4 (D) 3

18. If A = $\frac{4}{5}$ of B and B = $\frac{5}{2}$ of C, then find A : C.

(A) 1 : 2 (B) 2 : 1

(C) 2 : 3 (D) 1 : 3

19. $33\frac{1}{3}\%$ of B = 1.5 of B = $\frac{1}{8}$ of C, then A : B : C equals to– **(NCERT)**

(A) 24 : 2 : 9 (B) 2 : 9 : 24

(C) 9 : 2 : 24 (D) 9 : 24 : 2

20. If $(3x - y) : (x + 5y) = 5 : 7$, then the ratio $(x + y) : (x - y)$ will be–

(A) 2 : 3 (B) 3 : 2

(C) 3 : 1 (D) 1 : 3

21. If b is mean proportion of a and c, then $(a - b)^3 : (b - c)^3$ equals to–

(A) $a^3 : c^3$ (B) $b^2 : c^2$

(C) $a^2 : c^2$ (D) $a^3 : b^3$

22. Solve : $6^{29} : 3^{29}$ **(NCERT)**

(A) 3^{29}

(B) 6^{29}

(C) 2^{29}

(D) None of these

23. The value of the angles of a triangle is in the ratio 2: 7: 11. What will be the measure of the angles?

(A) 16°, 56°, 88° (B) 18°, 63°, 99°

(C) 20°, 70°, 90° (D) 25°, 175°, 105°

24. Two numbers are 20% and 50% more than a third number respectively. The ratio in these two numbers will be–

(A) 2 : 5 (B) 4 : 5

(C) 6 : 7 (D) 3 : 5

25. A person takes a loan for 5 years. If loan amount: total interest = 5: 2, then the interest rate will be– **(NCERT)**

(A) 8% (B) 5%

(C) $8\frac{1}{2}\%$ (D) $5\frac{1}{2}\%$

26. If 1000 is divided in the ratio of 3 : 2 into A and B, then A will get–

(A) ₹ 400 (B) ₹ 500

(C) ₹ 600 (D) ₹ 800

27. If $p : q : r = 1 : 2 : 4$, then $\sqrt{5p^2 + q^2 + r^2}$ equals to–

(A) 5 (B) $2q$

(C) $5p$ (D) $4r$

28. Ratio between A and B is 4 : 5 and between B and C is 2 : 3. If A has 800, then C has– **(NCERT)**

(A) ₹ 1000 (B) ₹ 1200

(C) ₹ 1500 (D) ₹ 2000

29. Ratio of three numbers is 1 : 2 : 3 and sum of their squares is 126. Then, the numbers will be–

(A) 2, 4, 3 (B) 1, 2, 3

(C) 3, 6, 9 (D) 4, 8, 12

30. 1050 is distributed among 1500 males and females in such a way that each male got 1 and each female got 50p. Find the numbers of the females.

(A) 600 (B) 750

(C) 800 (D) 900

31. A, B, C hire a taxi for 1040 and travels for 14 hrs, 16 hrs and 22 hrs respectively. Find the share of C in the rent.**(NCERT)**

(A) ₹ 540 (B) ₹ 280

(C) ₹ 440 (D) ₹ 320

32. The sum and difference of two numbers is 40 and 4 respectively. What will be the ratio of the numbers?

(A) 21 : 19 (B) 22 : 9

(C) 11 : 9 (D) 11 : 18

SOLUTIONS

1. (D) Ratio between A and B = 6 : 5

and sum of their ages = 44 years

$\therefore$ A's present age = $\frac{6}{11} \times 44$

= 24 yrs

$\therefore$ B's present age = 44 – 24

= 20 yrs

After 8 years,

A's age = 32 years

B's age = 28 years

$\therefore$ Required ratio = 32 : 28 or 8 : 7

2. (B) $\because$ ₹ $\frac{2}{3}$ of 12.60 $= 12{\cdot}60 \times \frac{2}{3}$

$= ₹\ 8{\cdot}40$

and ₹ 0.6 of 21 $= 21 \times 0{\cdot}6$

$= ₹\ 12{\cdot}60$

$$\text{Required ratio} = \frac{8{\cdot}4}{12{\cdot}6} = \frac{2}{3}$$

$= 2 : 3$

3. (A) Sum of angles = 180°

Ratio of angles = 1 : 2 : 3

$$\text{Largest angle} = \frac{3}{1+2+3} \times 180°$$

$$= \frac{3}{6} \times 180° = 90°$$

$$= 90° \times \frac{\pi}{180°} \text{ radian}$$

$$= \frac{\pi}{2} \text{ radian}$$

4. (C) Let, Sheela and Shilpa had $5x$ and $3x$.

According to question,

$₹5x = 400$

$\Rightarrow$ $x = ₹\ 80$

$\therefore$ Shilpa's amount $= 3x = 3 \times 80$

$= ₹\ 240$

$\because$ Shilpa had all the money in form 5 rupee coins.

$\therefore$ 5 rupee coins $= \frac{240}{5} = 48$

5. (C) Correct ratio of A, B and C

$$= \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6 : 4 : 3$$

Incorrect ratio of A, B and C

$= 2 : 3 : 4$

Then, A's profit

$$= \frac{6}{13} \times 234 - \frac{2}{9} \times 234$$

$$= \frac{28}{13 \times 9} \times 234 = ₹\ 56$$

$$\text{B's profit} = \frac{4}{13} \times 234 - \frac{3}{9} \times 234$$

$$= \frac{-3}{13 \times 9} \times 234 = ₹\ (-6)$$

$$\text{C's profit} = \frac{3}{13} \times 234 - \frac{4}{9} \times 234$$

$= -25 \times 2 = -₹\ 50$

Hence, A got the maximum profit, *i.e.* 56.

6. (A) Given ratio = 4 : 9 : 16

Let, the numbers be $4x$, $9x$ and $16x$.

According to question,

$4x + 9x + 16x = 58$

$\Rightarrow$ $x = 2$

Sum $= 4x + 16x = 20x$

$= 20 \times 2 = 40$

7. (D) Let, x and y be the number of students in class A and B.

According to question,

$$\therefore \quad \frac{x \times 68{\cdot}4 + y \times 71{\cdot}2}{(x+y)} = 70$$

$\Rightarrow$ $x \times 68{\cdot}4 + y \times 71{\cdot}2 = 70x + 70y$

$\Rightarrow$ $y\,(71{\cdot}2 - 70) = x\,(70 - 68{\cdot}4)$

$\Rightarrow$ $1{\cdot}2 \times y = x \times 1{\cdot}6$

$\Rightarrow$ $x : y = 1{\cdot}2 : 1{\cdot}6$ **or** $3 : 4$

8. (B) If a, b, c, d are in proportion, then $ac = bd$

9. (A) $a : b = 4 : 5, b : c = 6 : 9, c : d = 15 : 19$

$a \quad : \quad b \quad : \quad c$

$4 \quad : \quad 5$

$\quad\quad\ 6 \quad : \quad 9$

$a : b : c = 24 : 30 : 45$

Again, $a \quad : \quad b \quad : \quad c \quad : \quad d$

$24 \quad : \quad 30 \quad : \quad 45$

$\quad\quad\quad\quad\ 15 \quad : \quad 19$

$\therefore$ $a : b : c : d = 360 : 450 : 675 : 855$

$= 8 : 10 : 15 : 19$

10. (C) Required ratio $= \frac{7}{100} \times \frac{2}{1} = \frac{7}{50}$

11. (C) Let, X's income and expenses are $5a$ and $3b$ respectively. Y's income and expenses are $4a$ and $2b$ respectively.

According to question,

$5a - 3b = 1600$ ...(1)

$4a - 2b = 1600$ ...(2)

On solving eq. (1) and (2),

$a = 800$

So, A's income $= 5a$

$= 5 \times 800 = ₹\ 4000$

12. (A) Total no. of students in school

= 2500

$\because$ Total no. of girls in school

= 1400

$\therefore$ Total no. of girls in school

$= 2500 - 1400$

$= 1100$

$\therefore$ Required ratio = 1100 : 1400

$= 11 : 14$

13. (B) X : Y = 3 : 4 and Y : Z = 8 : 9

$\therefore$ X : Y : Z = 3 × 8 : 4 × 8 : 4 × 9

$= 24 : 32 : 36$

$= 6 : 8 : 9$

14. (A) $\frac{x}{y} = \frac{7}{3}$ ...(1)

$$\frac{xy + y^2}{x^2 - y^2}$$

On dividing numerator and denominator by y^2,

$$\frac{\frac{x}{y} + 1}{\left(\frac{x}{y}\right)^2 - 1} = \frac{\frac{7}{3} + 1}{\left(\frac{7}{3}\right)^2 - 1} = \frac{\frac{10}{3}}{\frac{49-9}{9}}$$

$$= \frac{10}{3} \times \frac{9}{40} = \frac{3}{4}$$

15. (B) A : B : C = 2 : 3 : 5

$$\frac{B+C}{A} : \frac{C+A}{B} : \frac{A+B}{C}$$

Put A = 2, B = 3 and C = 5,

$$\frac{3+5}{2} : \frac{5+2}{3} : \frac{2+3}{5}$$

$$= 4 : \frac{7}{3} : 1 = 12 : 7 : 3$$

16. (C) Here, $p = r = t = 2$

and $q = s = u = 3$

$(mp + nr + ot) : (mq + ns + ou)$

$= [m(2) + n(2) + o(2)] : [m(3) + n(3) + o(3)]$

$= 2(m + n + o) : 3(m + n + o)$

$= 2 : 3$

17. (D) Let, the required number be x.

$$\Rightarrow \quad \frac{6+x}{7+x} = \frac{15+x}{17+x}$$

$\Rightarrow$ $(6 + x)(17 + x) = (15 + x)(7 + x)$

$\Rightarrow$ $102 + 6x + 17x + x^2$

$= 105 + 15x + 7x + x^2$

$\Rightarrow$ $23x - 22x = 105 - 102$

$\Rightarrow$ $x = 3$

18. (B) A = $\frac{4}{5}$ of B and B = $\frac{5}{2}$ of C

$$A = B \times \frac{4}{5} = \frac{4B}{5}$$

and $B = C \times \frac{5}{2} = \frac{5}{2}C$

Put the value in eq. (1),

$$A = \frac{4}{5}\left(\frac{5}{2}C\right) \qquad \text{[From eq.(1)]}$$

A = 2C **or** $\frac{A}{C} = \frac{2}{1}$

A : C = 2 : 1

19. (C) $33\frac{1}{3}\%$ of B = 1.5 of B

$= \frac{1}{8}$ of C

$$A \times \frac{100}{3 \times 100} = B \times 1.5 = C \times \frac{1}{8}$$

$$\Rightarrow \quad \frac{A}{3} = \frac{15B}{10} = \frac{C}{8}$$

$$\Rightarrow \quad \frac{A}{3} = \frac{B}{\frac{2}{3}} = \frac{C}{8}$$

$$\Rightarrow \quad A : B : C = 3 : \frac{2}{3} : 8$$

$$= 9 : 2 : 24$$

20. (C) $\frac{3x - y}{x + 5y} = \frac{5}{7}$

$$\Rightarrow \quad 21x - 7y = 5x + 25y$$

$$\Rightarrow \quad 16x = 32y$$

$$\Rightarrow \quad \frac{x}{y} = \frac{32}{16}$$

$$\frac{x}{y} = \frac{2}{1}$$

$$\frac{x + y}{x - y} = \frac{2 + 1}{2 - 1}$$

$$= 3 : 1$$

21. (D) b is mean proportion of a and c.

$a : b :: b : c$

$$\frac{a}{b} = \frac{b}{c} = k \text{ (Let)}$$

$a = bk$ and $b = ck$

$$\frac{(a-b)^3}{(b-c)^3} = \frac{(bk-b)^3}{(ck-c)^3}$$

$$= \frac{b^3(k-1)^3}{c^3(k-1)^3}$$

$$= \left(\frac{b}{c}\right)^3 = \left(\frac{\frac{a}{k}}{\frac{b}{k}}\right)^3$$

$$= \frac{a^3}{b^3} = a^3 : b^3$$

22. (C) $6^{29} : 3^{29} = \left(\frac{6}{3}\right)^{29} = 2^{29}$

23. (B) $2 + 7 + 11 \longrightarrow 180°$

$20° \longrightarrow 180°$

$1° \longrightarrow \frac{180}{20} = 9°$

1st angle, $2 \longrightarrow 9 \times 2 = 18°$

2nd angle, $7 \longrightarrow 9 \times 7 = 63°$

3rd angle, $11 \longrightarrow 9 \times 11 = 99°$

24. (B) $\frac{120}{100} : \frac{150}{100} = 12 : 15$

$\Rightarrow \quad = 4 : 5$

25. (A) According to question,

$$\frac{P}{\frac{P \times R \times T}{100}} = \frac{5}{2}$$

$$\frac{100}{R \times 5} = \frac{5}{2}$$

$$R = 8\%$$

26. (C) A's part $= \frac{3}{3+2} \times 1000$

$= 3 \times 200 = ₹\ 600$

27. (A) $\sqrt{5p^2 + q^2 + r^2}$

$= \sqrt{5(1)^2 + 2^2 + 4^2}$

$= \sqrt{5 + 4 + 16}$

$= \sqrt{25} = 5$

28. (C) A : B : C

4 : 5

2 : 3

$4 \times 2 : 5 \times 2 : 5 \times 3$

$A : B : C = 8 : 10 : 15$

C's part $= \frac{15}{8} \times 800 = ₹\ 1{,}500$

29. (C) Let, the three numbers be x, $2x$ and $3x$ respectively.

According to question,

$$x^2 + (2x)^2 + (3x)^2 = 126$$

$$\Rightarrow \quad x^2(1 + 4 + 9) = 126$$

$$\Rightarrow \quad x^2 = \frac{126}{14}$$

$$x = \sqrt{9}$$

$$x = 3$$

Numbers = x, $2x$ and $3x$

$= 3,\ 2 \times 3$ and 3×3

$= 3, 6, 9$

30. (D) Let, Total no. of females be x.

Males $= 1500 - x$

According to question,

$$(1500 - x) \times 1 + x \times \frac{1}{2} = 1050$$

$$\Rightarrow \quad 1500 - x + \frac{x}{2} = 1050$$

$$\Rightarrow \quad -\frac{x}{2} = 1050 - 1500$$

$$-\frac{x}{2} = -450$$

$$x = 900$$

31. (C) $14 : 16 : 22 = 7 : 8 : 11$

C's part $= \frac{11}{7+8+11} \times 1040$

$= \frac{11}{26} \times 1040$

$= ₹\ 440$

32. (C) $x + y = 40$ ...(1)

$x - y = 4$ ...(2)

$2x = 44$

$x = 22$

[From eq. (1)]

$22 + y = 40$

$y = 18$

$x : y = 22 : 18 = 11 : 9$

Chapter 8

Percentage

1. Introduction

When we express an amount as a part of 100, it is called a percentage. Percent means 'per hundred' or 'hundredth'.

Percent is an abbreviated form of the Latin language parcentum.

To understand, usually when we say that Amit got 80 marks in the 12th examination, it means that Amit got 80 marks for every 100 marks. Also the percentage can also be understood in such a way that for every 100 or a fraction whose denominator will be 100.

Generally, $x\% = \frac{x}{100}$, $y\%$ of $x = \frac{x \times y}{100}$

Example : $25\% = \frac{25}{100}$, 25% of $200 = \frac{200 \times 25}{100} = 50$

2. Conversion

2.1 Express a fraction into percentage :

Example : Express $\frac{16}{25}$ into percentage -

(A) 32% (B) 64 % (C) 4/625 (D) 50%

Sol (B): To solve these type of questions, multiply the numerator directly by 100. Put % sign next to the answer.

$$\frac{16}{25} \times 100 = 64\%$$

2.2 Express percetage into a fraction or decimal :

Example : Express $87\frac{1}{2}\%$ into fraction.

(A) $\frac{6}{7}$ (B) $\frac{9}{8}$ (C) $\frac{7}{8}$ (D) $\frac{1}{4}$

Sol (C): When a percentage has to be converted to a fraction, divide it by 100.

$$87\frac{1}{2}\% = \frac{175}{2}\% = \frac{175}{2 \times 100} = \frac{7}{8}$$

3. Some Numbers and their Percentage Values

$\frac{1}{2} = 50\%$	$\frac{1}{3} = 33.33\%$
$\frac{1}{4} = 25\%$	$\frac{1}{5} = 20\%$
$\frac{1}{6} = 16.66\%$	$\frac{1}{7} = 14.28\%$
$\frac{1}{8} = 12.5\%$	$\frac{1}{9} = 11.11\%$
$\frac{1}{11} = 9.09\%$	$\frac{1}{13} = 7.69\%$
$\frac{1}{15} = 6.66\%$	$\frac{1}{25} = 4\%$

4. Important Formulae

(I) $x\% = \frac{x}{100}$

(II) $y\%$ of $x = x \times \frac{y}{100} = \frac{xy}{100}$

(III) What percentage of y is x? $= \left(\frac{x \times 100}{y}\right)$

(IV) If there is an increase of $x\%$ in the price of a commodity, then the percentage decrease in its consumption so that there is no increase in the expenditure will be

$$= \left\{\frac{x}{(100 + r)} \times 100\right\}\%$$

(V) If there is a decrease of $y\%$ in the price of a commodity, then the percentage increase in its consumption so that the expenses remain the same

$$= \left\{\frac{x}{(100 - y)} \times 100\right\}\%$$

(VI) If the population of a city is N and the annual growth rate is x%, then

(i) Population after t years, $= N\left(1 + \frac{x}{100}\right)^t$

(ii) Population before t years, $= \frac{N}{\left(1 + \frac{x}{100}\right)^t}$

(VII) If the population of a city is N, in the first year the population increases at the rate of x%, in the second year at the rate of y% and in the third year the population increases at the rate of z%, then after 3 years the population

$$= N\left(\frac{100 + x}{100}\right)\left(\frac{100 + y}{100}\right)\left(\frac{100 + z}{100}\right)$$

Note : $=$ for increases $x\%$ $\left(\frac{100 + x}{10}\right)$

$=$ for decreases $y\%$ $\left(\frac{100 - y}{100}\right)$

(VIII) If A has an amount of x. Out of which the first person is given $a\%$, $b\%$ of the remaining to the second person and $c\%$ of the remaining to the third and A has 5 leftover money. Then,

$$R = 5 \times \frac{100}{(100-x)} \times \frac{100}{(100-y)} \times \frac{100}{(100-z)}$$

(IX) If A has $x\%$ more amount than B, then B has less percentage than $A = \left\{\left(\frac{x}{100+x}\right) \times 100\right\}\%$

(X) If A has $x\%$ less amount than B, then B has more percentage than $A = \left\{\left(\frac{x}{100-x}\right) \times 100\right\}\%$

(XI) If A and B are $x\%$ and $y\%$ more than the third number C, then what percentage of B is $A = \frac{(100+x)}{(100+y)} \times 100$

Important Questions

1. If numerator of a fraction is decreased by 20% and its denominator increased by 10%, then the value of the fraction becomes $\frac{16}{55}$. Find the original fraction—

(A) $\frac{2}{5}$ (B) $\frac{5}{2}$

(C) $\frac{3}{5}$ (D) $\frac{5}{3}$

2. There are 60% males out of 1000 residents in a town and out of which 20% are literate. If the total 25% of residents are literate, then what percentage of females of the town are literate? **(NCERT)**

(A) 30 (B) 37.5

(C) 35 (D) 32.5

3. If ? % of $(4)^? = 51.2$, then find the value of question mark (?). **(NCERT)**

(A) 4 (B) 5

(C) 8 (D) 10

4. A's monthly income is 20% more than B's monthly income. B's income is 30% of C's income. If the gross monthly income of them is 74700, then find the C's income.

(A) ₹ 50000 (B) ₹ 45000

(C) ₹ 25000 (D) ₹ 35000

5. If $x\%$ of x is 36, then find x.

(A) 45 (B) 50

(C) 60 (D) 75

6. Radhika bought a car for 250000. its price has fallen by 10% in the next year. and 12% of fall occurred in the subsequent year. The total fall percentage in these two years is—

(A) 3.2% (B) 22%

(C) 20.8% (D) 8%

7. 16% of a number is added to 21, then it gets itself. Find the number.

(A) 81 (B) 25

(C) 18 (D) 64

8. There are 20 liters diesel in a container. Due to leakage, there is a loss of 3 liters diesel. What percentage of diesel has remained in the container?

(A) $66\frac{2}{3}\%$ (B) 68%

(C) 82% (D) 85%

9. If x is 25% less than y, then how much percentage y is more than x?

(A) 25 (B) $33\frac{1}{3}$

(C) $66\frac{1}{3}$ (D) 75

10. What percentage of 1 kg is 1gm?

(A) 1% (B) 10%

(C) 0.1% (D) 0.01%

11. 5% of 0.4 is —

(A) 0.08 (B) 0.8

(C) 0.02 (D) 0.2

12. If a man earns 2400 and saves 600, what percentage of his income is spent? **(NCERT)**

(A) 75% (B) 80%

(C) 40% (D) 36%

13. A's income is 25% more than B, then what percentage of B's income to the A's income?

(A) 90% (B) 80%

(C) 75% (D) 125%

14. There are 25 liters petrol in a vessel. Due to leakage, 5 liters petrol losses. What percentage of petrol is remained in the vessel?

(A) 80% (B) 68%

(C) 82% (D) 85%

15. 40% of (100 – 20% of 300) = ?

(A) 64 (B) 140

(C) 16 (D) 20

16. In a class, 75% passes in English, 60% in Math and 20% fails in both subjects. Find the passing percentage.

(A) 55% (B) 65%

(C) 67.5% (D) 68.2%

17. Express 0·33 ÷ 0·11 in percentage. **(NCERT)**

(A) $\frac{1}{300}$ (B) $\frac{1}{3}$

(C) 30 (D) 300

18. Two positive numbers x and y are inversely proportional. If x is increased by 10%, then y will be decresed by —

(A) $\frac{100}{11}\%$ (B) $\frac{10}{11}\%$

(C) 10% (D) $\frac{2}{11}\%$

19. There are 40% girls in a class of 50 students. Find the number of boys in the class. **(NCERT)**

(A) 20 (B) 10

(C) 25 (D) 30

20. An institute has 2500 members. If it increases 10% in the first year and decreases 10% in the second year, what will be the increment or decrement in the number of members after these two years?

(A) No increase no decrease

(B) Increase of 25 members

(C) Decrease of 25 members

(D) None of these

21. The value of a machine is devalued at the rate of 10% per annum. It was purchased three years ago. If its present value is 1,45,800, how much was it bought for?

(A) ₹ 1,80,000 (B) ₹ 2,00,000

(C) ₹ 2,10,000 (D) ₹ 1,75,800

22. A has 20% more money than B and C has 20% less money than B. What percentage of money A has more than C?

(A) 50 (B) 17

(C) 43 (D) 30

23. If 60% of X is 30 less than 75% of X, find X. **(NCERT)**

(A) 200 (B) 600

(C) 750 (D) 300

24. What percentage of 16×8 is 4×24.

(A) 96% (B) 24%

(C) 75% (D) 32%

25. If 200% of a number is 90, what will be 80% of the number?

(A) 36 (B) 72

(C) 81 (D) 144

26. In an examination, 65% pass out of total candidates. If the number of failed candidates is 420, find the total number of students.

(A) 1600 (B) 840

(C) 1000 (D) 1200

27. What will be the sum of 30% of 20 and 20% of 30.

(A) 50% of 600 (B) 10% of 1200

(C) 1% of 1200 (D) 60% of 500

28. If 48% of X is 60% of Y, Find X : Y. **(NCERT)**

(A) 5: 4 (B) 2: 3

(C) 6: 7 (D) None of these

29. Which of the following is equal to 6.25%?

(A) 0.00625 (B) 0.0625

(C) 0.625 (D) 6.25

30. If 15% of an amount is 45, then find the amount. **(NCERT)**

(A) ₹ 300 (B) ₹ 325

(C) ₹ 350 (D) ₹ 250

31. What percentage of 5% is 3%?

(A) 60% (B) 50%

(C) 40% (D) 30%

32. If 8% of a = 4% of b, then 20 % of a = ?

(A) 10% of b (B) 16% of b

(C) 80% of b (D) None of these

33. If 40% of 40% of x = 40, find x.

(A) 100 (B) 250

(C) 400 (D) 1000

34. If 905 of x = 30% of y and $y = p\%$ of x, then find the value p?

(A) 300 (B) 600

(C) 800 (D) 900

35. If $x\%$ of a equals to $y\%$ of b, then find $z\%$ of b—

(A) $\frac{yz}{x}\%$ of a (B) $\frac{xy}{z}\%$ of a

(C) $\frac{xz}{y}\%$ of a (D) None of these

36. $x\%$ of $y + y\%$ of x = ?

(A) $x\%$ of y (B) $y\%$ of x

(C) $xy\%$ of 3 (D) 2% of xy

37. If 90% of y is x,then what percentage of x is y?

(A) 90% (B) 190%

(C) $101\frac{1}{9}\%$ (D) $111\frac{1}{9}\%$

38. If 80% of y is x, then what percentage of $2x$ is y ?

(A) 40% (B) $62\frac{1}{2}\%$

(C) 80% (D) 160%

39. If 50% of $(x - y)$ = 30% of $(x + y)$, then what percentage of x is y?

(A) 25% (B) $33\frac{1}{3}\%$

(C) 40% (D) 400%

40. If $x\%$ of 1400 = 119, then find x.

(A) 17 (B) 28

(C) 8.5 (D) 7.5

41. If $5\frac{1}{2}$ of 240 = 150% of x, find x.

(A) 1320 (B) 660

(C) 880 (D) 1980

42. If $x\%$ of 350 is 21, find x.

(A) 5 (B) 7

(C) 6 (D) 8

43. If $x\%$ of $\frac{25}{2}$ is 150, then the value of x is—

(A) 1000 (B) 1200

(C) 1400 (D) 1500

44. What will be 40% of the number whose 2005 is 90?

(A) 18 (B) 45

(C) 16 (D) 36

45. If 20% of a number is 120, then 120% of the number will be—

(A) 20 (B) 120

(C) 360 (D) 720

46. 50% of a number is 39 more than 35% of the number. Find the 115% of the number.

(A) 179 (B) 299

(C) 85 (D) 215

47. If 35% of a number is 12 less than 50% of that number, then find the number.

(A) 80 (B) 60

(C) 50 (D) 40

48. A number is 42 more than its 16%. Find the number.

(A) 58 (B) 50

(C) 52 (D) 60

49. If 35% of a number is 63, what will be the number?

(A) 180 (B) 98

(C) 135 (D) 150

50. If 16% of 40% of a number is 8, then what will be the number?

(A) 200 (B) 225

(C) 125 (D) 320

51. If $\frac{1}{3}$ of $\frac{3}{7}$ of $\frac{2}{9}$ of a number is 16, then what will be 25% of the number?

(A) 504 (B) 126

(C) 252 (D) 378

52. If 15% of 30% of a number is 18, then what is the number?

(A) 90 (B) 360

(C) 380 (D) 400

53. 8% of 36 = 72% of x, find x.

(A) 2.88 (B) 4

(C) 16 (D) 40

54. If 25% of a number is 2 more than 15% of 40, then find the number.

(A) 16 (B) 20

(C) 24 (D) 32

55. If a number is 20% more than 50, find the number.

(A) 10 (B) 20

(C) 40 (D) 60

56. Adding 24 to a number is equal to adding 20% to that number. What is 120% of that number?

(A) 240 (B) 144

(C) $115\frac{1}{5}$ (D) $172\frac{4}{5}$

57. If 40 is subtracted from a number, then that number decreases to 60% of itself. What is 100% of that number?

(A) 60 (B) 100

(C) 40 (D) 80

58. Decreasing 40% of a number gives 30. The number is—

(A) 28 (B) 50

(C) 52 (D) 70

59. If 50 is subtracted from 50% of a number, 50 is left as remainder. Find the number

(A) 150 (B) 400

(C) 200 (D) 300

60. 8% of x is added to x. After this, 8% of the result is then added to the result, in the final answer multiple of x is—

(A) 1166.4 (B) 116.64

(C) 11.664 (D) 1.1664

61. If 5% of x is added to x, then how many times of x will the number obtained be?

(A) 105 (B) 10.5

(C) 0.105 (D) 1.05

62. A is 5 times of B. How much percent is B less than A?

(A) 20% (B) 25%

(C) 75% (D) 80%

SOLUTIONS

1. (A) Required fraction $= \frac{16}{55} \times \frac{(100+10)}{(100-20)}$

$$= \frac{16}{55} \times \frac{110}{80} = \frac{2}{5}$$

2. (D) Total residents

$$= 1000 \xrightarrow[\text{Literate}]{25\%} 250$$

$$60\% \text{ males} \rightarrow 600 \xrightarrow[\text{Literate}]{20\%} 120$$

$$40\% \text{ females} \rightarrow 400$$

$\therefore$ $x\%$ of $400 = (250 - 120)$ Lit. females

$$400 \times \frac{x}{100} = 130$$

$$\Rightarrow \quad x = \frac{130 \times 100}{400} = 32{\cdot}5\%$$

3. (B) Let, $x\%$ of $(4)^x = 51{\cdot}2$

$$4^x \times x = 5120$$

$$= 4 \times 4 \times 4 \times 4 \times 4 \times 5$$

$$= 4^5 \times 5$$

$$\Rightarrow \quad x = 5$$

4. (B) Let, C's income $= ₹\ 100$

$\therefore$ B's income $= ₹\ 30$ and

A's income $= 30 + 6 = ₹\ 36$

Total income $= 100 + 30 + 36$

$= 166$

According to question,

$$₹\ 166 \equiv 74700$$

$$\therefore \quad 100 \equiv \frac{74700}{166} \times 100$$

$$= ₹\ 45000$$

5. (C) $x\%$ of $x = 36$

$$\Rightarrow \quad \frac{x \times x}{100} = 36$$

$$\Rightarrow \quad x^2 = 3600$$

$$\Rightarrow \quad x = 60$$

6. (C) Let, $x = -10\%$, $y = -12\%$

Required fall percentage

$$= \left[x + y + \frac{xy}{100}\right]\%$$

$$= \left[-10 - 12 + \frac{120}{100}\right]\%$$

$$= (-22 + 1{\cdot}2)\% = -20{\cdot}8\%$$

7. (B) 16% of $x + 21 = x$

$$\Rightarrow \quad \frac{16x}{100} + 21 = x$$

$$\Rightarrow \quad \frac{4x}{25} + 21 = x \Rightarrow x - \frac{4x}{25} = 21$$

$$\Rightarrow \quad \frac{21x}{25} = 21 \Rightarrow x = 25$$

8. (D) Diesel quantity in the container

$= 20$ L

Loss due to leakage $= 3$ L

Remaining diesel quantity in the container $= 20 - 3 = 17$

$\therefore$ Required $\% = \frac{17}{20} \times 100\%$

$= 85\%$

9. (B) $\because$ Value of x is 25% less than y. So, y is greater than x.

$$= \frac{25}{100 - 25} \times 100\%$$

$$= \frac{25}{75} \times 100 = \frac{100}{3} = 33\frac{1}{3}\%$$

10. (C) $\because$ 1 kg = 1000 gm

$\therefore$ Required percentage

$$= \frac{1}{1000} \times 100 = 0.1\%$$

11. (C) 0·4 of $5\% = 0{\cdot}4 \times \frac{5}{100} = \frac{2}{100}$

$= 0{\cdot}02$

12. (A) Total income of a person $= 2400$

Saving $= ₹\ 600$

$\therefore$ Total expenses $= ₹\ (2400 - 600)$

$= ₹\ 1800$

So, required $\% = \frac{1800}{2400} \times 100 = 75\%$

13. (B) Let, B's income $= ₹\ 100$

$$\therefore \quad \text{A's income} = 100 + \frac{25}{100} \times 100$$

$$= 125$$

So, required percentage

$$= \frac{100}{125} \times 100\%$$

$$= 80\%$$

14. (A) Remaining petrol in the container

$= 25 - 5$

$= 20$ L

$\therefore$ Remaining percentage $= \frac{20}{25} \times 100$

$= 80\%$

15. (C) 40% of (100 – 20% of 300)

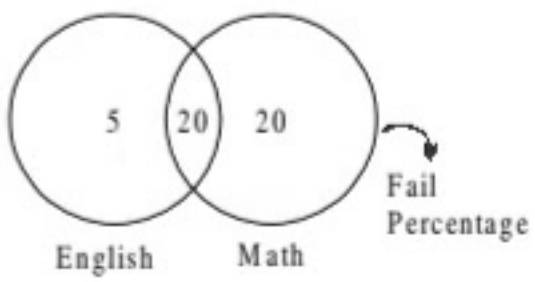
$$= \left(100 - 300 \times \frac{1}{5}\right) \times \frac{2}{5}$$

$$= (100 - 60) \times \frac{2}{5}$$

$$= 40 \times \frac{2}{5} = 16$$

16. (A) Pass percentage $= 100 - (5 + 20 + 20)$

$= 100 - 45^\circ = 55\%$



17. (D) Percentage $= \frac{033}{011} \times 100 = 300$

18. (A) Since, x and y are inversely proportional. So, let

$xy = n$

$\therefore \quad \left(\frac{100+10}{100}\right)y = n$

$\Rightarrow \quad y = \frac{100}{110}n$

So, decrease $= n - \frac{100}{110}n = \frac{1}{11}n$

Percentage $= \frac{n/11}{n} \times 100$

$= \frac{100}{11}\%$

19. (D) Total students = 50

$\because$ Girls percentage= 40%

$\therefore$ Boys percentage = 100 – 40 = 60%

$\Rightarrow$ Number of boys $= 50 \times \frac{60}{100} = 30$

20. (C) Total members after 2 years

$= 2500\left(1+\frac{10}{100}\right)\left(1-\frac{10}{100}\right)$

$= 2500 \times \frac{110}{100} \times \frac{90}{100}$

$= 2475$

$\therefore$ Decrease = 2500 – 2475 = 25

21. (B) Let, 3 years ago, the value of machine was x. So, we have

$x \times \frac{90}{100} \times \frac{90}{100} \times \frac{90}{100} = 1{,}45{,}800$

$\Rightarrow \quad x = \frac{145800 \times 100 \times 100 \times 100}{90 \times 90 \times 90}$

$\Rightarrow \quad x = ₹\ 2{,}00{,}000$

22. (A) Let, B has an amount = 100

$\therefore$ A has = 100 + 20 = ₹ 120

$\because$ C has 20% less money than B.

$\therefore$ C has money = 100 – 20 = ₹ 80

$\therefore$ A has more money than C

$= 120 - 80 = 40$

$\Rightarrow$ Required percentage $= \frac{40}{80} \times 100\%$

$= 50\%$

23. (A) 60% of X = 75% of X – 30

$\frac{60X}{100} = \frac{75X}{100} - 30$

$\Rightarrow \quad \frac{75X}{100} - \frac{60X}{100} = 30$

$\Rightarrow \quad \frac{15X}{100} = 30 \Rightarrow X = 200$

24. (C) Required % $= \frac{4 \times 24}{16 \times 8} \times 100\%$

$= 3 \times 25 = 75\%$

25. (A) Let, the number = x

200% of x = 90

$x \times \frac{200}{100} = 90$

$\Rightarrow \quad x = \frac{90 \times 100}{200}$

$\Rightarrow \quad x = 45$

Again, 80 % of 45 $= 45 \times \frac{80}{100} = 36$

26. (D) $\because$ Percentage of passed students

= 65%

$\therefore$ Percentage of failed students = 35%

According to question,

Total number of students

$= \frac{420 \times 100}{35}$

= 1200

27. (C) According to question,

$20 \times \frac{30}{100} + 30 \times \frac{20}{100} = 1200 \times \frac{1}{100}$

$\Rightarrow$ 6 + 6 = 12 $\Rightarrow$ 12 = 12

28. (A) 48% of X = 60% of Y

$\frac{48X}{100} = \frac{60Y}{100} \Rightarrow 4X = 5Y$

$\therefore \quad X : Y = 5 : 4$

29. (B) 6·25% $= \frac{6{\cdot}25}{100} = 0{\cdot}0625$

30. (A) Let, the number = x

$\therefore$ 15% of x = 45

$\Rightarrow \quad x = \frac{45 \times 100}{15} = ₹\ 300$

31. (A) Let, the required percentage = x%.

According to question,

x% of 5% = 3%

$\therefore x = \frac{3}{100} \times \frac{100}{5} \times 100 = 60\%$

32. (A) $a \times \frac{8}{100} = b \times \frac{4}{100}$

or, $a = \frac{1}{2}b$

$\therefore$ 20% of $a = a \times \frac{20}{100}$

$= \frac{1}{2}b \times \frac{20}{100} = 10\%$ of b

33. (B) $x \times \frac{40}{100} \times \frac{40}{100} = 40$

$\therefore x = \frac{40 \times 100 \times 100}{40 \times 40}$

$x = 250$

34. (A) $x \times \frac{90}{100} = y \times \frac{30}{100} = x \times \frac{p}{100} \times \frac{30}{100}$

or, $\frac{9}{10} = \frac{3}{1000}p$

$\therefore p = \frac{1000 \times 9}{10 \times 3} = 300$

35. (C) $\because$ x% of a = y% of b

or, $b = \frac{x}{y}$ of a

$\therefore$ z% of b = z% of $\frac{x}{y}$ of a

$= \frac{xz}{y}\%$ of a

36. (D) x% of y + y% of $x = \frac{y \times x}{100} + \frac{xy}{100}$

$= \frac{2xy}{100} = 2\%$ of xy

37. (D) $\because$ 90% of $y = x$

$\therefore \quad y = \frac{100x}{90}$

Let, y is z% of x, then z% of $x = y$

or, $\frac{xz}{100} = \frac{100x}{90}$

or, $z = \frac{100x}{90} \times \frac{100}{x}$

$= \frac{1000}{9} = 111\frac{1}{9}\%$

38. (B) $\therefore$ Required % $= \frac{y}{2x} \times 100$

$= \frac{y \times 100}{2 \times y \times 80} \times 100$

$= \frac{125}{2}\% = 62\frac{1}{2}\%$

39. (A) $(x - y) \times \frac{50}{100} = (x + y) \times \frac{30}{100}$

or, $2x = 8y \Rightarrow \quad x = 4y$

$$\therefore \text{Required } \% = \frac{y}{x} \times 100$$

$$= \frac{y}{4y} \times 100 = 25\%$$

40. (C) $\because x\%$ of $1400 = 119$

$$\text{or, } x = \frac{119 \times 100}{1400} = 8.5$$

41. (C) $240 \times \frac{11}{2} = x \times \frac{150}{100}$

$$\text{or, } x = \frac{240 \times 11 \times 100}{2 \times 150} = 880$$

42. (C) $x\%$ of $350 = 21$

$$\text{or, } x = 21 \times \frac{100}{350} = 6$$

43. (B) $x\%$ of $\frac{25}{2} = 150$

$$\text{or, } x = \frac{150 \times 100 \times 2}{25} = 1200$$

44. (A) Let, the number $= x$

According to question,

$$\frac{x \times 200}{100} = 90 \text{ or, } x = 45$$

$$\therefore 40\% \text{ of } 45 = 45 \times \frac{40}{100} = 18$$

45. (D) The number $= 120 \times \frac{100}{20} = 600$

$$\therefore 120\% \text{ of } 600 = 600 \times \frac{120}{100} = 720$$

46. (B) Let, the number $= x$

$$\therefore \left(x \times \frac{50}{100}\right) - \left(x \times \frac{35}{100}\right) = 39$$

$$\text{or, } \frac{x}{2} - \frac{7x}{20} = 39$$

$$\frac{10x - 7x}{20} = 39$$

$$3x = 39 \times 20$$

$$= \frac{39 \times 20}{3}$$

$$x = 260$$

$$\therefore 115\% \text{ of } 260 = 260 \times \frac{115}{100}$$

$$= 299$$

47. (A) Let, the number $= x$

$$\therefore \left(x \times \frac{50}{100}\right) - \left(x \times \frac{35}{100}\right) = 12$$

$$\text{or, } x = 80$$

48. (B) Let, the number $= x$

$$\therefore \left(x - \frac{16x}{100}\right) = 42$$

$$\frac{84x}{100} = 42$$

$$x = \frac{42 \times 100}{84}$$

$$\text{or, } x = 50$$

49. (A) Let, the number x, then $x \times \frac{35}{100} = 63$

$$\therefore x = \frac{63 \times 100}{35} = 180$$

50. (C) Let, the number is x.

$\therefore 16\%$ of 40% of $x = 8$

$$\text{or, } \frac{x \times 40}{100} \times \frac{16}{100} = 8$$

$$\therefore x = \frac{100 \times 100 \times 8}{40 \times 16} = 125$$

51. (B) Let, the number $= x$

$$\therefore x \times \frac{2}{9} \times \frac{3}{7} \times \frac{1}{3} = 16$$

$$\text{or, } x = 504$$

$$504 \times \frac{25}{100} = 126$$

TRICK : $16 \times \frac{9}{2} \times \frac{7}{3} \times \frac{3}{1} \times \frac{25}{100}$

$$= 126$$

52. (D) Let, the number $= x$

$$x \times \frac{30}{100} \times \frac{15}{100} = 18$$

$$\text{or, } x = 400$$

53. (B) The number $= \frac{36 \times 8}{72} = 4$

54. (A) Let, the number $= x$

$\therefore 15\%$ of $40 - 25\%$ of $x = 2$

$$\text{or, } \frac{40 \times 15}{100} - \frac{x \times 25}{100} = 2$$

$$6 - \frac{x}{4} = 2$$

$$\text{or, } \frac{x}{4} = 6 - 2 = 4$$

$$\therefore x = 16$$

55. (D) 20% of $50 = 50 \times \frac{1}{5} = 10$

$\therefore$ The number $= 50 + 10 = 60$

56. (B) $\because 20\% = 24$

$$1\% = \frac{24}{20}$$

$$\therefore 120\% = \frac{24}{20} \times 120 = 144$$

57. (B) Let, the number $= x$

$$\therefore x - 40 = x \times \frac{60}{100}$$

$$\text{or, } x - \frac{3}{5}x = 40 \text{ or, } x = 100$$

58. (B) Let, the number $= x$

$$\therefore x - \frac{40x}{100} = 30$$

$$\therefore \frac{60x}{100} = 30$$

$$\therefore x = \frac{30 \times 100}{60} = 50$$

59. (C) Let, the number $= x$

$$\therefore \frac{50x}{100} - 50 = 50$$

$$\frac{50x}{100} = 100$$

$$x = \frac{100 \times 100}{50}$$

$$\text{or, } x = 200$$

60. (D) Last result $= 108\%$ of $[108\%$ of $x]$

$$= x \times \frac{108 \times 108}{100 \times 100} = 1.1664x$$

Required answer $= 1.1664$

61. (D) $x + x \times 5\% = x + \frac{x}{20} = \frac{21x}{20} = 1.05x$

$\therefore$ Multiple $= 1.05$

62. (D) Decrease $\% = \frac{4}{5} \times 100 = 80\%$

●●

Chapter 9 Profit-Loss and Discount

1. Introduction

Business transactions have become nowadays common feature of life. The prime aim of any business is to earn profit. The amount with which any commodity is purchased is called cost price and amount in which commodity is sold is called selling price.

When goods are sold for more than that of the cost they are said to be sold at a profit or gain, when they are sold for less than what they cost, they are said to be sold at a loss.

- Gain = Selling price – Cost price
- Loss = Cost price – Selling price

The cost price is also called outlay or the prime cost. The selling price and the cost price are usually abbreviated as S.P. and C.P.

2. Important Points and Formulae

I. The price at which an article is bought is called Cost Price (C.P.).

II. The price at which the article is sold, is called Selling Price (S.P.).

III. The overhead expenses are added to the Cost Price.

IV. Profit = S.P. – C.P.
Loss = C.P. – S.P.

V. Profit and Loss are always calculated on Cost Price.

VI. (i) $\text{Profit \%} = \frac{(\text{S.P.} - \text{C.P.})}{\text{C.P.}} \times 100$

(ii) $\text{Loss \%} = \frac{(\text{C.P.} - \text{S.P.})}{\text{C.P.}} \times 100$

VII. $\text{S. P.} = \frac{\text{C.P.} \times (100 - \text{Loss \%})}{100}$

VIII. $\text{S.P.} = \frac{\text{C.P.} \times (100 - \text{Profit \%})}{100}$

IX. $\text{C.P.} = \frac{\text{S.P.} \times 100}{(100 + \text{Profit \%})}$

X. $\text{C.P.} = \frac{\text{S.P.} \times 100}{(100 - \text{Loss \%})}$

Examples :

- If a man buys a pen for ₹ 25 and sells it for ₹ 30, then he makes a Profit of 30 – 25 = ₹ 5
- If a man buys a pen for ₹ 25 and sells it for ₹ 20, then he makes a Loss of 25 – 20 = ₹ 5
- A man buys a pen for ₹ 25 and sells it for ₹ 30, then his gain % $\frac{30-25}{25} \times 100 = \frac{5}{25} \times 100 = 20\%$
- A man buys a pen for ₹ 25 and sells it for ₹ 20, then loss % $= \frac{25-20}{25} \times 100 = 20\%$
- A man bought a cycle for ₹ 250 for how much should be sell it so as to gain 10% ?

$$\text{S.P.} = \frac{100+10}{100} \times 250 = \frac{110}{100} \times 250 = ₹\ 275$$

- A man bought a cycle for ₹ 250. For how much should be sale it so as to loss of 20% ?

$$\text{S.P.} = \frac{100-20}{100} \times 250 = \frac{80}{100} \times 250 = ₹\ 200$$

- If by selling an article for ₹ 187.47 p a shopkeeper gains $12\frac{1}{2}$ %. Find his Cost Price.

$$\text{C.P.} = \frac{100}{100 + \frac{25}{2}} \times 187.47 = ₹ 166.64$$

Sign Convention for Change

For Profit + sign for change
For Loss – sign for change

- Quality Price (C.P. or S.P.)

$$\%\text{Profit} = \left(\frac{AX}{ZY} - 1\right) \times 100\%$$

3. Some Important Shortcut Methods

Rule 1 : If C.P. of x goods = S.P. of y goods, then

(i) $\text{Gain \%} = \frac{x-y}{y} = 100$ (in case of $x > y$)

(ii) $\text{Loss \%} = \frac{x-y}{y} \times 100$ (in case of $x < y$)

Rule 2 : If X_1 and X_2 both are the rate of gain or both are the rate of loss, then C. P. $= \left(\frac{100}{x_1 - x_1}\right)$ × amount of difference between selling price

Rule 3 : When a man buys two things on equal price each and in those things, one is sold on the profit of x% and another is sold on the loss of x%, then there is no loss or gain per cent

Example : If Ram buys two cows at ₹ 824 each and sells one at a gain of 14% and another at a loss of 14%. How much does he gain or lose in the whole transaction ?

Sol. No loss no gain.

Reason. Whatever he gains on the first cow, the same he loses on the other.

Rule 4 : When a man sells two things at the same price each and in this process his loss on first thing is $x\%$, and gain on second things is $x\%$, then in such a type of questions, there is always a loss. Loss $\% = \left(\frac{x}{10}\right)^2 \%$

Example : A man sold two watches at ₹ 450 each. He sold one at a loss of 15% and the other at a gain of 15%. His loss or gain is

(A) 15% gain (B) 30% gain

(C) 2.25% loss (D) no loss no gain

Sol. (C) Loss $\% = \left(\frac{15}{10}\right)^2 \% = 2{\cdot}25\%$.

Rule 5 : Dishonest dealer and less weight

$$\text{Gain}\% = \frac{\text{Error}}{\text{True value} - \text{Error}} \times 100$$

where Error = 1000 grams – used weight of goods

Example : A dishonest dealer professes to sell his goods at cost price, but used a weight of 950 g for a kilogram weight. His real gain per cent is

(A) 5% (B) 5·26%

(C) 4% (D) 4·75%

Sol. (B) $\text{Gain}\% = \frac{50}{1000-50} \times 100 = \frac{500}{95} = 5.26\%$

Rule 6 : If A sells a thing to B at a profit of R_1 %, B sells it to C at a profit of R_2 %, C sells it to D at a profit of R_3 %, then Cost price of D

$$= (\text{Cost Price of A}) \left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$$

Example : A sells a watch to B at a gain of 20%, B sells it to C at gain of 25% and C sells it to D at a gain of 10%. If D pays ₹ 330, what did it cost A ?

(A) ₹ 250 (B) ₹ 300

(C) ₹ 200 (D) ₹ 225

Sol. (C) Let Cost Price of A be ₹ x and Cost Price of D = ₹ 330

$$\Rightarrow 330 = x\left(1+\frac{12}{100}\right)\times\left(1+\frac{25}{100}\right)\times\left(1+\frac{10}{100}\right)$$

$$\Rightarrow x = \frac{330\times100\times100\times100}{120\times125\times110} = ₹\,200$$

Rule 7 : If A sells a thing to B at a loss of R_1%, B sells it to C at a loss of R_2 % C, sells it to D at a loss of R_3 %, then Cost Price of D

$$= (\text{Cost Price of A}) \left(1-\frac{R_1}{100}\right)\left(1-\frac{R_2}{100}\right)\left(1-\frac{R_3}{100}\right)$$

Example : If A sells a radio to B at a loss of 20%, B sells it to C at a loss of 10%. If cost of A was ₹ 2000, then cost price of D is

(A) ₹ 1280 (B) ₹ 1190

(C) ₹ 1305 (D) ₹ 1008

Sol. (D) $\text{Cost price of D} = 2000\left(1-\frac{20}{100}\right)\left(1-\frac{30}{100}\right)\left(1-\frac{10}{100}\right)$

$$= 2000\times\frac{80}{100}\times\frac{70}{100}\times\frac{90}{100} = ₹\,1008$$

Rule 8 : When there are two successive profits of $x\%$ and $y\%$, then Resultant profit per cent

$$= \left(x+y+\frac{xy}{100}\right)$$

When there is a profit of $x\%$ and loss of y% in a transaction, the Resultant profit or loss per cent = $\left(x-y+\frac{xy}{100}\right)$ according to the + ve and – ve sign respectively

Example : Mr. Lal sales his watch to Mr. Saha at a profit of 20 % and Mr. Saha sells it to Mr. Sharma at a profit of 30%. If Mr. Sharma pays ₹ 560 for it, then what is the Cost Price of Mr. Lal ?

(A) 358·97 (B) 320·97

(C) 540·97 (D) 680·97

Sol. (A) $\text{Resultant profit} = 20+30+\frac{20+30}{100} = 50+6 = 56$

$$\text{C.P.} = 560\times ₹\,\frac{100}{156} = ₹\,358{\cdot}97$$

Rule 9 : When cost price and sale price are reduced by the same amount (say x), then Cost Price

$$= \frac{[\text{Initial profit}\% + \text{Increase in profit}\%]}{\text{Increase in profit}\%} \times x$$

Example : If a horse is sold at a profit of 25%. If both the cost price and selling price are ₹ 200 less, the profit will be 5% more. Find the cost price.

(A) ₹ 1100 (B) ₹ 1200

(C) ₹ 1000 (D) ₹ 900

Sol. (B) $\text{Cost Price} = \frac{(25+5)\times200}{5}$

$$= \frac{30\times200}{5} = ₹1200$$

Rule 10 : If Cost price of Z articles is equal to the selling price of X articles, then

$$\text{Profit percentage} = \frac{Z-X}{X} \times 100$$

Example : The cost price of 10 pens is equal to the selling price of 9 pens, then profit percentage is

(A) $12\frac{1}{9}\%$ (B) $9\frac{1}{9}\%$

(C) $10\frac{1}{9}\%$ (D) $11\frac{1}{9}\%$

Sol. (D) Profit $\% = \frac{10-9}{9} \times 100 = \frac{100}{9} = 11\frac{1}{9}\%$

Rule 11 : If x part is sold at $m\%$ profit, y part is sold at $n\%$ profit, z part is sold at $p\%$ and s is earned as overall profit, then

$$\text{Value of total consignment} = \frac{s \times 100}{xm + ny + pz}$$

Example : $\frac{1}{4}$ of goods is sold at 12% profit, $\frac{1}{3}$ is sold at 15% profit and rest at 36% profit. If a total profit of ₹ 100 is earned, then what is the value of goods ?

(A) ₹ 434.78 (B) ₹ 232.48

(C) ₹ 230 (D) ₹ 540

Sol. (A) $$\text{Value of goods} = \frac{100 \times 100}{\frac{1}{4} \times 12 + \frac{1}{3} \times 15 + \frac{5}{12} \times 36}$$

$$= \frac{10000}{3+5+15} = \frac{10000}{23} = ₹ 434{\cdot}78$$

Rule 12 : A man purchases a certain number of articles at x a rupee and the same number at y a rupee. He mixes them together and sells them a 'z' a rupee, then

$$\text{Gain or Loss} = \left[\frac{2xy}{z(x+y)} - 1\right] \times 100$$

(according as the sign is + ve or – ve)

Example : A man purchases a certain number of oranges at 4 per rupee and the same number at 5 per rupee. He mixes them together and sells them at 4 per rupee, then gain or loss percent

(A) $11\frac{1}{9}\%$ gain (B) $11\frac{1}{9}\%$ loss

(C) 11% gain (D) 9% loss

Sol. (A) $$\text{Profit or loss } \% = \left[\frac{2 \times 4 \times 5}{4(4+5)} - 1\right] \times 100$$

$$= \left[\frac{40}{36} - 1\right] \times 100 = \frac{4}{36} \times 100$$

$$= \frac{100}{9} = 11\frac{1}{9}\% \text{ (+ve sign, so gain)}$$

Rule 13 : If a trademarks his goods at $x\%$ above his Cost Price and allows purchasers a discount of $y\%$ for cash, then

$$\text{Profit or Loss } \% = x - y - \frac{xy}{100}$$

(according to + ve or – ve sign respectively).

Example : If a trademan marks his goods at 30% above his Cost Price and allows purchases a discount of 10% for cash. What profit % does he make?

(A) 27% (B) 10%

(C) 17% (D) 15%

Sol. (C) $$\%\text{ profit} = 30 - 10 - \frac{30 \times 10}{100}$$

$$= 30 - 10 - 3$$

$$= 17\% \qquad \text{(+ve sign, so profit)}$$

Rule 14 : When each of the two articles is sold at the same price, and a profit of $x\%$ is made on the first and a loss of $y\%$ is made on the second, then

$$\text{Profit or Loss } \% = \frac{100(x-y) - (2xy)}{(100+x) + (100-y)}$$

(according to the + ve or – ve sign)

Example : Each of the two watches is sold for ₹ 500. The first one is sold at 25% profit and the other one at 25% loss. What is the % loss or gain in this deal ?

(A) 6·25% profit (B) 6·25% loss

(C) 2·3% loss (D) 3·2% gain

Sol. (B) In such question it is always a loss and loss %

$$= \frac{(25)^2}{100} = \frac{625}{100} = 6{\cdot}25\%$$

Important Questions

1. A shopkeeper professes to sell all things at a discount of 10% but increases the selling price of each article by 20%. His gain on each article is :

(A) 6% (B) 8%

(C) 10% (D) 12%

2. If the selling price of an article is $\frac{4}{3}$ rd of its cost price, the profit in transaction is :

(A) 16.75% (B) 20.50%

(C) 25.50% (D) 33.33%

3. If the selling price is doubled, the profit triples. Find the profit per cent.

(A) 66.66 (B) 100

(C) 105.33 (D) 120

4. A shopkeeper offers his customers 10% discount and still makes a profit of 26%. What is the actual cost to him of an article marked ₹ 280?

(A) ₹ 175 (B) ₹ 200

(C) ₹ 225 (D) ₹ 215

5. What percentage of profit should be added in the cost price of an item so as to gain a profit of 33% after allowing 5% discount to the customer ?

(A) 45 (B) 40

(C) 52 (D) 48

6. If the manufacturer gains 10%, the wholesale dealer gains 15% and the retailer gains 25%, find the cost of production of a table. The retail price of table is ₹ 1265.

(A) ₹ 800 (B) ₹ 1000

(C) ₹ 950 (D) ₹ 1180

7. A loss of 19% on a belt gets converted into a profit of 17% when the selling price is increased by ₹ 162. What is the cost price of belt ?

(A) ₹ 540 (B) ₹ 450
(C) ₹ 600 (D) ₹ 360

8. When the price of a product was increased by 15%, the number of items sold was decreased by 20%. What was the net effect?

(A) 10%, gain (B) 6%, loss
(C) 8%, loss (D) 4%, gain

9. The difference between the cost price and sale price is ₹ 240. If the profit is 20%, the selling price is :

(A) ₹ 1200 (B) ₹ 1440
(C) ₹ 1800 (D) ₹ 2440

10. Samant bought a microwave oven and paid 10% less than Maximum Retail Price (MRP). He sold it with 30% profit on his purchase cost. What percentage of profit did he earn on MRP?

(A) 17% (B) 20%
(C) 27% (D) 32%

11. A horse is sold at a profit of 25%. If both the cost price and selling price are ₹ 200 less, the profit will be 5% more. The cost price is :

(A) ₹ 1100 (B) ₹ 1200
(C) ₹ 1000 (D) ₹ 900

12. By selling a chair for ₹ 368, a man lost 8%. For how much should he have sold it to gain 15%?

(A) ₹ 450 (B) ₹ 475
(C) ₹ 460 (D) ₹ 500

13. A man sold two watches for ₹ 3750 each, on one he gained 5% and on the other he lost 5%. What was his total gain or loss as a percentage ?

(A) 0.25% loss (B) 2.5% loss
(C) 25% gain (D) 12.5 gain

14. Anmol sold two items for ₹ 1000 each. On one, he gained 10% and on other, he lost 10%. How much did he gain or loss in the whole transaction ?

(A) Profit 0.95% (B) Loss 1%
(C) Profit 10% (D) Loss 10%

15. Two-third of a consignment was sold at a profit of 5% and the remainder at a loss of 2%. If the total profit was ₹ 400, what was the value of the consignment ?

(A) ₹ 13000 (B) ₹ 17000
(C) ₹ 15000 (D) ₹ 40000

16. A sells 2 TV sets, one at a loss of 15% and another at a profit of 15%. Find the loss/gain percentage in the overall transaction.

(A) 2.25 (B) 3
(C) 4 (D) No profit, no loss

17. A dishonest shopkeeper professes to sell his grocer at his cost price but uses a false weight of 900 g each kilogram. Find his gain percentage.

(A) 91/9 (B) 100/9
(C) 100/11 (D) 95/9

18. A man purchased a bullock and a cart for ₹ 1800. He sold the bullock at a profit of 20% and the cart at a profit of 30%. His total profit was 155/6%. Find the cost price of bullock.

(A) ₹ 650 (B) ₹ 750
(C) ₹ 900 (D) ₹ 800

19. A shopkeeper buys two cameras at the same price. He sells one camera at a profit of 18% and the other at a price 10% less than the selling price of the first. His total profit or loss per cent is—

(A) 12.1% profit (B) 12.1% loss
(C) 12.2% profit (D) 11.1% loss

20. Successive discounts of 20% and 10% are equivalent to a single discount of :

(A) 30% (B) 15%
(C) 28% (D) 25%

21. X purchased an item at a discount of 10% and sold it to Y at 10% profit. The marked price and the price for which Y purchased the item are in ratio :

(A) 1 : 1 (B) 10 : 99
(C) 20 : 99 (D) 100 : 99

22. The marked price of an item is twice the cost price. For a gain of 15%, the discount should be :

(A) 7.5% (B) 20.5%
(C) 32.5% (D) 42.5%

23. A man sold his watch at a loss of 5%. Had he sold it for ₹ 56.25 more, he would have gained 10%. What is the cost price (in ₹) of the watch ?

(A) 370 (B) 365
(C) 375 (D) 390

24. A double bed is marked at ₹ 7500. The shopkeeper allows successive discounts of 8%, 5% and 2% in it. What is the Net selling price ?

(A) ₹ 6500 (B) ₹ 6000
(C) ₹ 6423.90 (D) ₹ 6500.50

25. A shopkeeper allows a rebate 12% on the marked price of an article such that the selling price is ₹ 440. Then, the marked price of the article is :

(A) ₹ 490 (B) ₹ 500
(C) ₹ 600 (D) ₹ 550

26. If selling price of an article is $1\frac{1}{3}$ of its cost price, find gain percentage.

(A) 25 (B) $33\frac{1}{3}$
(C) 1.33 (D) $66\frac{2}{3}$

27. Mr. Y purchased a flat for ₹ 925000 and spent ₹ 35000 for its renovation. If he sold the flat for ₹ 1080000, then his profit per cent is :

(A) 15.0 (B) 17.5
(C) 20.0 (D) 12.5

28. Successive discounts of 50% and 50% is equivalent to :

(A) 100% (B) 75%
(C) 50% (D) 25%

29. Mohan sold his watch at 10% loss. If he had sold it for ₹ 45 more, he would have made 5% profit. The selling price (in ₹) of the watch was :

(A) 110 (B) 270
(C) 300 (D) 900

30. A, B, C enter into a pertnership with shares in the ratio $\frac{7}{2}:\frac{4}{3}:\frac{6}{5}$. After 4 months, A increases his share by 50%. If the total profit at the end of the year was ₹ 43200, then the B's share in the profit is :

(A) ₹ 4200 (B) ₹ 4800
(C) ₹ 7200 (D) ₹ 8000

31. A tradesman marks his goods at 25% above the cost price. If he reduces the marked price by $12\frac{1}{2}\%$, then his profit will be :

(A) $9\frac{3}{8}\%$ (B) $7\frac{3}{5}\%$
(C) $6\frac{3}{8}\%$ (D) $5\frac{1}{3}\%$

32. There is a difference of 22 when you give a discount of 35% in a bill and two times of 20% discount. Accordingly, what was the amount of that bill? **(NCERT)**

(A) ₹ 200 (B) ₹ 220
(C) ₹ 1,100 (D) ₹ 2,200

33. If 1 buy 11 books for 100 and sell 10 books for 110, what percentage of profit will I make on each book?

(A) 10 (B) 11.5
(C) 17.3 (D) 21

34. A clothing dealer sold half of his clothes at a profit of 40% and half of the remaining at a loss of 40% and the remaining at the cost price. How much profit or loss did he make in the whole transaction?

(A) 20% profit (B) 25% loss

(C) 10% profit (D) 15% loss

35. When the value of the cloth was reduced by 25%, its sales volume increased by 20%. Accordingly, what was the effect on the gross receipt of the shop? **(NCERT)**

(A) 5% increase (B) 5% loss

(C) 10% increase (D) 10% loss

36. What will be the difference of 40% discount on 1,000 and 30% and 10% discount on the same amount respectively?

(A) ₹ 0 (B) ₹ 20

(C) ₹ 30 (D) ₹ 40

37. There is a difference of 240 between the cost price and the selling price of an item. Accordingly, if there is a profit of 20%, then what is the selling price?

(A) ₹ 1,440 (B) ₹ 1,400

(C) ₹ 1,240 (D) ₹ 1,200

38. By selling 32 oranges for 30, a person loses 25%. Accordingly how many oranges should he sell for 24, so that he can get 20% profit in this deal?

(A) 16 (B) 24

(C) 32 (D) 40

39. If the price of an item is increased by 15% and the total number of sales of that item is reduced by 10%, then what will be the effect on the gross receipt of the respective shop accordingly?

(A) 5% loss (B) 1.5% increase

(C) 2.5% loss (D) 3.5% increase

40. A person sells an item for 960 and bears a loss of 4%. Accordingly, what was his purchase price? **(NCERT)**

(A) ₹ 1,000 (B) ₹ 784

(C) ₹ 498.4 (D) ₹ 300

41. A discount of 24% is given on the marked price of an item and after that the item is sold for 342. Accordingly, what is the marked price of that item?

(A) ₹ 500 (B) ₹ 490

(C) ₹ 450 (D) ₹ 430

42. A milkman bought 70 liters of milk at 630 and added 5 liters of water to it. After that if he sells it at the rate of 9 per liter, then what percentage of profit will he make? **(NCERT)**

(A) $8\frac{1}{5}\%$ (B) 7%

(C) $8\frac{2}{5}\%$ (D) $7\frac{1}{7}\%$

43. A shopkeeper keeps the marked price of his goods at 15% more than their cost price, but also gives a 20% discount on them. Accordingly, what is his net loss?

(A) 3% (B) 5%

(C) 8% (D) 10%

44. A shopkeeper also gains 21% by giving a discount of 12% on the marked price of an item. Accordingly, its marked price is how much more than its cost price?

(A) 25% (B) 30%

(C) 37.5% (D) 42.5%

45. How much better the only single discount of 50% on an item of 10,000 cost than 40% and two successive discounts of 10% on the same?

(A) ₹ 400 (B) ₹ 1000

(C) ₹ 500 (D) ₹ 600

46. A shopkeeper sold 3/4 of an item at 20% profit and the remaining part at its cost price. Accordingly, how much was his actual profit in the whole deal?

(A) 10% (B) 15%

(C) 20% (D) 25%

SOLUTIONS

1. (B) Let C.P. be ₹ 100.

$\therefore$ M.P. = CP + 20% of C.P.
= ₹ 120

Therefore, $S.P. = \frac{120\times(100-10)}{100}$

$= \frac{120\times90}{100} = 108$

$\Rightarrow$ gain % $= \left[\frac{SP-CP}{CP}\times100\right]\%$

$= \left[\frac{108-100}{100}\times100\right]\%$

= 8%

2. (D) Let CP = ₹ 100

$SP = \frac{4}{3}\times100 = \frac{400}{3}$

$Profit = \frac{400}{3} - 100 = \frac{100}{3}$

$\therefore$ Profit percent $= \frac{100}{3\times100}\times100$

$= \frac{100}{3} = 33.33\%$

3. (B) According to question,

2 SP = CP + 3 P

$\Rightarrow$ 2(CP + P) = CP + 3P

$\Rightarrow$ 2CP + 2P = CP + 3P

$\Rightarrow$ CP = P

$\therefore$ Profit % = 100

($\because$ Cost Price is same as the profit)

4. (B) Cost price = ₹ x

and Marked price = ₹ 280

SP = 90% of ₹ 280

$= \frac{90}{100}\times280$

= ₹ 252

Given, 126% of x = 252

$\therefore$ $x = \frac{252\times100}{126}$

= ₹ 200

5. (B) Let the original value of an item = ₹ x.

$\therefore$ 95% of $x = \frac{95}{100}x = \frac{19}{20}x$

CP of an item $= \frac{100}{133}\times\frac{19x}{20} = \frac{5}{7}x$

$\therefore$ Gain $= x - \frac{5x}{7} = \frac{2x}{7}$

$\therefore$ Percentage profit $= \frac{2x}{7}\times\frac{7}{5x}\times100$

= 40%

6. (B) Let cost of production of a table

= ₹ 1000

CP of whole sale $= \frac{10}{100}\times1000$

= ₹ 100 = ₹ 1100

CP of retalier $= 1100\times\frac{15}{100} + 1100$

= 1100 + 165

= ₹ 1265

Hence, cost of production = ₹ 1000

7. (B) Let C.P. = 100

$\because$ Discount = 100 – 19 = 81

and Profit = 100 + 17 = 117

$\therefore$ Difference = 117 – 81 = 36

$\Rightarrow$ Required C.P. $= \frac{\text{Old CP}\times\text{SP}}{\text{Difference}}$

$= \frac{100\times162}{36}$

= ₹ 450

8. (C) Net effect

$= \frac{(100+15)(100-20)}{100}$

$= \frac{115\times80}{100} = 23\times4 = 92$

Loss = 100 – 92 = 8%

9. (B) SP – CP = ₹ 240

$\Rightarrow$ Profit = ₹ 240

SP = CP + 240

$$\text{Profit per cent} = \frac{\text{Profit}}{\text{CP}} \times 100$$

$$\Rightarrow \quad 20 = \frac{240}{\text{CP}} \times 100$$

$$\Rightarrow \quad \text{CP} = \frac{240 \times 100}{20} = ₹\ 1200$$

$$\text{SP} = 1200 + 240 = ₹\ 1440$$

10. (A) Let, $\text{MRP} = ₹\ 100$

$$\text{CP} = 100 - 10 = ₹\ 90$$

$$\text{SP} = 90 + \frac{30}{100} \times 90$$

$$= 90 + 27 = ₹\ 117$$

Profit per cent on MRP

$$= 117 - 100 = 17\%$$

11. (B) Let the cost price of horse be x and selling price

$$= x \times \frac{125}{100} = \frac{5x}{4}$$

According to the question,

$$\frac{\left(\frac{5x}{4} - 200\right) - (x - 200)}{x - 200} \times 100 = 30$$

$$\frac{125x - 200 - x + 200}{x - 200} = \frac{3}{10}$$

$$\frac{0.25x}{x - 200} = \frac{3}{10}$$

$$2.5x = 3x - 600$$

$$0.5x = 600,\ x = 1200$$

So, the Cost Price of horse

$$= ₹1200$$

12. (C) Let the cost price of a chair be ₹ x.

According to the question

$$368 = x \times 92\%$$

$$\therefore \quad x = \frac{368 \times 100}{92} = 400$$

So, the selling price at the profit of 15%

$$= 115\% \text{ of cost price}$$

$$= \frac{115}{100} \times 400$$

$$= ₹\ 460$$

13. (A) The cost price of first watch

$$\Rightarrow \quad 105\% = 3750$$

$$\therefore \quad 100\% = \frac{375000}{105} = 3571.42$$

The cost price of second watch

$$\Rightarrow \quad 95\% = 3750$$

$$\therefore \quad 100\% = 3947.36$$

The cost price of both watchs

$$= 3571.42 + 3947.36$$

$$= 7518.8$$

and selling price = 7500, loss

$$= ₹\ 18.8$$

$$\% \text{ loss} = \frac{18.8 \times 100}{7518.8}$$

$$= 0.25\%$$

14. (B) In this case, the result will always be 1% loss.

15. (C) Suppose value of total consignment

$$= ₹\ 100$$

$\therefore$ Selling value of two third consignment

$$= \frac{2}{3} \times 100 \times \frac{105}{100} = ₹\ 70$$

And selling value of one third (remaining) consignment

$$= \frac{1}{3} \times 100 \times \frac{98}{100}$$

$$= ₹\ \frac{98}{3}$$

So, Total selling value of consignment

$$= ₹\left(70 + \frac{98}{3}\right) = ₹\ \frac{308}{3}$$

$$\text{Profit} = ₹\left(\frac{308}{3} - 100\right) = ₹\ \frac{8}{3}$$

$\because$ When profit is ₹ $\frac{8}{3}$ value of consignment = ₹ 100

$\therefore$ When profit is ₹ 400 value of consignment

$$= 100 \times \frac{3}{8} \times 400$$

$$= ₹\ 15000$$

16. (A) When an article is sold at the same percent of profit and loss then always becomes a loss.

$$\therefore \quad \text{Loss percentage} = \frac{(15)^2}{100}$$

$$= \frac{225}{100} = 2.25\%$$

17. (B) True weight = 1000 g

Selling weight = 900 g

Error weight = (1000 – 900) g

= 100 g

$\therefore$ Gain percentage

$$= \frac{\text{Error weight}}{\text{Selling weight}} \times 100$$

$$= \frac{100}{900} \times 100 = \frac{100}{9}\%$$

18. (B) Suppose Cost Price of bullock = ₹ x and Cost Price of cart = ₹ $(1800 - x)$

Then, $x \times \frac{120}{100} + (1800 - x) \times \frac{130}{100}$

$$= 1800 \times \frac{\left(100 + \frac{155}{6}\right)}{100}$$

$$x \times \frac{120}{100} + (1800 - x) \times \frac{130}{100}$$

$$= 1800 \times \frac{755}{6 \times 100}$$

$$\Rightarrow \quad \frac{120x}{100} + \frac{234000 - 130x}{100} = 3 \times 755$$

$$\Rightarrow \quad \frac{120x + 234000 - 130x}{100} = 2265$$

$$\Rightarrow \quad 10x = 234000 - 226500$$

$$\Rightarrow \quad 10x = 7500$$

$$\Rightarrow \quad x = \frac{7500}{10}$$

$$= ₹\ 750$$

19. (A) Let the cost of each camera = x

$\therefore$ S.P. of both camera

$$= \frac{118}{100}x + \frac{118x \times 90}{100 \times 100}$$

$$= \frac{2242}{1000}x$$

$\therefore$ Cost price of both camera = $2x$

$$\therefore \quad \text{Profit \%} = \frac{(2.242 - 2)x}{2x} \times 100$$

$$= \frac{0.242}{2} \times 100$$

$$= 12.1\%$$

20. (C) Equivalent to a single discount of successive discount

$$= \left(A + B - \frac{A \times B}{100}\right)$$

$$= \left(20 + 10 - \frac{20 \times 10}{100}\right)$$

$$= (30 - 2) = 28$$

21. (D) Let the marked price = ₹ 100

$\therefore$ S.P. for X = 90% of 100

= ₹ 90

$\therefore$ S.P. for Y = 110% of 100

$$= ₹\ \frac{110}{100} \times 90$$

$$= 99$$

$\therefore$ Ratio = 100 : 99

22. (D) Let the discount be x%

According to question,

$(100 - x)\%$ of 200 = 115

$$\Rightarrow \quad \frac{(100 - x)200}{100} = 115$$

$$\Rightarrow \quad 100 - x = \frac{115}{2} = 57.5$$

$$\Rightarrow \quad x = 100 - 57.5$$

$$= 42.5\%$$

23. (C) Let C.P. of watch = x

$\therefore$ 110% of x – 95% of x = 56.25

$\Rightarrow$ $1.1x - 0.95x = 56.25$

$\Rightarrow$ $0.15x = 56.25$

or $x = ₹\ 375$

24. (C) Net selling price

$$= \frac{98}{100}\times\frac{95}{100}\times\frac{92}{100}\times 7500$$

$$= ₹\ 6423.90$$

25. (B) Let the M.P. be ₹ 100.

Rebate per cent = 12%

$\therefore$ Rebate = 12% of M.P.

$$= \frac{12}{100}\times 100 = ₹\ 12$$

$\therefore$ S.P. = M.P. – Rebate (discount)

= (100 – 12) = ₹ 88

$\because$ If S.P. is ₹ 88, then the M.P.

= ₹ 100

$\therefore$ If S.P. is ₹ 1, then the M.P.

$$= ₹\left(\frac{100}{88}\right)$$

$\therefore$ If S.P. is ₹ 440, then the M.P.

$$= \left(\frac{100}{88}\times 440\right)$$

= ₹ 500

26. (B) Let C.P. of an object be ₹ x.

$$\therefore \quad \text{S.P.} = \frac{4}{3}x$$

$$\therefore \text{Gain \%} = \frac{\left(\frac{4}{3}x - x\right)}{x}\times 100$$

$$= \frac{x}{3x}\times 100 = 33\frac{1}{3}\%$$

27. (D) C.P. of a flat = ₹ 925000

Expenditure for renovation = ₹ 35000

$\therefore$ Net C.P. = 925000 + 35000

= ₹ 960000

S.P. of a flat = ₹ 1080000

$\because$ S.P. > C.P.

$\therefore$ Profit = S.P. – C.P.

= [{(1080000 – 960000)}]

= ₹ 120000

$$\therefore \text{Percent profit} = \frac{\text{Profit}}{\text{CP}} = 100$$

$$= \frac{120000}{960000}\times 100$$

= 12.5%

28. (B) Required percentage discount

$$= \left(50 + 50 - \frac{50\times 50}{100}\right)\%$$

= (100 – 25)% = 75%

29. (B) (10 + 5)% ≡ ₹ 45

$$\therefore 100\% \quad \frac{45}{15}\times 100 = ₹\ 300$$

$\therefore$ S.P. of the watch = 90% of 300

= ₹ 270

30. (D) $A : B : C = \frac{7}{2} : \frac{4}{3} : \frac{6}{5}$

= 105 : 40 : 36

Ratio of the profits of A, B and C

$$\left(105\times 4 + 105\times\frac{3}{2}\times 8\right)$$

$$: 40\times 12 : 36\times 12$$

$$= (420 + 1260) : 480 : 432$$

$$= 1680 : 480 : 432$$

$$= 210 : 60 : 54$$

$$= 35 : 10 : 9$$

$$\therefore \text{Share of B} = \frac{10}{54}\times 43200 = ₹\ 8000$$

31. (A) Let the C.P. be ₹ 100.

M.P. = 100 + 25 = ₹ 125

M.P. reduced by $12\frac{1}{2}\%$

i.e., $\frac{25}{2}\%$ of $125 = \frac{25}{2\times 100}\times 125$

$$= ₹\ \frac{125}{8}$$

$$\therefore \quad \text{New S.P.} = \left(125 - \frac{125}{8}\right)$$

$$= \left(\frac{1000 - 125}{8}\right)$$

$$= ₹\ \frac{875}{8}$$

Since, S.P. > C.P.

$\therefore$ Profit = S.P. – C.P.

$$= \frac{875}{8} - 100$$

$$= \frac{875 - 800}{8}$$

$$= ₹\ \frac{75}{8}$$

$$\therefore \text{Per cent profit} = \frac{\text{Profit}}{\text{CP}}\times 100$$

$$= \frac{75}{8}\times\frac{100}{100}$$

$$= \frac{75}{8} = 9\frac{3}{8}\%$$

32. (D) Discount = 35%

Equivalent single discount

$$= \left(20 + 20 - \frac{20\times 20}{100}\right)\%$$

= (40 – 4)% = 36%

According to question,

36% – 35% = ₹ 22

$\Rightarrow$ 1% = ₹ 22

$\therefore$ 100% = ₹ 22 × 100

= ₹ 2,200

So, amount of the bill = ₹ 2,200

33. (D) C.P. of a book = $\frac{100}{11}$

S.P. of a book = $\frac{110}{10} = 11$

Profit = $11 - \frac{100}{11}$

$$= \frac{121 - 100}{11} = \frac{21}{11}$$

$$\therefore \text{Percent profit} = \frac{\frac{21}{11}}{\frac{100}{11}}\times 100$$

$$= \frac{21}{11}\times\frac{11}{100}\times 100 = 21\%$$

34. (C) Let, the dealer bought 100 m cloth for 100.

$\therefore$ Total S.P.

$$= \left(100\times\frac{1}{2}\times\frac{100+40}{100}\right) + \left(100\times\frac{1}{2}\times\frac{1}{2}\times\frac{100-40}{100}\right) + \left(100\times\frac{1}{2}\times\frac{1}{2}\times\frac{100}{100}\right)$$

= 70 + 15 + 25 = ₹ 110

$\therefore$ Profit percent = 110 – 100 = 10%

35. (D) Net percent effect

$$= \left(-25 + 20 - \frac{25\times 20}{100}\right)\%$$

= – 25 + 20 – 5 = –10%

So, there will be 10% loss.

36. (C) According to question,

40% discount on 1,000

$$= 1{,}000\times\frac{40}{100} = ₹\ 400$$

30% discount on 1,000

$$= 1{,}000\times\frac{30}{100} = ₹\ 300$$

Remainder = 1,000 – 300 = 700

10% discount on 700 = $700\times\frac{10}{100}$

= ₹ 70

Total discount = 300 + 70 = ₹ 370

∴ Required difference = 400 – 370

= ₹ 30

37. (A) Let, C.P. = x

$$\text{S.P.} = x \times \frac{100+20}{100}$$

$$= x \times \frac{120}{100} = \frac{6x}{5}$$

$$\Rightarrow \quad \frac{6x}{5} - x = 240$$

$$\Rightarrow \quad \frac{6x-5x}{5} = 240$$

$$\Rightarrow \quad \frac{x}{5} = 240$$

$$\therefore \quad x = 240 \times 5 = ₹\ 1{,}200$$

$$\text{S.P.} = \frac{6x}{5}$$

$$= \frac{6 \times 1200}{5}$$

$$= ₹\ 1{,}440$$

38. (A) S.P. of an orange on 25% loss

$$= ₹\ \frac{30}{32}$$

C.P. of an orange

$$= \frac{30}{32} \times \frac{100}{100-25}$$

$$= \frac{30}{32} \times \frac{100}{75} = ₹\ \frac{5}{4}$$

S.P. of an orange on 25% profit

$$= \frac{5}{4} \times \frac{100+20}{100}$$

$$= \frac{5}{4} \times \frac{120}{100} = ₹\ \frac{3}{2}$$

∴ ₹Required oranges for selling in 24

$$= 24 \times \frac{2}{3} = 16$$

39. (D) Net percent effect

$$= (15 - 10) - \frac{15 \times 10}{100}$$

= 5 – 1.5 = 3.5% increase

40. (A) C.P. of an item

$$= \frac{\text{S.P} \times 100}{100 - \text{Loss}}$$

$$= \frac{960 \times 100}{100-4}$$

$$= \frac{960 \times 100}{96} = ₹\ 1{,}000$$

41. (C) M.P. of an item = $\frac{342 \times 100}{100-24}$

$$= \frac{342 \times 100}{76}$$

= ₹ 450

42. (D) C.P. of 70 L milk = ₹ 630

S.P. of (70 + 5L water = 75 L) mixture = 75 × 9 = ₹ 675

∴ Percent profit = $\frac{675-630}{630} \times 100$

$$= \frac{45}{630} \times 100 = \frac{50}{7} = 7\frac{1}{7}\%$$

43. (C) C.P. of an item = ₹ 100

Marked price = ₹ 115

$$\text{S.P.} = \frac{115 \times 80}{100} = ₹\ 92$$

∴ Loss percent = 8%

44. (C) C.P. of an item = ₹ 100

Marked price = ₹ x

$$\therefore \quad \frac{x \times 88}{100} = 121$$

$$\Rightarrow \quad x = \frac{121 \times 100}{88} = ₹\ 137.5$$

∴ So, answer = 37.5% more than C.P.

45. (A) Single equivalent discount

$$= \left(40 + 10 - \frac{40 \times 10}{100}\right)\% = 46\%$$

Percent difference = 4%

∴ Saving = 4% of 10000

$$= \frac{10000 \times 4}{100} = ₹\ 400$$

46. (B) C.P. of an item = ₹ 100

$$\text{S.P. of the item} = \frac{75 \times 120}{100} + 25$$

= 90 + 25 = ₹ 115

∴ Net profit = ₹ 15 or 15%

●●

Chapter 10

Average

1. Introduction

If in a class there are 60 students, instead of knowing the age of individual students we usually talk about aggregate age. Aggregate or Average is calculated by summing, the age of all the students and dividing the sum by the number of students.

$$\text{Average} = \frac{\text{Sum of all items}}{\text{Number of items}}$$

Average is classified as follows :

(i) Arithmetic Average (Arithmetic Mean)

(ii) Geometric Mean

(iii) Harmonic Mean

(i) Arithmetic Mean

It is defined as the sum of total to all values of items divided by the total number of items. Average Mean of $x_1, x_2, x_3, \ldots x_n$ is denoted by AM.

$$\text{A.M.} = \frac{x_1 + x_2 + x_3 \ldots + x_n}{n}$$

Example : If 5 students, having marks 50, 60, 70, 80, 20. Then, their average marks are

(A) 55 (B) 56

(C) 57 (D) 58

Sol. (B) Average Marks

$$= \frac{50 + 60 + 70 + 80 + 20}{5}$$

$$= \frac{280}{5} = 56$$

(ii) Geometric Mean

Geometric Mean of $x_1, x_2, x_3, \ldots x_n$ is denoted by GM.

$$\text{G.M.} = \sqrt[n]{x_1 \times x_2 \times x_3 \times \ldots \times x_n}$$

Example : Geometric mean of 2, 9, 12 is

(A) 2 (B) 7

(C) 5 (D) 6

Sol. (D) $\text{GM} = \sqrt[3]{2 \times 9 \times 12}$

$$= \sqrt[3]{216} = 6$$

(iii) Harmonic Mean—Harmonic Mean of $x_1, x_2, x_3, \ldots x_n$ is denoted by HM.

$$\text{H.M} = \frac{1}{\frac{1}{n}\left[\frac{1}{x} + \frac{1}{x} + \ldots \frac{1}{x_1}\right]}$$

Example : Harmonic Mean of 2, 4, 5 and 10 is

(A) $\frac{80}{21}$ (B) $\frac{21}{80}$

(C) $\frac{8}{21}$ (D) $\frac{21}{8}$

Sol. (A) $\text{H.M.} = \frac{1}{\frac{1}{4}\left[\frac{1}{2} + \frac{1}{4} + \frac{1}{5} + \frac{1}{10}\right]} = \frac{4}{\frac{21}{20}}$

$$= \frac{80}{21}$$

2. Important Rules

Rule 1 : $\text{Average} = \frac{\text{Total travelled distance}}{\text{Total time-taken}}$

Example : A man walks 2000 metres in 30 minutes. 1500 metres in 40 minutes and 500 metres in 10 minutes. Then what is the average speed for whole walking distance or journey ?

(A) 50 m/min (B) 60 m/min

(C) 55 m/min (D) 65 m/min

Sol. (A) $\text{Average Speed} = \frac{(2000 + 1500 + 500) \text{ metres}}{(30 + 40 + 10) \text{ minutes}}$

$= 50$ m/min.

Rule 2 : If equal distance are travelled at the rate of x and y, then average speed $= \frac{2xy}{x+y}$

Example : Rajeev goes to his school at 5 km per hour and returns at 8 km per hour crossing same route. Then his average speed is

(A) $7\frac{1}{3}$ km/hr (B) $5\frac{3}{5}$ km/hr

(C) $6\frac{2}{3}$ km/hr (D) $6\frac{2}{13}$ km/hr

Sol. (D) $\text{Average speed} = \frac{2 \times 5 \times 8}{5+8}$

$= 6\frac{2}{13}$ km/hr.

Rule 3 : When a person leaves a group and another person joins the group in the place of separated person, then

- In the case of increasing of average age, the age of the new person
  = Age of separated person group + [no. of persons in the group × increase in average age]
- In the case of decreasing of average age, the age of the new person
  = Age of separated person group – [no. of persons in the group × increase in average age]

Example 1 : The average age of 8 men is increased by 4 years when one of them whose age is 30 years is replaced by a new man. What is the age of new man ?

(A) 55 years (B) 62 years
(C) 42 years (D) 69 years

Sol. (B) The age of the fresh man $= 30 + (8 \times 4)$
$= 62$ years.

Example 2 : The average age of 45 persons decreases by $\frac{1}{9}$ year when one of them whose age is 60 years is replaced by a newcomer. What is the age of newcomer ?

(A) 40 years (B) 62 years
(C) 55 years (D) 59 years

Sol. (C) Age of newcomer $= 60 - (45 \times \frac{1}{9}) = 55$ years.

Rule 4 : When a person joins a group without replacing any previous person from that group, then

- In the case of increasng of average age, the age of the new person
  = Previous average of the age + [no. of persons (including new person) × increase in average age]
- The the case of decreasing of average age, the age of the new person
  = Previous average of the age – [no. of persons (including new person) × decrease in average age]

Example 1. The average age of 6 women is 32 years which is increased by 1 year when a new women joins the group. What is the age of new women ?

(A) 42 yrs (B) 35 yrs
(C) 45 yrs (D) 39 yrs

Sol. (D) Age of new woman $= 32 + (6 + 1) \times 1 = 39$ years.

Example 2 : The average age of 20 teachers is 45 years which is decreased by $\frac{6}{7}$ years when a student joins this group. What is the age of that students ?

(A) 15 yrs (B) 27 yrs
(C) 18 yrs (D) 25 yrs

Sol. (B) Age of the students

$= 45 - (20 + 1) \times \frac{6}{7} = 27$ years.

Rule 5 : When a person leaves the group but nobody joins this group, then

- In the case of increasing of average age, the age of separated man
  = Previous average of the age – [no. of all present persons × increase in average age]
- In the case of decreasing of average age, the age of separated man
  = Previous average of the age + [no. of all present persons × increase in average age]

Example 1 : The average age of 10 girls in a hostel is 19 years. But one girl left the hostel and average age is increased by $\frac{1}{2}$ years. Then how many years she is old ?

(A) $14\frac{1}{2}$ yrs (B) 15 yrs
(C) $15\frac{1}{2}$ yrs (D) 18 yrs

Sol. (A) Age of left girl $= 19 - (10 - 1) \times \frac{1}{2}$

$= 14\frac{1}{2}$ yrs

Example 2 : The average age of 26 labours is 30 years. It is decreased by $\frac{1}{5}$ years. When a labour went home. Then the value of age of that labour is

(A) 30 years (B) 32 years
(C) 24 years (D) 35 years

Sol. (D) Age of the labour $= 30 + (26 - 1) \times \frac{1}{5}$

$= 30 + 5 = 35$ yrs.

Rule 6 : Ratio of past, present and future age are given

	Past	Present	Future
A : B	$n_1 : 1$	$n_2 : 1$	$n_3 : 1$
Time	t_1 years ago	Present	t_3 years hence

- The ratio of past and present are given

$$\text{B's present age} = \left(\frac{n_1 - 1}{n_1 - n_2}\right) t_1$$

$$\text{A's present age} = \left(\frac{n_1 - 1}{n_1 - n_2}\right) n_2 t_1$$

- The ratio of present and present are given

$$\text{B's present age} = \left(\frac{n_3 - 1}{n_2 - n_3}\right) t_3$$

$$\text{A's present age} = \left(\frac{n_3 - 1}{n_2 - n_3}\right) n_2 t_3$$

- The ratio of past and future age are given

$$\text{B's present age} = \frac{(n_1 - 1)\, t_1 + (n_3 - 1)\, t_3}{n_1 - n_3}$$

$$\text{A's present age} = \frac{n_3 (n_1 - 1)\, t_1 + n_1 (n_3 - 1)\, t_3}{n_1 - n_3}$$

Rule 7 : Average of consecutive natural number till n

$$= \frac{n + 1}{2}$$

Example : Average of consecutive natural number till 7

$$= \frac{7+1}{2} = 4$$

Rule 8 : Average of square of number till

Average of square of number till $n = \frac{(n+1)(2n+2)}{6}$

Example : Average of squares of consecutive natural number till 11

$$= \frac{(11+1)(2\times11+1)}{6} = 46$$

Rule 9 : Average of cube of number till $n = \frac{n(n+1)^2}{4}$

Example : Average of $1^3, 2^3, 3^3, 4^3 = \frac{4(4+1)^2}{4} = 25$

Rule 10 : Average of Consecutive even number $s = n + 1$

Example : Average of 4 consecutive even numbers

$= 4 + 1 = 5.$

Rule 11 : Average of consecutive even number $n = \frac{n}{2} + 1$

Example : Average of consecutive even numbers till 10

$$= \left(\frac{10}{2} + 1\right) = 6$$

Rule 12 : Average of squqares of n consective even number

$$= \frac{2(n+1)(2n+1)}{3}$$

Example : Average of $2^2, 4^2, 6^2, 8^2$

$$= \frac{2(4+1)(2\times4+1)}{3} = 30$$

(Square of 4 consecutive even numbers). Here n = 4.

Rule 13 : Average of squares of consecutive even number till

$$n = \frac{(n+1)(n+2)}{3}$$

Example : Average of square of consecutive even number till 16

$$= \frac{(16+1)(16+2)}{3} = 102$$

Rule 14 : Average of n consecutive odd numbers = n.

Example : Average of 7 consecutive odd numbers = 7.

Rule 15 : Average of n consecutive odd numbers till n

$$= \frac{(n+1)}{2}$$

Example : Average of consecutive odd number till 13

$$= \frac{13+1}{2} = 7$$

Rule 16 : Average of squares of consecutive odd

Average of n consecutive odd numbers till n

$$= \frac{n(n+1)}{3}$$

Example : Average of squares of consecutive odd number till 9

$$= \text{average of } 1^2, 3^2, 5^2, 7^2, 9^2 = \frac{9(9+2)}{3} 33$$

Important Questions

1. The average age of 8 men is increased by 4 yr when one of them whose ages is 30 yr is replaced by a new man. What is the age of new man ?

(A) 55 yr (B) 62 yr

(C) 42 yr (D) 69 yr

2. A man's average monthly expenditure for the first 4 months of the year was ₹ 231.25. For the next 5 months, the average monthly expenditure was ₹ 22.75 more than what it was during the first 4 months. If the person spent ₹ 605 in all during the remaining 3 months of the year, find what percentage of his annual income of ₹ 3500 did he save in the year ?

(A) 10 (B) 15

(C) 20 (D) 25

3. The average age of students of a class is 15.8 yr. The average age of boys in the class is 16.4 yr and that of the girls is 15.4 yr. The ratio of the number of boys to the number of girls in the class is :

(A) 1 : 2 (B) 2 : 3

(C) 3 : 4 (D) 3 : 5

4. Find the average of all the numbers between 6 and 34 which are divisible by 5.

(A) 18 (B) 20

(C) 24 (D) 30

5. The average of first 80 natural numbers is :

(A) 40 (B) 41

(C) 40.5 (D) 142

6. If the sum of a few numbers is 450 and their mean is 50 and if another number 100 is included, the mean would become :

(A) 55 (B) 60

(C) 75 (D) 150

7. A man's average monthly expenditure for the first four months of the year was ₹ 225.25. For the next five months, the average monthly expenditure was ₹ 20.7 more than what it was during the first four months ? If the person spent ₹ 700 in all during the remaining three months of the year find what percentage of his annual income of ₹ 3500 he saved in the year ?

(A) 10 (B) 15

(C) 19.11 (D) 25

8. The average age of a man and his son is 40 yr. The ratio of their ages is 7 : 3, respectively. What is the man's age ?

(A) 70 yr (B) 63 yr

(C) 56 yr (D) 49 yr

9. The average of five consecutive odd numbers is 61. What is the difference between highest and lowest number ?

(A) 2 (B) 5

(C) 8 (D) 12

10. A boy has an average of 30 runs in 14 innings. How many runs should he score in his next innings to achieve an average of 32 runs ?

(A) 65 (B) 60

(C) 55 (D) 50

11. The average age of 35 students in a class is 16 yr. Out of these students the average age of 21 students is 14 yr. The average age of remaining students is :

(A) 15 yr (B) 17 yr

(C) 20 yr (D) 19 yr

12. After replacing an old member by a new member, it was found that the average age of five members of a club is the same as it was 3 yr ago. What is the difference between the age of replaced member and new member ?

(A) 2 yr (B) 8 yr

(C) 15 yr (D) 25 yr

13. The average salary of all the workers in a workshop is ₹ 8000. The average salary

of seven technicians is ₹ 12000 and average salary of others is ₹ 6000. The total number of workers in the workshop are
(A) 20 (B) 21
(C) 22 (D) 23

14. The mean of 50 observations was 36. It was found later that an observation 48 was wrongly taken 23. The corrected new mean is :
(A) 35.2 (B) 34.1
(C) 36.5 (D) 39.1

15. The average score of a cricketer for ten matches is 38.9 runs. If the average for the first six matches is 42, the average for the last four matches is :
(A) 33.25 (B) 33.5
(C) 34.25 (D) 35

16. The average weight of 50 boys in a class is 45 kg. When one boy leaves the class, the average reduces by 100 g. Find the weight of the boy who left the class.
(A) 50 kg (B) 50.8 kg
(C) 49 kg (D) 49.9 kg

17. In Arun's opinion his weight is greater than 65 kg but less than 72 kg. His brother does not agree with Arun and he thinks that Arun's weight is greater than 60 kg but less than 70 kg. His mother's view is that his weight cannot be greater than 68 kg. If all of them are correct in their estimation, what is the average of different probable weights of Arun ?
(A) 71 kg (B) 67 kg
(C) 67.5 kg (D) 68 kg

18. In three annual examinations, of which the aggregate marks of each was 500, a student secured average marks 45% and 55% in the first and the second yearly examinations respectively. To secure 60% average total marks, it is necessary for him in third yearly examination to secure ..... marks.
(A) 300 (B) 350
(C) 355 (D) 400

19. The average weight of 5 men is increased by 2 kg when one of the men whose weight is 60 kg is replaced by a new man. The weight of the new man is :
(A) 50 kg (B) 65 kg
(C) 68 kg (D) 70 kg

20. There were 35 students in a hostel. If the number of students be increased by 7, the expenditure on food increases by ₹ 42 per day while the average expenditure of students is reduced by ₹ 1. What was the initial expenditure on food per day ?
(A) ₹ 432 (B) ₹ 442
(C) ₹ 420 (D) ₹ 400

21. There were 24 students in a class. One of them, who was 18 yr old, left the class and his place was filled up by a new comer. If the average of the class was thereby lowered by 1 month, the age of new comer is :
(A) 14 yr (B) 15 yr
(C) 16 yr (D) 17 yr

22. 19 persons went to a hotel for a combined dinner party. 13 of them spent ₹ 79 each on their dinner and the rest spent ₹ 4 more than the average expenditure of all the ₹ 19. What was the total money spent by them ?
(A) ₹ 1628.4 (B) ₹ 1534
(C) ₹ 1492 (D) None of these

23. Anshuman aims to score an average of 95 marks in quarterly and half yearly exams. But his average in quarterly is 3 marks less than his target and that in half yearly is 2 marks more than his aim. The difference between the total marks scored in both the exams is 20. Total marks aimed by Anshuman is—
(A) 380 (B) 400
(C) 410 (D) 420

24. A Library has an average number of 510 visitors on sunday and 240 on other days. The average number of visitors per day in a month of 30 days beginning with Sunday is :
(A) 290 (B) 285
(C) 295 (D) 300

25. Given that the average of Five numbers is 27. If one of them is excluded, the average gets reduced by 2. Determine the excluded number.
(A) 45 (B) 55
(C) 25 (D) 35

26. The average age of 100 workers in a factory is 36.5. The average age of the men is 45 and that of the women is 28. The number of women working in the factory is :
(A) 50 (B) 45
(C) 40 (D) 60

27. If the average of x and $\frac{1}{x}(x \neq 0)$ is M, then the average of x^2 and $\frac{1}{x^2}$ is :
(A) $1 - M^2$ (B) $1 - 2M$
(C) $2M^2 - 1$ (D) $2M^2 + 1$

28. The average value of 20 observations was found to 75, but later on it was detected that 97 was misread as 79. Find the correct average.
(A) 75.7 (B) 75.8
(C) 75.9 (D) 75.6

29. A tabular while calculating the average marks of 100 students of an examination, by mistake enters 68, instead of 86 and obtained the average as 58, the actual average marks of these student is :
(A) 58.18 (B) 57.82
(C) 58.81 (D) 57.28

30. The average of all odd numbers less than 100 is :
(A) 49.5 (B) 50
(C) 50.5 (D) 51

31. The average age of a hunsband and a wife was 27 years when they married 4 years ago. The average age of the husband, the wife and a new born child is 21 years now. The present age of the child is :
(A) 4 years (B) 3 years
(C) 2 years (D) 1 year

32. The average age of eleven cricket players is 20 years. If the age of the coach is also included, the average age increase by 10%. The age of the coach is :
(A) 48 years (B) 44 years
(C) 40 years (D) 36 years

33. The average of first five numbers is 15 and that of another five numbers is 20. The average of those ten numbers is :
(A) 17.5 (B) 18.5
(C) 19.5 (D) 20

34. The average of $1^2, 2^2, 3^2, \ldots\ldots, 10^2$ is :
(A) 46.5 (B) 50.5
(C) 38.5 (D) 42.5

35. The average marks of 32 boys of section A of Class X is 60 whereas the average marks of 40 boys of section B of class X is 33. The average marks for both the sections combined together is :
(A) 44 (B) 45
(C) $46\frac{1}{2}$ (D) $45\frac{1}{2}$

36. The average expenditure of a man for the first five months is ₹ 1200 and for the next seven months is ₹ 1300. If he saves ₹ 2900 in that year, his monthly average income is :
(A) ₹ 1400 (B) ₹ 1500
(C) ₹ 1600 (D) ₹ 1700

37. The average of the first 100 positive integers is :
(A) 100 (B) 51
(C) 50.5 (D) 49.5

38. The average age of P, Q and R is 5yr. more than R's age. If the total age's of P and Q together is 39yr., then R's age is :
(A) 16 yr. (B) 14 yr.
(C) 12 yr. (D) 24 yr.

39. The average age of a panel of 5 judges is 40. If a member of the panel aged 35 resigns and a man aged 25 becomes a member of the panel, then the average age of the new panel is :
(A) 30 yr. (B) 38 yr.
(C) 40 yr. (D) 42 yr.

40. A cricket player after playing 10 tests scored 100 runs in the 11^{th} test. As a result, the average of his runs is increased by 5. The present average of runs is :
(A) 45 (B) 40
(C) 50 (D) 55

SOLUTIONS

1. (B) Let the average age of 8 men be M yr

According to question,

Age of 7 men + 30 = 8 M ...(i)

According to new condition,

Age of 8 men = 8 (M + 4)

Age of 7 men + age of new man

= 8 M + 32

[From Eq. (i)]

8 M – 30 + age of new man

= 8 M + 32

$\Rightarrow$ Age of new man = 32 + 30

= 62 yr

2. (C) Expenditure for the first 4 months

= 4 × 231.25 = ₹ 925

Expenditure for the next 5 months

= 5 × 254 = ₹ 1270

Expenditure remaining 3 months

= ₹ 605

Total expenditure

= 925 + 1270 + 605

= ₹ 2800

Saving = 3500 – 2800

= ₹ 700

Percentage saving

$$= \frac{700}{3500} \times 100$$

= 20%

3. (B) Let number of boys = x

and number of girls = y

Total age of the students = 15.8 $(x + y)$

Total age of boys = 16.4 x

and total age of girls = 15.4 y

$\therefore$ $16.4x + 15.4y = 15.8(x + y)$

$\Rightarrow$ $16.4x + 15.4y = 15.8x + 15.8y$

$\Rightarrow$ $16.4x - 15.8x = 15.8y - 15.4y$

$.6x = .4y \Rightarrow 6x = 4y$

$\Rightarrow$ $3x = 2y$

$\Rightarrow$ $x : y = 2 : 3$

4. (B) $$\text{Average} = \frac{10+15+20+25+30}{5}$$

$$= \frac{100}{5} = 20$$

5. (C) Average of first n natural numbers

$$= \frac{n+1}{2} = \frac{80+1}{2} = 40.5$$

6. (A) $$n = \frac{450}{50} = 9$$

$$\text{New Mean} = \frac{550}{10} = 55$$

7. (C) Total expenditure four months

= 225.25 × 4 = 901

Total expenditure five months

= (225.25 + 20.75) × 5

= 246 × 5

= ₹ 1230

Total expenditure in 12 months

= 901 + 1230 + 700

= ₹ 2831

Saving = 3500 – 2831

= ₹ 669

$$\text{Percentage saving} = \frac{669}{3500} \times 100$$

= 19.11%

8. (C) The total age of man and his son

= 40 × 2 = 80 yr

Total ratio = 7 + 3 = 10 Man's age

$$= \frac{80}{10} \times 7 = 56 \text{ yr}$$

9. (C) $x + 1 + x + 3 + x + 5 + x + 7 + x + 9$

= 61 × 5

$5x + 25 = 61 \times 5$

$x + 5 = 61$

$x = 56$

$\therefore$ Difference = 65 – 57 = 8

10. (B) Total runs in 14 innings = 30 × 14

= 420

Total runs in 15 innings = 32 × 15

= 480

$\therefore$ Number of runs = 480 – 420

= 60

11. (D) Total age of 35 students = 35 × 16

= 560 yr.

Total age of 21 students = 21 × 14

= 294 yr

Total age of 14 students = 560 – 294

= 266 yr

$$\text{Average of 14 students} = \frac{266}{14}$$

= 19 yr

12. (C) $\because$ Average age of five members = x – 15 (before 3 yr) and Let the new average age of 4 old members and a new member = y

According to question,

$x - 15 = y$

$\Rightarrow$ $x - y = 15$ yr

13. (B) Let total number of workers = x

Total salary of x workers = ₹ $8000x$

Total salary of 7 workers

= 7 × 12000 = ₹ 84000

Total salary of remaining $(x - 7)$ × 6000

According to the question,

$8000x = 6000x - 42000 + 84000$

$\Rightarrow$ $2000x = 42000$

$x = 21$

Hence, Total number of workers = 21

14. (C) Average of 50 observation = 36

So, the sum of observation of 50.

Total value of 50 observation

= 50 × 36 = 1800

But 48 was wrongly taken 23.

So, difference = 48 – 23 = 25

Hence, total correct value of 50 observation

= 1800 + (25)

= 1825

$$\therefore \text{ Required mean} = \frac{1825}{50} = 36.5$$

15. (C) The average of runs of 10 matches

= 38.9

$\therefore$ Total no. of runs = 10 × 38.9

= 389

but no. of runs for the first six matches

= 42 × 6 = 252

and no. of runs for last 4 matches

= 389 – 252 = 137

So, the required mean/average

$$= \frac{137}{4}$$

= 34.25

16. (D) Average of 50 students in a class

= 45 kg

Total weight = 45 × 50

= 2250 kg

= 2250000g

Now, one boy leaves the class.

So, new average = 44.9 kg

= 44900 g

Total weight of remaining student

= 44900 × 49

= 2200100 g

Weight of student who left

= (2250000 – 2200100) g

= 49900 g

= 49.9 kg

17. (B) According to Arun

→ 66, 67, 68, 69, 70, 71

According to his brother

→ 61, 62, 63, 64, 65, 66, 67, 68, 69,

According to mother

→ !> 68 (not greather than 68)

$\therefore$ Required average weight = 67 kg

18. (D) Marks in the first year Examination

$$= 500 \times \frac{45}{100} = 225$$

Marks in the second year Examination

$$= 500 \times \frac{55}{100} = 275$$

$\because$ 60% marks of total marks

$$= (500 + 500 + 500) \times \frac{60}{100}$$

$$= 900$$

$\therefore$ Required marks $= 900 - (225 + 275)$

$$= 900 - 500$$

$$= 400$$

19. (D) Total increase weight $= 5 \times 2$ kg

$$= 10 \text{ kg}$$

$\therefore$ Weight of the new man $= (60 + 10)$

$$= 70 \text{ kg}$$

20. (C) Suppose initial expenditure on food per day $= ₹\ x$

Then, $\frac{x}{35} - \frac{(x+42)}{42} = 1$

$$\Rightarrow \frac{42x - 35x - 35 \times 42}{35 \times 42} = 1$$

$$\Rightarrow 7x - 1470 = 1470$$

$$\Rightarrow 7x = 2940$$

$$\Rightarrow x = \frac{2940}{7}$$

$$= ₹\ 420$$

21. (C) Age of new comer $= 18 - \left(24 \times \frac{1}{2}\right)$

$$= (18 - 2) \text{ yr}$$

$$= 16 \text{ yr}$$

22. (D) Suppose average money spent by 19 persons $= ₹\ x$

Then, $19 \times x = 13 \times 79 + (19 - 13) \times (x + 4)$

$$\Rightarrow 19x = 1027 + 6x + 24$$

$$\Rightarrow 13x = 1051$$

$$\Rightarrow x = ₹\ \frac{1051}{13}$$

$\therefore$ Total money spend by them

$$= 19 \times \frac{1051}{13} = ₹\ 1536.07$$

23. (B) Let Anshuman has x exams.

$\Rightarrow$ Total marks $= 100x$

and his aimed marks $= 95x$

$\because$ He got 3 marks less in quarterly than his target

$\therefore$ Total less marks $= (80x - 3x) = 77x$

and he got 2 marks more in half yearly then his target

$\therefore$ Total more marks $= (80x + 2x) = 82x$

$\therefore$ Difference $= 20$

$$82x - 77x = 20$$

$$5x = 20$$

$$\Rightarrow x = 4$$

$\Rightarrow$ Total exam marks $= 100x$

$$= 100 \times 4 = 400$$

24. (B) Required average

$$= \frac{510 \times 5 + 240 \times 25}{30}$$

$$= \frac{2550 + 6000}{30}$$

$$= \frac{8550}{30} = 285$$

25. (D) Average of 5 numbers $= 27$

$$\text{Sum} = 27 \times 5 = 135$$

$\because$ 1 excluded

$\therefore$ Average of 4 numbers $= 25$

$\therefore$ $\text{Sum} = 25 \times 4 = 100$

$\therefore$ Excluded number $= 135 - 100$

$$= 35$$

26. (A) Let the number of women $= x$

$\Rightarrow$ the number of men

$$= (100 - x)$$

According to statement,

$$\frac{28x + 45(100 - x)}{100} = 36.5$$

$$\Rightarrow 28x + 4500 - 45x = 3650$$

$$\Rightarrow 17x = 850$$

$$\Rightarrow x = \frac{850}{17} = 50$$

27. (C) Given that,

$$\left(x + \frac{1}{x}\right) \times \frac{1}{2} = M$$

$$\therefore x + \frac{1}{x} = 2M$$

Now, $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

$$(2M)^2 - 2 = x^2 + \frac{1}{x^2}$$

Therefore, Average of $x^2 + \frac{1}{x^2}$

$$= \frac{1}{2} \times \left(x^2 + \frac{1}{x^2}\right)$$

$$= \frac{1}{2} \times (4M^2 - 2)$$

$$= \frac{1}{2} \times 2(2M^2 - 1)$$

$$= 2M^2 - 1$$

28. (C) Correct average

$$= \frac{75 \times 20 + 97 - 79}{20}$$

$$= \frac{1500 + 18}{20}$$

$$= \frac{1518}{20} = 75.9$$

29. (A) Actual average

$$= \frac{\text{Sum of marks of 100 stuents + correct value - incorrect value}}{\text{Total number of students}}$$

$$= \frac{5800 + 86 - 68}{100}$$

$$= \frac{5818}{100} = 58.18$$

30. (B) Since, there are 50 odd numbers till 100.

$\therefore$ Sum of first n odd numbers $= n^2$

$\therefore$ Average of first 50 odd numbers

$$= \frac{(50)^2}{50} = 50$$

31. (D) The average of husband and wife before 4yr. = 27 yr.

$\Rightarrow$ Sum of age of them 4 yr. ago = 54 yr.

$\Rightarrow$ At Present, Sum of age of them $= 54 + 8 = 62$ yr.

$\because$ The average of husband, (H), wife (W) and child (C) at present = 21 yr.

$$\Rightarrow \frac{H, W, C}{3} = 21$$

$$\Rightarrow 62 + C = 21 \times 3$$

$$\Rightarrow C = 63 - 62 = 1 \text{ yr.}$$

32. (B) Let Age of the coach $= x$ yr.

Average age of 11 players = 20 yr.

$\Rightarrow$ Sum of age of 11 players $= 20 \times 11$

$$= 220 \text{ yr.}$$

$\therefore$ New, Average age of players including coach

$$\Rightarrow \frac{220 + x}{12} = 20 \times \frac{110}{100}$$

$$\Rightarrow \frac{220 + x}{12} = 22$$

$$\Rightarrow 220 + x = 264$$

$$\Rightarrow x = 264 - 220 = 44 \text{ yr.}$$

33. (A) Required Average $= \frac{5 \times 15 + 5 \times 20}{10}$

$$= \frac{75 + 100}{10}$$

$$= \frac{175}{10} = 17.5$$

34. (C) Sum of squares of 'n' natural numbers

$$= \frac{1}{6} n(n+1)(2n+1)$$

$$\text{Average} = \frac{\frac{n}{6}(n+1)(2n+1)}{n}$$

$$= \frac{1}{6}(n+1)(2n+1)$$

$\because$ $n = 10$

$\therefore$ $= \frac{1}{6} \times 11 \times 21$

$$= \frac{231}{6} = 38.5$$

35. (B) Average marks of section A = 60

$\Rightarrow$ Sum of marks in section A $= 60 \times 32 = 1920$

and, Average marks of section B = 33

$\Rightarrow$ Sum of marks in section B $= 33 \times 40 = 1320$

$\therefore$ Total marks in section A and B

$= 1920 + 1320 = 3240$

$\therefore$ Average $= \dfrac{3240}{72} = 45$

36. (B) Monthly average income

$$= \frac{5\times1200+7\times1300+2900}{12}$$

$$= \frac{6000+9100+2900}{12}$$

$$= \frac{18000}{12} = 1500$$

37. (C) The average of the first 'n' positive integers

$$= \frac{n+1}{2}$$

$\therefore$ Average $= \dfrac{100+1}{2}$

$$= \frac{101}{2} = 50.5$$

38. (C) Let the age of P, Q and R be x yr., y yr. z yr

$\therefore$ According to statement ,

$$\frac{x+y+z}{3} = z + 5$$

$\Rightarrow$ $x + y - 2z = 15$ ...(i)

and $x + y = 39$ ...(ii)

From eq. (i) and eq. (ii)

$39 - 2z = 15$

$\Rightarrow$ $2z = 39 - 15 = 24$

$\Rightarrow$ $z = 12$ yr.

39. (B) Required average

$$= \frac{5\times40-35+25}{5}$$

$$= \frac{200-10}{5} = \frac{190}{5}$$

$= 38$ yr.

40. (C) Let the average of runs in 10^{th} test was x

According to statement,

Sum of 10^{th} innings $= 10x$

Sum of 11 innings $= 11(x + 5)$

$\therefore$ $10x + 100 = 11x + 55$

$\Rightarrow$ $x = 100 - 55 = 45$

$\therefore$ Present average of runs

$= 45 + 5 = 50$

●●

Chapter 11

Simple Interest

1. Introduction

Interest is the money paid for the use of money borrowed. It is generally a percentage of sum borrowed. It is paid quarterly, half-yearly or annually as agreed upon.

2. Principal

The sum borrowed or lent out for a certain period is called the *Principal* (P).

3. Amount

The sum of interest and principal is called the *Amount* (A).

Amount = Principal + Interest

4. Simple Interest

If the interest is payable on the principal amount only, it is called Simple Interest. It is denoted by the symbol (SI). If principal, rate of interest and time are P, R, and T respectively, then

$$SI = \frac{P \times R \times T}{100}$$

Example : ₹ Find the simple interest on 2,000 for 5 years at 10% per annum.

(A) ₹ 1,100 (B) ₹ 200
(C) ₹ 1,000 (D) ₹ 500

Solution : (C) $S.I. = \frac{PRT}{100}$

$$S.I. = \frac{2000 \times 5 \times 10}{100} = ₹\,1{,}000$$

With the help of the following formulae, we can find out the principal, rate of interest and time.

$$P = \frac{SI \times 100}{R \times T}$$

$$R = \frac{SI \times 100}{T \times P}$$

$$T = \frac{SI \times 100}{P \times R}$$

5. Important Formulae

1. If an amount becomes n times itself, then

- $\text{Time} = \frac{(n-1)\times 100}{R}$
- $\text{Rate} = \frac{(n-1)\times 100}{R}$

2. If an amount becomes n times at $r\%$ interest

$$\text{Time} = \frac{(n-1)\times 100}{r}$$

3. If an amount becomes n_1 times at a certain rate then for the same period of time the amount will be n_2 times at the rate of interest

$$\text{rate} = \left(\frac{n_2 - 1}{n_1 - 1}\right) \times n_1$$

6. Important Rules

Rule 1 : An amount P becomes n times itself at R% simple interest per annum in T years. Then,

$$T = \frac{(n-1)\times 100}{R} \text{ years}$$

Rule 2 : An amount P becomes n times itself at R% simple interest per annum in T years. Then,

$$R = \frac{(n-1)\times 100}{T}$$

Rule 3 : On a certain amount P, the simple interest at the end of T years becomes n times itself, then the rate of interest p.c.p.a will be

$$R = \left(\frac{n}{T} \times 100\right)\%$$

Rule 4 : On a certain amount P, the simple interest at the end of T years becomes n times itself at the rate of interest R p.c.p.a, then

$$T = \frac{n}{R} \times 100 \text{ years}$$

Rule 5 : If interest is gained x more by changing the rate from $R_1\%$ to $R_2\%$ in T years, then the principal will be

$$P = \frac{x \times 100}{(R_2 - R_1)\% \text{ of } T}$$

Rule 6 : An amount becomes n_1 times of itself in T_1 years and n_2 times of itself in T_2 years at simple interest, then the time in which it became n_2 times itself will be–

$$T_2 = \frac{(n_2 - 1)T_1}{(n_1 - 1)} \text{ years}$$

Rule 7 : An amount becomes n_1 times of itself in T_1 years and n_2 times of itself in T_2 years at simple interest R, then

$$R = \frac{(n_2 - 1)\times 100}{T_2}$$

Rule 8 : An amount becomes n_1 times of itself at simple interest R_1 in a certain time, then the rate of interest at which the amount becomes n_2 times of itself in the same time will be–

$$R_2\% = \frac{(n_2 - 1)\times R_1}{(n - 1)}\%$$

Important Examples

Example 1. In how many years will an amount double itself at 10% simple interest per annum?

Solution: $$T = \frac{(A-1)\times 100}{R}$$

$$= \frac{(2-1)\times 100}{10}$$

$$= 10 \text{ years}$$

Example 2. If an amount doubles itself in 15 years, then find the rate of interest per annum.

Solution: $$R = \frac{(A-1)\times 100}{T}:$$

$$= \frac{(2-1)\times 100}{15}$$

$$= \frac{100}{15} = \frac{20}{3} = 6\frac{2}{3}:$$

Example 3. On a certain amount for certain period of time, the simple interest becomes $\frac{16}{25}$ of the amount. If rate of interest per annum and the years are numerically equal, then find the rate.

Solution: Let, Principal = ₹ P

Here, $T = R$

$$\frac{P\times R\times R}{100} = \frac{16}{25}P$$

$$R^2 = \frac{16}{25}\times 100$$

$$R = \sqrt{16\times 4}$$

$$R = 4\times 2$$

$$= 8\%$$

Example 4. In how many years an amount will give the double interest of the amount at 10% simple interest per annum?

Solution : $$T = \frac{A}{R}\times 100$$

$$= \frac{2}{10}\times 100$$

$$= 20 \text{ years}$$

Example 5. If interest is gained 50 more by changing the rate from 10% to 12% in 5 years, then find the principal.

Solution : $$P = \frac{SI\times 100}{(R_2 - R_1):\ T}$$

$$= \frac{50\times 100}{(12\ \ 10)\ \ 5}$$

$$= \frac{5000}{2\times 5} = ₹\,500$$

Example 6. An amount becomes double itself at simple interest in 10 years. In how many years will it be 4 times of itself?

Solution : $$\frac{P\times R\times 10}{100} = 100$$

$$PR = 1000 \qquad ...(1)$$

Similarly, $$\frac{P\times R\times T}{100} = 300$$

$$\frac{1000\times T}{100} = 300 \qquad [\text{from eq. (1)}]$$

$$T = 30 \text{ years}$$

Example 7. If an amount becomes double itself in 5 years and 3 times in 10 years at simple interest, then find the annual rate of interest.

Solution: Required rate $= \frac{(A_2 - 1)\times 100}{T_2}$

$$= \frac{(3-1)\times 100}{10}$$

$$= \frac{2}{10}\times 100 = 20:$$

Example 8. A bank doubles money in a fixed time at an annual rate of 10%. What will be the annual rate to make 3 times the money in the same time?

Solution : Required rate $= \left(\frac{A_2 - 1}{A_1 - 1}\right)\times R_1:$

$$= \left(\frac{3-1}{2-1}\right)\times 10:$$

$$= 2\times 10\% = 20\%$$

Example 9. If the simple interest of lent amount is 1/4 of it at the same annual rate and the same time, then find the time.

Solution : Required time $= \sqrt{\frac{A_1}{A_2}}\times 10$

$$= \sqrt{\frac{1}{4}}\times 10$$

$$= \frac{1}{2}\times 10$$

$$= 5 \text{ years}$$

Important Questions

1. The simple interest accrued on a sum of money at the end of 4 yr is 1/5th of its principal. What is the rate of interest per annum ?

(A) 4% (B) 5%

(C) 6% (D) Inadequate data

2. A sum of money lent out at simple interest doubled itself in 20 yr. In how many years will it triple itself ?

(A) 28 (B) 30

(C) 40 (D) 35

3. The simple interest on a sum of money is $\frac{1}{9}$ of the sum. The number of years is numerically equal to the rate per cent per annum. The rate per cent per annum is :

(A) 3.33 (B) 5

(C) 6.66 (D) 10

4. A certain sum of money becomes three times of itself in 20 yr simple interest. In how many years will the initial sum become double at the same rate of simple interest ?

(A) 8 (B) 10

(C) 12 (D) 14

5. Ram borrows ₹ 8000 at 12% per annum simple interest and Mohan borrows ₹9100 at 10% per annum simple interest. In how many years will their borrowed amounts (debt) be equal ?

(A) 18 (B) 20

(C) 22 (D) 24

6. Reena took a loan of ₹ 1200 with simple interest for a certain number of years. The number of years are same as the interest rate. If she has paid ₹ 432 as interest at the end of the loan period, what was the rate of interest ?

(A) 3.6% (B) 6%

(C) 12 % (D) None of these

7. ₹ 800 becomes ₹ 956 in 3 yr at a certain rate of interest. If the rate of interest is increased by 4%, what amount will ₹800 become in 3 yr ?

(A) ₹ 1020 (B) ₹ 1052

(C) ₹ 1282 (D) ₹ 1080

8. How much time will it take for an amount of ₹ 450 to gain ₹ 81 as interest, if rate of interest is 4.5% per annum on simple interest ?

(A) 4.5 yr (B) 3.5 yr

(C) 5 yr (D) 4 yr

9. At what rate of annual simple interest will ₹10000 double in 15 yr ?

(A) 5.5% (B) 8%

(C) 6.75% (D) 7.25%

10. How long will it take for a sum of money invested at 5% p.a. at simple interest to increase its value by 40% ?

(A) 5 years (B) 6 years

(C) 7 years (D) 8 years

11. On a certain sum, the simple interest at the end of $6\frac{1}{4}$ years becomes $\frac{3}{8}$ th of the sum. The rate percent is :

(A) 7 % (B) 6%

(C) 5 % (D $5\frac{1}{2}$%

12. A man lent a sum of money at the rate of simple interest of 4%. If the interest for 8 yr is ₹ 340 less than the principal, the principal is :

(A) ₹ 500 (B) ₹ 520

(C) ₹ 540 (D) ₹ 560

13. A sum was invested, for 3 yr at simple interest at a certain rate. Had it been invested at 4% higher rate of interest, it would have fetched ₹ 600 more. The sum is :

(A) ₹ 4000 (B) ₹ 4950

(C) ₹ 5000 (D) ₹ 5150

14. A banker lent ₹ 6000 at 10%, and ₹5000 at 12% at the same time and for same period of time. The banker received ₹2400 as total interest on both loans. Find the period for which the banker had lent the amount.

(A) 3 yr 6 months (B) 3 yr

(C) 2 yr 6 months (D) 2 yr

15. If a sum become double in 16 yr, how many times will it be in 8 yr ?

(A) $1\frac{1}{2}$ times (B) $1\frac{1}{3}$ times

(C) $1\frac{3}{4}$ times (D) $1\frac{1}{4}$ times

16. If a person repaid ₹ 22500 after 10 yr of borrowing a loan, at 10% per annum simple interest, find out what amount did he take as a loan ?

(A) ₹ 11225 (B) ₹ 11250

(C) ₹ 10000 (D) ₹ 7500

17. A sum of money invested at simple interest triples itself in 8 yr. How many times will it become in 20 yr time ?

(A) 8 (B) 7

(C) 6 (D) 9

18. A certain scheme of investment in simple interest declares that it triples the investment in 8 yr. If you want to quadruple your money through that scheme, you have to invest it for :

(A) 12 yr.

(B) 11 yr. 6 months

(C) 10 yr. 8 months

(D) 10 yr.

19. In certain years a sum of money is doubled itself at $6\frac{1}{4}$% simple interest p.a., then required time will be :

(A) 16 yr. (B) $10\frac{2}{3}$ yr.

(C) $12\frac{1}{2}$ yr. (D) 8 yr.

20. The simple interest on ₹4000 in 3 yr. at the rate of x% p.a. equals to the simple interest on ₹5000 at the rate of 12% p.a. in 2 yr. The value of x is :

(A) 6% (B) 8%

(C) 9% (D) 10 %

21. The interest on a certain sum of money is ₹22 and the discount on the same sum for the same time and at the same rate is ₹20. Find the sum.

(A) ₹ 220 (B) ₹ 200

(C) ₹ 210 (D) ₹ 212

SOLUTIONS

1. (B) Let P = ₹ 100

$$SI = \frac{P \times r \times t}{100}$$

$$= \frac{100 \times 4 \times t}{100} = 4r$$

$$\Rightarrow \quad 4r = \frac{1}{5}(P)$$

$$= \frac{1}{5} \times 100 = 20$$

$$\Rightarrow \quad r = \frac{20}{4} = 5\%$$

$\therefore$ Rate of interest = 5% per annum

2. (C) According to question,

$$\frac{n_1 - 1}{T_1} = \frac{n_2 - 1}{T_2}$$

$$\frac{2-1}{20} = \frac{3-1}{T_2}$$

$$\Rightarrow \quad \frac{1}{20} = \frac{2}{T_2}$$

$$\Rightarrow \quad T_2 = 40 \text{ years}$$

3. (A) $SI = \frac{P}{9}, r = t$

$$SI = \frac{P \times r \times t}{100}$$

$$\frac{P}{9} = \frac{P \times r \times r}{100}$$

$$r^2 = \frac{100}{9}$$

$$r = \frac{10}{3} = 3.33\%$$

4. (B) Let in n years the initial sum will be double.

$\therefore$ According to question,

$$\frac{3-1}{20} = \frac{2-1}{n}$$

$$\Rightarrow \quad \frac{2}{20} = \frac{1}{n}$$

or $\quad n = 10$ yr

5. (C) $$SI = \frac{8000 \times 12 \times t}{100}$$

$$= 960t$$

$$\Rightarrow \quad A = 8000 + 960t$$

Again, $$SI = \frac{9100 \times 10 \times t}{100} = 910t$$

$$A = 9100 + 910t$$

According to the question,

$$960t + 8000 = 910t + 9100$$

$$\Rightarrow \quad 50t = 1100$$

$$t = \frac{1100}{50} = 22 \text{ yr.}$$

6. (B) $$SI = \frac{P \times r \times t}{100}$$

$$432 = \frac{1200 \times t \times t}{100}$$

$$\Rightarrow \quad 12t^2 = 432$$

$$t^2 = 36$$

$$\Rightarrow \quad t = 6$$

$$\therefore \quad t = r = 6$$

Hence, $\quad r = 6\%$

7. (B) $$r = \frac{SI \times 100}{P \times t}$$

$$= \frac{156 \times 100}{800 \times 3} = \frac{13}{2}\%$$

$$\text{New rate} = 4 + \frac{13}{2} = \frac{21}{2}\%$$

$$SI = \frac{800 \times 3 \times 21}{2 \times 100} = ₹ 252$$

$$\text{Amount} = P + SI = 800 + 252$$

$$= ₹ 1052$$

8. (D) $$t = \frac{81 \times 100 \times 2}{450 \times 9} = 4 \text{ yr}$$

9. (C) $$r = \frac{SI \times 100}{P \times t}$$

$$= \frac{10000 \times 100}{10000 \times 15}$$

$$= \frac{20}{3} = 6.66\% \cong 6.75\%$$

10. (D) Let the principle amount be P.

According to the condition,

$$(100 + 40)\% \text{ of } P = \frac{140 \times P}{100} = \frac{7P}{5}$$

and $$\frac{7P}{5} = P\left(\frac{T \times 5}{100} + 1\right)$$

$$\frac{7}{5} - 1 = \frac{T \times 5}{100},$$

$$\frac{2}{5} = \frac{T}{20}$$

$$T = 8 \text{ years}$$

11. (B) Let the principle amount is p, rate $= r\%$

According to the question,

$$\frac{3}{8}p = p \times 6\frac{1}{4} \times \frac{r}{100},$$

$$\frac{3}{8} = \frac{25 \times r}{400},$$

$$r = 3 \times 2 = 6\%$$

12. (A) Let the required sum = P

$r = 4\%$ time

$= 8$ yr (given)

and $\quad$ interest = P – 340

$$\text{Simple interest} = \frac{PTR}{100}$$

$$\Rightarrow \quad P - 340 = \frac{P \times 8 \times 4}{100}$$

$$\Rightarrow \quad 100P - 34000 = P \times 32$$

$$\Rightarrow \quad 68P = 34000$$

$$\therefore \quad P = \frac{34000}{68}$$

$$= ₹ 500$$

13. (C) Time = 3 yr, $R_1 = r_1\%$,

$R_2 = (r_1 + 4)\%$

Interest = ₹ 600

$$\because \quad \frac{P \times 3(r_1 + 4)}{100} - \frac{P \times 3 \times r_1}{100} = 600$$

$$\Rightarrow \quad \frac{3P(r_1 + 4) - 3Pr_1}{100} = 600$$

$$\Rightarrow \quad 3Pr_1 + 12P - 3Pr_1 = 60000$$

$$\Rightarrow \quad 12P = 60000$$

$$\therefore \quad P = ₹ 5000$$

14. (D) Let $\quad$ time = n yr

Then,

$$\frac{6000 \times 10 \times n}{100} + \frac{5000 \times 12 \times n}{100}$$

$$= 2400$$

$$\Rightarrow 600n + 600n = 2400$$

$$\Rightarrow \quad 1200n = 2400$$

$$\Rightarrow \quad n = \frac{2400}{1200} = 2 \text{ yr}$$

15. (A) Let the sum will be n times in 8 years

$\therefore$ According to question,

$$\frac{2-1}{16} = \frac{n-1}{8}$$

$$\Rightarrow \quad n - 1 = \frac{8}{16} \text{ or } \frac{1}{2}$$

$$\Rightarrow \quad n - 1 = \frac{1}{2}$$

$$\Rightarrow \quad n = 1\frac{1}{2} \text{ times}$$

16. (B) Suppose loan amount = ₹ x

Then, $$x + \frac{x \times 10 \times 10}{100} = 22500$$

$$\Rightarrow \quad 2x = 22500$$

$$\Rightarrow \quad x = ₹ 11250$$

17. (C) Suppose principal amount = ₹ P

Then, $$3P - P = \frac{P \times r \times 8}{100}$$

$$\Rightarrow \quad 2P = \frac{P \times r \times 8}{100}$$

$$\Rightarrow \qquad r = \frac{2\times100}{8} = 25\%$$

$$\therefore \quad \text{After 20 yr} = P+\left\{\frac{P\times25\times20}{100}\right\}$$

$$= P + 5P = 6P$$

18. (A) Let the principal be ₹ x.

Then, amount = ₹ $3x$

$$\Rightarrow \qquad \text{S.I.} = 3x - x = ₹\ 2x$$

Now, $$\text{S.I.} = \frac{x\times R\times8}{100}$$

or $$R = \frac{2x\times100}{8x} = 25\%$$

Let to quadruple it takes n yr.

According to condition,

$$\therefore \qquad \text{S.I.} = \frac{P\times R\times n}{100}$$

or $$n = \frac{3x\times100}{x\times25}$$

$$= 12 \text{ yr.}$$

19. (A) $$T = \frac{\text{S.I.}\times100}{P\times R}$$

$$= \frac{(2P-P)\times100}{P\times\frac{25}{4}}$$

$$= \frac{4\times100}{25} = 16 \text{ yr.}$$

Short Trick : If any amount becomes n times in x years, then the amount will take "xy" years for change n times to n^y times.

Solution—Any amount becomes double in 15 years. Having 8 times of it, it will take time = 15 × 3 = 45 years.

20. (D) $$\frac{4000\times3\times x}{100} = \frac{5000\times12\times2}{100}$$

or $$x = 10\%$$

21. (A) Let the sum be ₹ x.

$$\therefore \qquad 22 = \frac{x\times R\times T}{100}$$

or $$xRT = 2200 \qquad \text{...(i)}$$

Now, Interest – Discount

$$= \frac{\text{Discount}\times R\times T}{100}$$

$$22 - 20 = \frac{20\times R\times T}{100}$$

or $$RT = 10$$

From eq. (i),

$$x \times 10 = 2200$$

$$x = \frac{2200}{10}$$

or $$x = ₹\ 220$$

●●

Chapter 12

Compound Interest

1. Compound Interest

Compound interest is basically interest on the principal amount plus whatever interest has already accrued. It means, we have two factors that add up to make compound interest: interest paid on the principal and interest paid on the accrued interest.

The first year amount (principal + interest) is considered the principal for the next year.

Amount– (principal + interest) is called amount. It is denoted by the symbol 'A'. It is calculated by –

- Amount (A) = $P\left(1+\frac{R}{100}\right)^{T}$

  Where, P = Principal

  r = Rate of interest (in percentage)

  n = Time

- Compound Interest (C. I.) = A – P

2. Important Points

(I) The rate of interest is always calculated in percentage.

(II) If rate of interest is compounded annually, then the principal changes in subsequent years.

(III) Simple interest and compound interest are same for the first year on the same principal at the same rate of interest.

(IV) If the rate of interest is compounded quarterly, then the time becomes 4 times and the rate of interest is one-fourth.

(V) If the rate of interest is compounded half-yearly, then the time becomes 2 times and the rate of interest is half.

(VI) If time is given rational number or mixed fraction, *i.e.* $3\frac{1}{2}$ years,

then, $A = P\left(1+\frac{R}{100}\right)^{3}\times\left(1+\frac{\frac{R}{2}}{100}\right)$

3. Important Types

Type 1: When the rates of interest for first, second, and third year are $r_1\%$, $r_2\%$ and $r_3\%$ respectively, then amount after three years will be:

$$A = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$$

Example : Anshuman take a loan of 50000 from Gramin bank at the rates of interest for first year 5%, second year 7% and third year 10% compounded. At the end of the third year, what amount will be paid ?

Sol : Required amount $= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$

$= 50000\left(1+\frac{5}{100}\right)\left(1+\frac{7}{100}\right)\left(1+\frac{10}{100}\right)$

$= 50000 \times \frac{105}{100}\times\frac{107}{100}\times\frac{110}{100}$

$= ₹\ 61792.50$

Type 2: If the amount on the principal P at the rate of $r\%$ per annum compound interest in n years is A, then the rate of interest will be :

$$r\% = \left[\left(\frac{A}{P}\right)^{\frac{1}{n}} - 1\right]\times 100:$$

Example : At what rate per cent per annum will the compound interest on 2304 for 2 years be 204?

Sol : Here, $A = P + r = 2500 + 204$

$A = 2704$

$$r\% = \left[\left(\frac{2704}{2500}\right)^{\frac{1}{2}} - 1\right]\times 100\%$$

$$= \left[\left(\frac{52}{50}\right) - 1\right]\times 100:$$

$$= \left(\frac{52-50}{50}\right)\times 100\% = 4\%$$

Type 3: If principal P becomes N times itself in n years at $r\%$ per annum compound interest, then the rate of interest will be :

$$r\% = \left[(N)^{\frac{1}{n}} - 1\right]\times 100\%$$

Example : If a sum of money compounded annually becomes $\frac{27}{8}$ times itself in 3 years, then the rate of interest per annum will be :

Sol : $$r\% = \left[\left(\frac{27}{8}\right)^{\frac{1}{3}} - 1\right]\times 100\%$$

$$= \left[\frac{3}{2} - 1\right]\times 100\%$$

$$= \frac{1}{2}\times 100\% = 50\%$$

Type 4: An amount of money at compound interest grows up to A_1 in n_1 years and up to A_2 in n_2 years. The rate of interest will be :

$$r\% = \left[\left(\frac{A_2}{A_1}\right)^{\frac{1}{n_2 - n_1}} - 1\right]\times 100\%$$

Example : Shikha invested some amount at compound interest. She got amount of 2420 for 2 years and 2662 for three years. Find the rate of interest.

Sol :

$$r = \left[\left(\frac{A_2}{A_1}\right)^{\frac{1}{n_2 - n_1}} - 1\right] \times 100\,\%$$

[$\because$ where, $A_1 = ₹\ 2420$, $A_2 = ₹\ 2662$

$n_2 = 3$ and $n_1 = 2$ years]

$$= \left[\left(\frac{2662}{2420}\right)^{\frac{1}{3-2}} - 1\right] \times 100\,\%$$

$$= \left(\frac{2662 - 2420}{2420}\right) \times 100\,\%$$

$$= \frac{1}{10} \times 100 = 10\%$$

Type 5: The difference between simple interest and compound interest (compounded annually) on a sum P for 2 years at 10% per annum is D, then

$$r\,\% = \sqrt{\frac{D}{P}} \times 100\%$$

Example : If the difference between simple interest and compound interest on 1600 for 2 years is 144, then find the rate of interest compounded annually.

Sol : Here, $P = ₹\ 1600$

$D = ₹\ 144$

$$r\,\% = \sqrt{\frac{D}{P}} \times 100\,\%$$

$$= \sqrt{\frac{144}{1600}} \times 100\,\%$$

$$= \frac{12}{40} \times 100$$

$$= 30\%$$

Type 6: If the interest are compounded quarter(every 4 months), then the amount will be : $A = ₹\left(\frac{300 + r}{300}\right)^{3n}$

Example : What will be the sum on 10,000 at 10% per annum compound interest at the end of the year if the interest compounded quarter (every 4 months)?

Sol : Here, $P = ₹\ 10{,}000$

$r = 10\%$

Condition $\longrightarrow$ Interest compounded quarter

$$A = P\left(\frac{300 + r}{300}\right)^{3n}$$

$$= 10{,}000\left(\frac{300 + 10}{300}\right)^{3\times 1}$$

$$= 10{,}000 \times \frac{310}{300} \times \frac{310}{300} \times \frac{310}{300}$$

$$= \frac{31 \times 31 \times 310}{27}$$

$$= ₹\ 11034 \text{ (Approx)}$$

Type 7: If the rate of interest is $r\%$ per annum, then the equivalent interest rate after 2 years will be :

$$\text{Equivalent interest rate (R)} = \left(2r + \frac{r^2}{100}\right)\%$$

Example : If the rate of interest is 10% per annum, then the equivalent interest rate after 2 years will be :

Sol : Equivalent interest rate

$$R = \left(2r + \frac{r^2}{100}\right)\%$$

$$= \left[2 \times 10 + \frac{(10)^2}{100}\right]\%$$

$$= (20 + 1)\,\% = 21\%$$

Type 8: On a certain amount, the difference between simple interest and compound interest at $r\%$ rate of interest for n years is D. Find the rate per cent per annum.

$$r = \frac{D \times 100}{\text{S.I. for 1 year}}$$

Example : On a certain amount, the simple interest and compound interest for 2 years are 120 and 150 respectively. If the rate of interest compounded annually, then find the rate of interest.

Sol : S.I. for a year $= \frac{120}{2} = ₹\ 60$

$D = (150 - 120) = ₹\ 30$

$$R = \frac{D \times 100}{\text{S.I. for 1 year}}$$

$$= \frac{30 \times 100}{60} = 50\%$$

Type 9: If simple interest and compound interest of a sum of amount P for 2 years are S and C respectively, then the principal will be :

$$P = \frac{S^2}{4(C - S)}$$

Example : On a certain amount, the simple interest and compound interest for 2 years are 50 and 60 respectively. Find the principal.

Sol :

$$P = \frac{S^2}{4(C - S)}$$

$$= \frac{(50)^2}{4(60 - 50)}$$

$$= \frac{2500}{4 \times 10} = ₹\ 62.50$$

Type 10: If simple interest and compound interest of a sum of amount P for the 2nd year are S and C respectively, then the principal will be : $P = \frac{S^2}{(C-S)}$

Example : If simple interest and compound interest of a sum of amount for the 2nd year are 200 and 210 respectively. Find the principal.

Sol : $P = \frac{(200)^2}{210-200} = \frac{40000}{10} = ₹\ 4000$

4. Important Examples

Example 1: The compound interest of an amount for 2 years is 2448 at the rate of 4% per annum. Find the simple interest of the same amount at the same rate of interest for 2 years.

Sol : Let the rate of interest = $r\%$

$$r = \frac{200(C-S)}{S}$$

$$4 = \frac{200(2448-S)}{S}$$

$$\Rightarrow \quad 4S = 489600 - 200S$$

$$\Rightarrow \quad 204S = 489600$$

$$\Rightarrow \quad S = \frac{489600}{204}$$

$$= ₹\ 2400$$

So, simple interest for 2 years = ₹ 2400

Example 2: Find the money whose compound interest for the second year becomes 1272 at 6% per annum.

Sol :

$$P = \frac{100C}{\left(1+\frac{r}{100}\right)\times r}$$

$$= \frac{100\times 1272}{\left(1+\frac{6}{100}\right)\times 6}$$

$$= ₹\ 20{,}000$$

Example 3: At the rate of compound interest, the principal increases to 700 in 2 years and to 770 in 3 years, then how much will that money increase in 4 years?

Sol :

$$r_1\% = \frac{(A_2-A_1)}{A_1}\times 100$$

$$= \frac{770-700}{700}\times 100$$

$$= \frac{70}{700}\times 100 = 10\%$$

$$\text{Amount} = P\left(1+\frac{r}{100}\right)^n$$

$$= 700\left(1+\frac{10}{100}\right)^2$$

$$= 700\times\frac{110}{100}\times\frac{110}{100}$$

$$= 7\times 121 = ₹\ 847$$

Example 4: A sum of money becomes 720 in 2 years and 1125 in 4 years. Find the annual rate of compound interest.

Sol :

$$\left(1+\frac{r}{100}\right)^{4-2} = \frac{1125}{720}$$

$$\therefore \quad \left(1+\frac{r}{100}\right)^2 = \frac{225}{144} = \left(\frac{15}{12}\right)^2$$

[On comparing both sides]

$$\therefore \quad 1+\frac{r}{100} = \frac{15}{12} = \frac{5}{4}$$

$$\therefore \quad \frac{r}{100} = \frac{5}{4} - 1$$

$$\therefore \quad \frac{r}{100} = \frac{1}{4}$$

$$r = \frac{1}{4}\times 100 = 25\ \%$$

Example 5: Find the difference between simple interest and compound interest on ₹ 2500 at 20% annual interest for 4 years.

Sol :

$$D = P\left[\left(1+\frac{r}{100}\right)^n - 1 - \frac{rn}{100}\right]$$

$$= 2500\left[\left(1+\frac{20}{100}\right)^4 - 1 - \frac{20\times 4}{100}\right]$$

$$= 2500\left[\left(\frac{6}{5}\right)^4 - 1 - \frac{4}{5}\right]$$

$$= 2500\left[\frac{1296}{625} - 1 - \frac{4}{5}\right]$$

$$= 2500\left[\frac{1296-625-500}{625}\right]$$

$$= 2500\times\frac{171}{625} = ₹\ 684$$

Example 6: Find out the money if you borrow a money at the rate of 10% per annum compound interest and pay it in three equal annual installments of 2500.

Sol :

$$P = \frac{2500}{\left(1+\frac{10}{100}\right)} + \frac{2500}{\left(1+\frac{10}{100}\right)^2} + \frac{2500}{\left(1+\frac{10}{100}\right)^3}$$

$$= 2500\left[\frac{1}{\frac{11}{10}} + \frac{1}{\left(\frac{11}{10}\right)^2} + \frac{1}{\left(\frac{11}{10}\right)^3}\right]$$

$$= 2500\left[\frac{10}{11} + \left(\frac{10}{11}\right)^2 + \left(\frac{10}{11}\right)^3\right]$$

$$= 2500\times\frac{10}{11}\left[1+\frac{10}{11}+\frac{100}{121}\right]$$

$$= 2500 \times \frac{10}{11} \times \left[\frac{121+110+100}{121}\right]$$

$$= 2500 \times \frac{10}{11} \times \frac{331}{121}$$

$$= ₹\ 6217.13$$

Example 7: If money doubles in 4 years at compound interest, then in how many years will that money become 8 times its value?

Sol : Here, $2^3 = n^m \Rightarrow m = 3$

Required time $= (m \times t)$ years $= 3 \times 4$

$= 12$ years

Example 8: If money becomes 3000 after 2 years and 3200 after 3 years at a fixed rate of compound interest, find that money.

Sol :
$$P = \frac{x^2}{y}$$

$$= \frac{(3000)^2}{3200} = 2812.50$$

Example 9: Find the compound interest of two years at 10% of ₹ 5000.

Sol : Required interest $= \dfrac{P\left(2r + \dfrac{r^2}{100}\right)}{100}$

$$= 5000 \frac{\left(2 \times 10 + \dfrac{10 \times 10}{100}\right)}{100}$$

$$= 5000 \times \frac{21}{100}$$

$$= ₹\ 1050$$

Example 10 : If the compound interest of two consecutive years on money is 100 and 120 respectively, find the rate of interest.

Sol :
$$r = \frac{C_2 - C_1}{C_1} \times 100$$

$$= \frac{120 - 100}{100} \times 100 = 20\%$$

Example 11: Find the sum for a year at the rate of 10% per annum compound interest of Rs. 10,000. If interest is compounded half yearly.

Sol :
$$A = P\left(1 + \frac{r}{200}\right)^{2n}$$

$$= 10{,}000\left(1 + \frac{10}{200}\right)^{2 \times 1}$$

$$= 10{,}000 \times \frac{210}{200} \times \frac{210}{200} = ₹\ 11025$$

Example 12 : Ram lent 4000 for a year at the rate of 10% compound interest. If interest is compounded quarterly, at the end of the year how much amount of money will Ram get?

Sol : $P = ₹\ 4000$, $r = 10\%$ and $n = 1$ year

$$A = P\left(1 + \frac{r}{400}\right)^{4n}$$

$$= 4000\left(1 + \frac{10}{400}\right)^{4 \times 1}$$

$$= 4000 \times \left(\frac{41}{40}\right)^4$$

$$= 4000 \times \frac{41 \times 41 \times 41 \times 41}{40 \times 40 \times 40 \times 40}$$

$$= ₹\ 4415.25 \text{ (appx.)}$$

Example 13 : If a sum doubles in 5 years at compound interest, then in how many years it will be eight times?

Sol : $n_1 = 5$ years, $n_2 = ?$

$A_1 = 2$, $A_2 = 8$

$I_1^{\text{ब}} = I_2^{\text{ब}}$

$2^{\text{ब}} = 8^5$

$\Rightarrow 2^{\text{ब}} = (2^3)^5$

$\Rightarrow 2^{\text{ब}} = 2^{15}$

$n_2 = 15$ years

Example 14: If a simple interest of 2 years is 800 at the rate of 8% compound interest on a sum, find the compound interest of 2 years on that amount.

Sol :
$$C.I. = S.I.\left(1 + \frac{r}{200}\right)$$

$$= 800\left(1 + \frac{8}{200}\right)$$

$$= 800 \times \frac{208}{200}$$

$$= 4 \times 208 = ₹\ 832$$

Example 15: If a sum becomes 121 after 2 years with compound interest and 133.10 after three years, find the amount.

Sol :
$$P = A_1\left(\frac{A_1}{A_2}\right)^2$$

$$= 121\left(\frac{121}{133.10}\right)^2$$

$$= \frac{121 \times 121 \times 121}{133.10 \times 133.10} = ₹\ 100$$

Important Questions

1. The compound interest on a certain amount at 10% annual rate is 331. What is that money?

(A) ₹ 900 (B) ₹ 1000

(C) ₹ 1050 (D) ₹ 1100

2. What will be the compound interest on 8000 at the 15% rate per annum for 2 years 4 months? **(NCERT)**

(A) ₹ 2980 (B) ₹ 3091

(C) ₹ 3109 (D) ₹ 3100

3. What will be the compound interest on 6000 at the 10% rate per annum for $1\frac{1}{2}$ years?

(A) ₹ 910 (B) ₹ 870

(C) ₹ 930 (D) ₹ 900

4. First year interest on a sum of money at the rate of 8% compounded interest is 48. What will be the interest for the second year?

(A) ₹ 48 (B) ₹ 51.84

(C) ₹ 56.48 (D) ₹ 45

5. The difference between C.I. and S.I. on 8,000 at the rate of 5% per annum for 3 years will be – **(NCERT)**

(A) ₹ 50 (B) ₹ 60

(C) ₹ 61 (D) ₹ 600

6. The difference between C.I. and S.I. on a sum at the rate of 8% per annum for 2 years is 768. Find the sum.

(A) ₹ 1,00,000 (B) ₹ 1,10,000

(C) ₹ 1,70,000 (D) ₹ 1,20,000

7. The amount of 12,000 deposited at compound interest doubles in 5 years. How much will it be after 20 years?

(A) ₹ 1,44,000 (B) ₹ 1,20,000

(C) ₹ 1,50,000 (D) ₹ 1,92,000

8. What is the amount on which x yields compound interest at x% per annum in x years? **(NCERT)**

(A) $\frac{x}{100}$ (B) ₹ x

(C) $\left(1+\frac{x}{100}\right)^n$ (D) $\frac{x}{\left(1+\frac{x}{100}\right)^x-1}$

9. A sum of money at 8% per annum compound interest becomes 2,916 in 2 years. The interest on the same amount at the rate of 9% per annum simple interest for 3 years will be :

(A) ₹ 625 (B) ₹ 600

(C) ₹ 675 (D) ₹ 650

10. A money becomes compounded at 4,950 in 19 years and 5,049 in 20 years at compound interest. Find the rate of compound interest.

(A) 2% (B) 1%

(C) 3% (D) 4%

11. If a simple interest of 2 years at 5% per annum is 100, then what will be the compound interest on that amount at the same number of years and rate? **(NCERT)**

(A) ₹ 102.50 (B) ₹ 25,010

(C) ₹ 20,510 (D) ₹ 10,025

12. A sum of money compounded annually at a rate of compound interest becomes 4,840 in 2 years and 5,324 in 3 years. The annual interest rate is:

(A) 10% (B) 9%

(C) 11% (D) 8%

13. An amount doubles in 4 years at a rate of compound interest compounded annually. At how much time will it be eight times itself?

(A) 10 yr (B) 12 yr

(C) 14 yr (D) 16 yr

14. B borrowed 5,000 from A at a rate of 6% per annum simple interest and he lent it to C at a rate of 10% compound interest annually, accordingly, if B got that amount back from C after 2 years and Returned to A, then how much profit did B get in this transaction? **(NCERT)**

(A) ₹ 600 (B) ₹ 1,050

(C) ₹ 500 (D) ₹ 450

15. At what time will the amount of 10,000 become 13,310 at the rate of 20% compounded interest compounded semi-annually?

(A) $1\frac{1}{2}$ yr (B) 2 yr

(C) $2\frac{1}{2}$ yr (D) 3 yr

16. At the rate of 5% annual interest on a sum of money, 1261 compound interest is received in 3 years. The amount is : **(NCERT)**

(A) ₹ 9,000 (B) ₹ 8,400

(C) ₹ 7,500 (D) ₹ 8000

17. The simple interest of 2 years is 80 at the rate of 4% per annum of an amount. The same amount will be compound interest for the same period of time—

(A) ₹ 82.60 (B) ₹ 82.20

(C) ₹ 81.80 (D) ₹ 81.60

18. An amount of 13,360 was borrowed at the rate of $8\frac{3}{4}$ percent compound interest annually and its repayment was made by two equal annual installments over two years. What was the amount of each installment?

(A) ₹ 5,769 (B) ₹ 7,569

(C) ₹ 7,009 (D) ₹ 7,500

19. A person takes a loan for 4 years at a rate of 10% annual compound interest of 10,000. How much interest will he have to pay?

(A) ₹ 4,371 (B) ₹ 4,581

(C) ₹ 14,641 (D) ₹ 4,641

20. 2000 were invested at the rate of 10% interest. Compound interest has been calculated every six months. What is the total money to be paid at the end of one year? **(NCERT)**

(A) ₹ 2,200 (B) ₹ 2,220

(C) ₹ 2,210 (D) ₹ 2,205

SOLUTIONS

1. (B) $C.I. = P\left[\left(1+\frac{r}{100}\right)^n-1\right]$

$$331 = P\left[\left(1+\frac{10}{100}\right)^3-1\right]$$

$$331 = P\left[\left(\frac{11}{10}\right)^3-1\right]$$

$$P = \frac{331\times1000}{11\times11\times11-1000}$$

$$= ₹\ 1{,}000$$

2. (C) Here, n = 2 years 4 months

$$= 2+\frac{4}{12} = 2\frac{1}{3}\ \text{yr}$$

$$C.I. = P\left[\left(1+\frac{r}{100}\right)^n\left(1+\frac{n_1 r}{100n_2}\right)-1\right]$$

$$= 8000\left[\left(1+\frac{15}{100}\right)^2\left(1+\frac{15}{300}\right)-1\right]$$

$$= 8000\left[\frac{115}{100}\times\frac{115}{100}\times\frac{315}{300}-1\right]$$

$$= 8000\left[\frac{23}{20}\times\frac{23}{20}\times\frac{63}{60}-1\right]$$

$$= 8000\left[\frac{33327-24000}{24000}\right]$$

$$= ₹\ 3{,}109$$

3. (C) C. I.

$$= 6000\left[\left(1+\frac{10}{100}\right)\left(1+\frac{5}{100}\right)-1\right]$$

$$= 6000\left[\frac{11}{10}\times\frac{21}{20}-1\right]$$

$$= 6000\left[\frac{231-200}{200}\right]$$

$$= 6000\times\frac{31}{200}$$

$$= 30 \times 31 = ₹\ 930$$

4. (B) Interest for 2nd year

$$= 48+\frac{48\times8\times1}{100}$$

$$= 48 + 3.84$$

$$= ₹\ 51.84$$

5. (C) Required difference

$$D = P\left[\left(1+\frac{r}{100}\right)^n - 1 - \frac{rn}{100}\right]$$

$$= 8000\left[\left(1+\frac{5}{100}\right)^3 - 1 - \frac{5\times3}{100}\right]$$

$$= 8000\left[\frac{105}{100}\times\frac{105}{100}\times\frac{105}{100} - 1 - \frac{15}{100}\right]$$

$$= 8000\left[\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20} - 1 - \frac{3}{20}\right]$$

$$= 8000\left[\frac{9261}{8000} - 1 - \frac{3}{20}\right]$$

$$= 8000\left[\frac{9261-8000-1200}{8000}\right] = ₹\ 61$$

6. (D) Required Principal $P = \frac{D}{r^2}\times10000$

$$= \frac{768\times10000}{8^2}$$

$$= ₹\ 1,20,000$$

7. (D) $12000\times\left(1+\frac{r}{100}\right)^5 = 24000$

$$\Rightarrow \left(1+\frac{r}{100}\right)^5 = 2$$

$$\Rightarrow \left[\left(1+\frac{r}{100}\right)^5\right]^4 = 2^4 = 16\ldots(1)$$

Amount after 20 years

$$= 12000\left(1+\frac{r}{100}\right)^{20}$$

$$= 12000 \times 16$$

[$\because$ From eq.(1),]

$$= ₹\ 1,92,000$$

8. (D) $C.I. = P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$

$$x = P\left[\left(1+\frac{x}{100}\right)^x - 1\right]$$

$$P = \frac{x}{\left(1+\frac{x}{100}\right)^x - 1}$$

9. (C) $2916 = P\left(1+\frac{8}{100}\right)^2$

$$\Rightarrow 2916 = P\left(\frac{108}{100}\right)^2$$

$$P = \frac{2916\times100\times100}{108\times108}$$

$$S.I. = \frac{2916\times10000\times9\times3}{108\times108\times100}$$

$$= ₹\ 675$$

10. (A) $r = \frac{A_2 - A_1}{A_1}\times100$

$$= \frac{5049-4950}{4950}\times100$$

$$= \frac{99}{4950}\times100 = 2\%$$

11. (A) $C.I. = S.I.\left(1+\frac{r}{200}\right)$

$$= 100\left(1+\frac{5}{200}\right)$$

$$= 100\times\frac{41}{40} = 102.50$$

12. (A) $A_1 = ₹\ 4840,\ A_2 = ₹\ 5324$

$$r\% = \left[\left(\frac{A_2}{A_1}\right)^{\frac{1}{n_2-n_1}} - 1\right]\times100\%$$

$$= \left[\left(\frac{5324}{4840}\right)^{\frac{1}{3-2}} - 1\right]\times100\%$$

$$= \left[\frac{5324-4840}{4840}\right]\times100$$

$$= \frac{484}{4840}\times100 = 10\%$$

13. (B) 4 years $\longrightarrow$ 2 times

8 years $\longrightarrow$ 4 times

12 years $\longrightarrow$ 8 times

So, required time = 12 years

14. (D) B's profit = C. I. – S. I.

$$= P\left[\left(1+\frac{r}{100}\right)^n - 1\right] - \frac{P\times R\times T}{100}$$

$$= 5000\left[\left(1+\frac{10}{100}\right)^2 - 1\right] - \frac{5000\times6\times2}{100}$$

$$= 5000\left[\frac{11}{10}\times\frac{11}{10} - 1\right] - 600$$

$$= 50\times21 - 600 = 1050 - 600$$

$$= ₹\ 450$$

15. (A) On compounded half yearly,

$$\text{Amount (A)} = P\left(1+\frac{r}{200}\right)^{2n}$$

$$= P\left(\frac{200+r}{200}\right)^{2n}$$

$$13310 = 10000\left(\frac{200+20}{200}\right)^{2n}$$

$$\therefore \frac{13310}{10000} = \left(\frac{220}{200}\right)^{2n}$$

$$\therefore \left(\frac{11}{10}\right)^3 = \left(\frac{11}{10}\right)^{2n}$$

$$\therefore 2n = 3$$

$$n = \frac{3}{2} = 1\frac{1}{2}\text{ yr}$$

16. (D) $r = 5\%,\quad n = 3$

C. I. = 1261

$$C.I. = P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$$

$\because$ $1261 = P\left[\left(1+\frac{5}{100}\right)^3 - 1\right]$

$\because$ $1261 = P\left[\frac{105}{100}\times\frac{105}{100}\times\frac{105}{100} - 1\right]$

$\because$ $1261 = P\left[\frac{157625}{1000000}\right]$

$\because$ $P = \frac{1261000000}{157625}$

$$= ₹\ 8000$$

17. (D) $\frac{P\times R\times T}{100} = I$

$$\frac{P\times4\times2}{100} = 80$$

$$P = ₹\ 1000$$

$$P = \frac{S^2}{4(C-S)}$$

$$1000 = \frac{(80)^2}{4C - 4\times80}$$

$$4C - 4 \times 80 = \frac{6400}{1000}$$

$$4C = 6.4 + 320$$

$$4C = 326.4$$

$$C = \frac{326.4}{4} = ₹\ 81.60$$

18. (B) Let, the each installlment be x.

$$\frac{x}{\left(1+\frac{35}{400}\right)^2} + \frac{x}{\left(1+\frac{35}{400}\right)} = 13360$$

$$\Rightarrow \quad \frac{x}{\left(\frac{435}{400}\right)^2} + \frac{x}{\left(\frac{435}{400}\right)} = 13360$$

$$\Rightarrow \quad \frac{80 \times 80x}{87 \times 87} + \frac{80x}{87} = 13360$$

$$\Rightarrow \quad \frac{80x}{87}\left(\frac{80}{87}+1\right) = 13360$$

$$\Rightarrow \quad x = \frac{13360 \times 87 \times 87}{80 \times 167} = ₹\ 7{,}569$$

19. (D) C. I.

$$= P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$$

$$= 10000\left[\left(1+\frac{10}{100}\right)^4 - 1\right]$$

$$= 10000\left[\frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} - 1\right]$$

$$= 10000\left[\frac{14641 - 10000}{10000}\right]$$

$$= ₹\ 4{,}641$$

20. (D) Here, rate $= \frac{10}{2} = 5\ \%$

Time $= 1 \times 2 = 2$ years

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$= 2000\left(1+\frac{5}{100}\right)^2$$

$$= 2000 \times \frac{21}{20} \times \frac{21}{20}$$

$$= ₹\ 2205$$

Chapter 13

Work and Time

1. Introduction

The questions related to time and work are mainly based on work efficiency and increasing and decreasing manpower for any work. There are the following important facts related to the time and work as–

(I) If A completes a job in n days, then A will do the job in a day $= \left(\frac{1}{n}\right)^{th}$ part

(II) If $\left(\frac{1}{n}\right)^{th}$ part is completed in a day, then the entire work will complete in n days.

(III) If A and B work together and A is twice efficient than B, then the working ratio between A and B is 2 : 1 and ratio between time taken will be 1 : 2.

(IV) Relation between Work, Time and Persons—More work requires more persons or more persons will produce more work.

$$\text{Work} \propto \text{People or } W \propto P \quad ...(1)$$

And, for completing a work in more days requires less persons or vice-versa.

$$\text{Time} \propto \frac{1}{\text{Persons}}$$

$$\Rightarrow T \propto \frac{1}{P} \text{ or } P \propto \frac{1}{T} \quad ...(2)$$

From eq. (1) and (2),

$$P \propto \frac{W}{T} \Rightarrow \frac{PT}{W} = \text{Constant}$$

(V) Relation among various entites :—

(i) Work and Persons—

More Work	More Persons	(Direct Proportion)
Less Work	Less Persons	(Direct Proportion)

(ii) Persons and Daily wages—

More Persons	More wages	(Direct Proportion)
Less Persons	Less wages	(Direct Proportion)

(iii) Work and Time—

More Work	More Time	(Direct Proportion)
Less Work	Less Time	(Direct Proportion)

(iv) Days and Persons—

More Days	Less Persons	(Indirect Proportion)
Less Days	More Persons	(Indirect Proportion)

(v) Days and Hours (Per day)—

More Hours	Less Days	(Indirect Proportion)
Less Hours	More Days	(Indirect Proportion)

(VI) The whole work is considers as 1 unit.

(VII) If there are fixed persons for a work, then the work done is directly proportional to the number of persons. If The work done is fixed, then the time will be indirectly proportional to the number of persons.

$$\frac{P_1T_1}{W_1} = \frac{P_2T_2}{W_2}$$

(VIII) A completes a job in x days and B completes in y days, then both will complete the work together in $= \left(\frac{xy}{x+y}\right)$ days

(IX) If A and B do a job in x days and B alone does the same job in y days, then A alone will complete the job in $= \left(\frac{xy}{x-y}\right)$ days

(X) A completes a job in x days, B completes in y days and C completes in z days, then all they will complete the work together in $= \left(\frac{xyz}{xy+yz+zx}\right)$ days

2. Pipes and Cisterns

(I) If a pipe fills a tank in x hours, then the part of the tank will be filled in an hour $= \left(\frac{1}{x}\right)$ part

(II) If a pipe empties the tank in y hours, then the tank will empty in 1 hour $= \left(\frac{1}{y}\right)$ part

(III) If two taps fill a tank in x hours and y hours respectively, then both taps will fill the tank in an hour $= \left(\frac{1}{x} + \frac{1}{y}\right)$

$$= \left(\frac{y+x}{xy}\right) \text{part}$$

(IV) If a tap fills a tank in x hours and another tap empties the tank in y hours, then the tank will empty in one hour (if both the taps are opened) $= \left(\frac{1}{x} - \frac{1}{y}\right) = \left(\frac{y-x}{xy}\right)$ part

(V) If $\left(\frac{1}{n}\right)^{th}$ part of a tank is filled in a day, then the entire tank will be filled in n days.

(VI) If a tap fills a tank in x hours, second tap fills in y hours and third tap fills the tank in z hours, then all the taps will fill the tank in $= \left(\frac{xyz}{xy + yz + zx}\right)$ hours

(VII) If the pipes A and B fill a tank in x hours and y hours respectively and the third pipe C empties the tank in z hours, then the tank will be filled in 1 hour $= \left(\frac{yz + xz - xy}{xyz}\right)$ part

Important Questions

1. If 15 men can do a work in 48 days, then how many men will be required to complete the wotk in 30 days?
(A) 21 (B) 20
(C) 24 (D) 22

2. A work is done by A and B in 4 days while A alone completes the job in 12 days, then in how many days will B alone complete?
(A) 5 (B) 6
(C) 7 (D) 8

3. A man completes a job in 15 days. His son completes the same job in 10 days. If both work together, then in how many days the work will be completed?
(A) 5 (B) 6
(C) 1/6 (D) 25

4. 15 labors make a water tank in 12 days, then in how mnay days will 20 labors make the tank?
(A) 16 (B) 10
(C) 8 (D) 9

5. 5 persons paint a house at 8 hours per day in 3 days. In how many days will 1 man paint the house if he works 4 hours per day?
(A) 30 (B) $\frac{15}{2}$
(C) $\frac{6}{5}$ (D) $\frac{3}{10}$

6. A man completes a work in 12 days. another man completes the same work in 10 days. If both work together, then in how many days will the work be completed?
(A) 5 (B) $5\frac{5}{11}$
(C) 1/11 (D) 11

7. A and B complete a work in 6 days. A alone can complete the work in 10 days. In how many days can B alone complete the work?
(A) 4 (B) 15
(C) $\frac{15}{4}$ (D) 16

8. A can finish a work in 12 days. B is 60% more efficient than A. How many days will B take to complete the same work?
(A) 6 (B) $7\frac{1}{2}$
(C) 8 (D) $8\frac{1}{2}$

9. A, B and C finish a work in 15 days, 20 days and 25 days. If all the three work for a day each in turn, then on which day the work will be completed?
(A) 19th day (B) 18th day
(C) 15th day (D) 10th day

10. There is 160 days ration for 540 persons in a fort. After 10 days, 60 more persons join them. For how many days will the ration sufficient at the same rate?
(A) 135 (B) 160
(C) 150 (D) 175

11. A and B took a contract for 4500 to complete a work. A alone can complete the work in 8 days can in 12 days. With the help of C, they finished the work in 4 days. Find the share of C in the contractual amount?
(A) ₹ 2,250 (B) ₹ 1,500
(C) ₹ 750 (D) ₹ 375

12. Rajni and Preeti can do a job in 15 days and 10 days respectively. They take contract to complete the work in 30,000. Find the Rajni' share in the contractual amount?
(A) ₹ 18,000 (B) ₹ 16,500
(C) ₹ 12,500 (D) ₹ 12,000

13. A works twice as fast as B and B works twice as fast as C. If A and B together complete a work in 4 days, then C alone can complete the work in—
(A) 6 (B) 8
(C) 24 (D) 12

14. A can complete a work in 18 days, B in 20 days and C in 30 days. B and C together start the work but after 2 days, they left the work. How many days will A take to complete the remaining work?
(A) 10 (B) 12
(C) 15 (D) 16

15. 7 men can complete a work in 12 days. How many additional men are required to complete two times the work in 8 days?
(A) 28 (B) 21
(C) 14 (D) 7

16. A man, a woman and a boy together complete a job in 3 days. If the man and the boy complete the work in 6 days and 18 days respectively, then in how many days will the woman complete the work?
(A) 9 (B) 21
(C) 24 (D) 27

17. If 72 persons make a 280 m long wall in 21 days. Similarly, How many persons will be required to make 100 m long wall in 18 days?
(A) 30 (B) 10
(C) 18 (D) 28

18. If 10 men finish 10 unit work in 10 days at 10 hours per day, then how many units of work will 5 men complete in 5 days at 5 hours per day?
(A) 1.25 units (B) 1.20
(C) 2.5 (D) None of these

19. 25 persons complete a work in 50 days. In how many days will 10 persons complete the work?
(A) 120 (B) 125
(C) 100 (D) 215

20. 100 workers finish a work in 20 days. How many workers are required to finish the work in 25 days?
(A) 100 (B) 60
(C) 80 (D) 40

21. $\frac{1}{4}$ th part of a tank is filled in 10 minutes. In how many minutes will the tank fill completely?

(A) 15 min (B) 40 min

(C) 30 min (D) 20 min

22. 12 men are able to complete a work in 18 days. All men work started together and after 6 days, 4 more men join them. In how many days will the remaining work be completed?

(A) 8 (B) 9

(C) 10 (D) 7

23. 12 cows together eats 756 Kg grass in 7 days. How much grass will 15 cows eat in 10 days?

(A) 1,500 Kg (B) 1,200 Kg

(C) 1,350 Kg (D) 1,400 Kg

24. If 18 bookbinders bind 900 books in 10 hours, then how many bookbinders are required to bind 660 books in 12 hours?

(A) 22 (B) 14

(C) 13 (D) 11

25. If 20 men build a 25 meter long wall in 5 days, then how many meters can 15 men build the same wall in 8 days?

(A) 22 m (B) 25 m

(C) 30 m (D) 20 m

26. If 30 workers complete a work in 10 days at 8 hours per day. How many workers will complete the work in 6 days at 10 hours per day?

(A) 30 (B) 40

(C) 20 (D) 25

27. If 400 persons complete one-fourth of work in 10 days at 9 hours per day, then how many additional persons are required so that they all can finish the remaining work in 20 days at 8 hours per day?

(A) 225 (B) 250

(C) 275 (D) 325

28. 500 Kg feed is sufficient for 25 horses for 30 days. For how many days will 600 Kg feed require for 20 horses?

(A) 45 (B) 25

(C) 30 (D) None of these

SOLUTIONS

1. (C) Days Persons

$48 \uparrow \quad 15 \downarrow$

$30 \quad\quad x$

$\therefore \quad \frac{x}{15} = \frac{48}{30} \Rightarrow x = \frac{48 \times 15}{30} = 24$

2. (B) Required time $= \frac{12 \times 4}{12 - 4}$

$= \frac{48}{8} = 6$ days

3. (B) $\because$ 1 day's work of a man $= \frac{1}{15}$

and 1 day's work of his son $= \frac{1}{10}$

$\therefore$ 1 day work of both $= \frac{1}{15} + \frac{1}{10}$

$= \frac{25}{150}$

$\therefore$ Both will complete the work in

$= \frac{150}{25} = 6$ days

4. (D) Workers Days

$15 \uparrow \quad 12 \downarrow$

$20 \quad\quad x$

$\Rightarrow \frac{x}{12} = \frac{15}{20}$

$\Rightarrow x = \frac{12 \times 15}{20} = 9$ days

5. (A) Persons Hours Days

(Per day)

$5 \uparrow \quad 8 \uparrow \quad 3 \downarrow$

$1 \quad\quad 4 \quad\quad x$

$\therefore \quad \frac{x}{3} = \frac{5}{1} \times \frac{8}{4}$

$\Rightarrow \quad x = \frac{5 \times 8 \times 3}{4} = 30$ days

6. (B) Required days $= \frac{xy}{x + y}$

$= \frac{10 \times 12}{10 + 12} = \frac{120}{22} = 5\frac{5}{11}$

7. (B) Required days $= \frac{d \times d_1}{d_1 - d}$

$= \frac{6 \times 10}{10 - 6}$

$= \frac{60}{4} = 15$

8. (B) A's work $= \frac{1}{12}$

B's work $= 160\%$ of $\frac{1}{12}$

$= \frac{1}{12} \times \frac{160}{100} = \frac{2}{15}$

Time taken by B to finish the work

$= \frac{15}{2} = 7\frac{1}{2}$

9. (A) Work done of a cycle of 3 days

$= \frac{1}{15} + \frac{1}{20} + \frac{1}{25}$

$= \frac{20 + 15 + 12}{300}$

$= \frac{47}{300}$

18 days' work $= 6 \times \frac{47}{300} = \frac{282}{300}$

So, the work will complete on $= 18 + 1$

$= 19$th day

10. (A) Here, $n = 540$

$d = 160$

$n_1 = 60$

$d_2 = 10$

Required no. of days $= \frac{n(d - d_1)}{n + n_1}$

$= \frac{540(160 - 10)}{540 + 60}$

$= \frac{540 \times 150}{600}$

$= \frac{540}{4} = 135$ days

11. (C) C's work $= \frac{1}{4} - \frac{1}{8} - \frac{1}{12}$

$= \frac{6 - 3 - 2}{24} = \frac{1}{24}$

A : B : C $= \frac{1}{8} : \frac{1}{12} : \frac{1}{24}$

$= 3 : 2 : 1$

C's part $= \frac{1}{3 + 2 + 1} \times 4500$

$= ₹\ 750$

12. (D) Rajni work efficiency : Preeti work efficiency

$= \frac{1}{15} : \frac{1}{10} = 2 : 3$

So, Rajni's share $= \frac{2}{2 + 3} \times 30,000$

$= 2 \times 6,000$

$= ₹\ 12,000$

13. (C) Let, A completes the work in x days.

Time taken by B = $2x$ days

Time taken by C = 2 (2x)

= $4x$ days

As per question, $\frac{1}{x}+\frac{1}{2x}=\frac{1}{4}$

$4+2=x$

$x=6$ days

Time taken by C= 4×6

= 24 days

14. (C) 2 days' work of (B + C)

$$=2\left(\frac{1}{20}+\frac{1}{30}\right)$$

$$=2\left(\frac{3+2}{60}\right)$$

$$=\frac{1}{6}\text{ part}$$

Remaining work = $1-\frac{1}{6}=\frac{5}{6}$

Time taken by A to complete the remaining work

$$=\frac{5}{6}\times18=15\ days$$

15. (C)

Men	Days	Work
7	12	1
(Let, x)	8	2

$1\times8\times x=2\times12\times7$

$$x=\frac{2\times12\times7}{8}=21$$

Additional men

= 21 – 7 = 14

16. (A) Men's efficiency = $\frac{1}{6}$

Boy's efficiency = $\frac{1}{18}$

Let, the woman alone completes the work in x days.

According to question,

$$\frac{1}{6}+\frac{1}{18}+\frac{1}{x}=\frac{1}{3}$$

$$\frac{1}{x}=\frac{1}{3}-\frac{1}{6}-\frac{1}{18}$$

$$=\frac{6-3-1}{18}$$

$$\frac{1}{x}=\frac{1}{9}$$

$x=9$ days

17. (A) Here, M_1=72, W_1 = 280 m

D_1 = 21, W_2 = 100 m

M_2 = ?

D_2 = 18

$$\frac{M_1D_1}{W_1}=\frac{M_2D_2}{W_2}$$

$$\frac{72\times21}{280}=\frac{M_2\times18}{100}$$

$$M_2=\frac{72\times21\times100}{280\times18}=30\text{persons}$$

18. (A) Required Work = $\frac{y^3}{x^2}=\frac{(5)^3}{10^2}$

$$=\frac{125}{100}=1.25\text{ units}$$

19. (B) Required day = $\frac{25\times50}{10}=125$

20. (C) Number of labors = $\frac{100\times20}{25}$

= 80

21. (C) $\frac{1}{4}$ part ⟶ 10 min

1 part ⟶ 10×4

= 40 min

$\left(1-\frac{1}{4}\right)\frac{3}{4}$ part ⟶ $40\times\frac{3}{4}$

= 30 min

22. (B) 1-day work of 12 men

$$=\frac{1}{18}$$

6-days' work of 12 men

$$=\frac{6}{18}=\frac{1}{3}$$

Workers	Work	Days
12	1	18
16	$\left(1-\frac{1}{3}\right)=\frac{2}{3}$	x

$12:16$

$\frac{2}{3}:1$ $= x:18$

$16\times1\times x=12\times\frac{2}{3}\times18$

$$x=\frac{8\times18}{16}=9\text{ days}$$

23. (C)

Cows	Days	Grass
12	7	756
15	10	x

12 सा 5
7 सा 0 $= 756 : x$

$12\times7\times x=15\times10\times756$

$$x=\frac{15\times10\times756}{12\times7}$$

= 1,350 Kg

24. (D)

Time 12 : 10 = 6 : 5

Books 900 : 660 = 15 : 11 $:: 18 : x$

$6\times15\times x=5\times11\times18$

$$x=\frac{5\times11\times18}{6\times15}=11$$

25. (C) $L_2=\frac{N_2D_2L_1}{N_1D_1}$

$$=\frac{15\times8\times25}{20\times5}$$

= 30 m

26. (B) $N_2=\frac{T_1D_1N_1}{D_2T_2}$

$$=\frac{8\times10\times30}{6\times10}$$

= 8 × 5 = 40 workers

27. (C) Time = 8 : 9

Work = $\frac{1}{4}$ रु $\frac{3}{4}$ = 1 : 3

Days = 20 : 10 = 2 : 1

8 रु
1 रु $::400:x$
2 रु

$8\times1\times2\times x=9\times3\times1\times400$

$$x=\frac{9\times3\times400}{16}$$

= 675

Additional manpower :

= 675 – 400 = 275

28. (A) $D_2=\frac{N_1D_1W_2}{N_2W_1}$

$$=\frac{25\times30\times600}{20\times500}$$

= 45 days

Chapter 14 Speed, Time and Distance

1. Important Terminology

I. Speed—If can be defined as rate at which distance is covered during the motion. It's measured in terms of distance per unit time. It is denoted by capital letter 'S' or small letter (s).

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \text{ or } S = \frac{D}{T}$$

II. Time—The duration in which an object covers a unit of distance is called time. It is denoted by T or t.

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} \text{ or } T = \frac{D}{S}$$

III. Distance—Displacement of the object during the motion. It is denoted by D or d. generally.

Distance = Time × Speed or $D = T \times S$

IV. Average Speed—It is the total distance travelled by the object divided by the elapsed time to cover that distance.

$$\text{Average Speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

Key points : When the distance is constant. Then

$$\text{Average Speed} = \frac{2xy}{x+y}$$

Where, x and y be the speeds at which the same distance has been covered.

Example 1 : Rajat drove a car at an average speed of 30 km/h and then returned at an average speed of 60 km/h. If he reached his destination without any stoppage in the entire journey, what was his average speed (in km/h) for the entire journey ?

(A) 35 (B) 40
(C) 45 (D) 50

Solution : $\text{Average speed} = \frac{2xy}{x+y}$

$$= \frac{2 \times 30 \times 60}{30+60}$$

$$= \frac{3600}{90} = 40 \text{ km/h}$$

Example 2 : When time is constant. Then

$$\text{Average Speed} = \frac{x+y}{2}$$

V. Conversion of Units

(i) $1 \text{ km/h} = \frac{5}{18} \text{ m/sec}$

(ii) $1 \text{ m/sec} = \frac{18}{5} \text{ km/h}$

Example : If Swati walks at the rate of km/h, what is its speed in meters per second ?

Solution : $18 \text{ km/h} = 18 \times \frac{5}{18} \text{ m/sec}$

$= 5 \text{ m/sec}$

VI. Special Rules

(i) More distance, more time at same speed.

(ii) More speed, less time for the same distance.

(iii) More speed, more distance in the same time.

(iv) If the speed of an object is changed in the ratio $a : b$, then ratio of time-taken to change in the ratio $b : a$

VII. Train Concepts

- The time-taken by a train in passing a railway bridge or a platform or a tunnel or a train at rest

$$= \frac{m+n}{\text{Speed}}$$

where m = length of the train

n = length of the bridge/platform/tunnel/standing train

- Length of the train

$$= \frac{\text{Difference of speed of two men} \times \text{Product of times}}{\text{Difference of the times}}$$

- Time-taken by faster train to pass the slower train in the same direction $= \frac{m_1 + m_2}{x_1 - x_2}$ $[x_1 > x_2]$

where m_1 = length of the first train
m_2 = length of the second train
x_1 = speed of first train
x_2 = speed of second train

- Time-taken by the trains in passing each other in opposite direction $= \frac{m_1 + m_2}{x_1 - x_2}$

- Time-taken by the train to cross the man = $\frac{m}{x-y}$

  where both are in the same direction and

  m = Length of the train

  x = Speed of the train and

  y = Speed of the man

- Time-taken by the train to cross the man running in the opposite direction = $\frac{m}{x-y}$

- Length of train = $\frac{\text{Length of platform}}{\text{Difference in time}}$ × Time-taken to cross a stationary pole or man

VIII. Boat & stream concepts

(i) Upstream

If a person/swimmer, boat or ship moves against the stream *i.e.* in the direction opposite to that of the stream, it is called upstream.

(ii) Downstream

If a person/swimmer, boat or ship moves with the stream *i.e.* along the direction of the stream it is called downstream.

Suppose

x = Speed of a boat or a man in still water.

y = Speed of the stream or the current or the river.

IX. Boats and Stream Formulae

- Speed of boat or man with the stream (downstream) = Down rate = $x + y$
- Speed of boat or man against the stream (upstream) = Up rate = $x - y$
- Speed of boat = $x = \frac{1}{2}$(Down Rate + Up Rate)
- Speed of stream = $y = \frac{1}{2}$ (Down Rate – Up Rate)
- If a body covers a distance at the rate of x and another equal distance at the rate of y, then

  Average speed = $\frac{2xy}{x-y}$
- If a man changes his speed in the ratio of $u : v$, then the ratio of the time-taken to cover the same distance is $v : u$
- A man can rows certain downstream in t_1 hours and returns the same distance upstream in t_2 hours. Then

  speed of stream is S $\left(\frac{t_2 + t_1}{t_2 - t_1}\right)$ km/hr.
- A man can rows a boat in still water at Z km/hr. In a stream flowing at S km/hr if takes him t hours to row to a point and come back, then

  distance between the two points = $t\left(\frac{z^2 + s^2}{2z}\right)$
- A person can rows a boat u_1 km upstream and d_1 downstream in t_1 hours. Also he can row u_2 km upstream and d_2 downstream in t_2 hours. Then his

  upstream speed = $\frac{u_1 d_2 - u_2 d_1}{t_1 d_2 - t_2 d_1}$

  and downstream speed = $\frac{u_1 d_2 - u_2 d_1}{t_1 u_1 - t_2 u_2}$

X. Precautions

- In a question, unit of all given quantities (speed or distance) will be in the same manner *i.e.* either in km/hr (speed) and km (distance) or in m/sec. (speed) or m (distance).
- Sometimes we have to change unit of given quantities according to need of question. It should be remembered.

Example 1 : If a coach moves at the rate of 2 km per hour, what is its speed in metres per second ?

(A) 259.2 m/s (B) 25.92 m/s

(C) 20 m/s (D) 25 m/s

Solution (C) : 72 km per hour = $72 \times \frac{5}{18} = 20$ m/sec.

Example 2 : A train passes a signal post in 6 seconds moving at the rate of 90 km/hr. The length of the train is—

(A) 120 metres (B) 150 metres

(C) 200 metres (D) 100 metres

Solution (B) : Speed of train = 90 km/hour

$= \left(90 \times \frac{5}{18}\right)$ m / s = 25 m / sec

Length of train = Time × Speed = 6 × 25 = 150 metres.

Example 3 : A train 60 metres long takes 30 seconds in crossing a tunnel 440 metres long. The speed of the train is—

(A) 60 km/hr (B) 46 km/hr

(C) 97 km/hr (D) 65 km/hr

Solution (A) : Speed = $\frac{m+n}{\text{time}} = \frac{(440+60)\text{ metres}}{30\text{ seconds}}$

where m = length of the train, n = length of the tunnel

$\therefore$ Speed = $\frac{500}{30}$ m / s = $\left(\frac{500}{30} \times \frac{18}{5}\right)$ km/hr

= 60 km/hr.

Example 4 : Two trains 130 metres and 80 metres long are running in opposite direction, are at the rate of 75 km/hr and another are at the rate of 33 km/hr. From the moment they meet, they will cross each other in—

(A) 10 sec (B) 9 sec

(C) 8 sec (D) 7 sec

Solution (D) : Relative speed of trains = (75 + 33) km/hr

$$= 108 \times \frac{5}{18} = 30 \text{ m/s.}$$

$$\text{Time} = \frac{m_1 + m_2}{x_1 \times x_2} \text{ because of opposite directions}$$

where m_1 and m_2 are their lengths

and $x_1 + x_2$ = Relative speed

$$\therefore \quad \text{Time} = \frac{130 + 80}{30} = 7 \text{ sec.}$$

Important Questions

1. A 150 m long train crosses a mile-stone in 15 s and a train of same length coming from opposite direction in 12 s, The speed of other train is :

(A) 36 km/h (B) 54 km/h

(C) 50 Km/h (D) 45 km/h

2. A worker may claim ₹ 1.5 for each km which he travels by taxi and 50 paise for each km he drives his own car. If in one week he claimed ₹ 50 for travelling 80 km, how many kms did he travel by taxi ?

(A) 20 km (B) 14 km

(C) 12 km (D) 10 km

3. A man rows upstream a distance of 9 km or downstream a distance of 18 km taking 3 hours each time. The speed of the boat in still water is :

(A) $7\frac{1}{2}$ km/h (B) $6\frac{1}{2}$ km/h

(C) $5\frac{1}{2}$ km/h (D) $4\frac{1}{2}$ km/h

4. A man can row 30 km upstream and 44 km downstream in 10 hours. He can also row 40 km upstream and 55 km downstream in 13 hours. Find the rate of current.

(A) 3 km/h (B) 2 km/h

(C) 4 km/h (D) 5 km/h

5. Two trains travel in opposite directions at 36 km/h and 45 km/h respectively. A man sitting in slower train passes the faster train in 8 seconds. The length of the faster train is :

(A) 80 m (B) 120 m

(C) 160 m (D) 180 m

6. A train 110 metres long is running with a speed of 60 km/h. In what time will it pass a-man who is running at 6 km/h in the direction opposite to that of train ?

(A) 5s (B) 6s

(C) 7s (D) 10s

7. A boat travels up stream from B to A and downstream from A to B in 3 h. If the speed of the boat in still water is 9 km/h and the speed of the current is 3 km/h, the distance between A and B is :

(A) 9 km (B) 10 km

(C) 11 km (D) 12 km

8. A steamer moves with a speed of 4.5 km/h in still water to a certain upstream point and comes back to the starting point in a river which flows at 1.5 km/h. The average speed of steamer for the total journey is :

(A) 12 km/h (B) 9 km/h

(C) 6 km/h (D) 4 km/h

9. A train covers a distance of 10 km in 12min. If its speed is decreased by 5 km/h, what is the time taken by the train to cover the same distance ?

(A) 14 min 33 s (B) 13 min 33 s

(C) 13 min 20 s (D) 15 rnin 20 s

10. A train is moving at a speed of 132 km/h. If the length of the train is 110 m. how long will it take to cross a railway platform 165 m long ?

(A) 6.0 s (B) 7.5 s

(C) 7.0 s (D) 8.5 s

11. Speed of a boat in still water is 9 km/h and the speed of the stream is 1.5 km/h. A man rows to a place at a distance of 105 km and comes back to the starting point. What will be the total time taken by him ?

(A) 16 h (B) 18 h

(C) 24 h (D) 28 h

12. A man can row $9\frac{1}{3}$ km/h in still water and finds that ittakes him thrice as much time to row up than as to row down the same distance in the river. The speed of the current is :

(A) $3\frac{1}{3}$ km/h (B) $3\frac{1}{9}$ km/h

(C) $4\frac{2}{3}$ km/h (D) $4\frac{1}{3}$ km/h

13. The speed of a boat in still water is 10 km/h. If it can travel 26 km downstream and 14 km upstream in the same time, the speed of the stream is :

(A) 2 km/h (B) 2.5 km/h

(C) 3 km/h (D) 4 km/h

14. A man travelled from a point A to B at the rate of 25 km/h and walked back at the rate of 4 km/h. If the whole journey took 5 h 48 min, the distance between A and B is :

(A) 30 km (B) 24 km

(C) 20 km (D) 51.6 km

15. A train travelling at a uniform speed clears a platform 200 m long in 10 s and passes a telegraph post in 5 s. The speed of the train is :

(A) 36 km/h (B) 39 km/h

(C) 144 km/h (D) 78 km/h

16. Two trains, of same length , are running in parallel tracks in opposite directions with speed 65 km/h and 85 km/h respectively. They cross each other in 6s. The length of each train is :

(A) 100 m (B) 115 m

(C) 125 m (D) 150 m

17. A train is moving at a speed of 80 km/h and covers a certain distance in 4.5 h. The speed of the train to cover the same distance in 4 h is :

(A) 70 km/h (B) 85 km/h

(C) 90 km/h (D) 100 km/h

18. A car covers a certain distance going at a speed of 60 km/h and returns to the starting point at a speed of 40 km/h. The average speed for the whole journey is :

(A) 48 km/h (B) 50 km/h

(C) 45 km/h (D) 40 km/h

19. The speed of a bus is 72 km/h. The distance covered by the bus in 5 s is :

(A) 50 m (B) 74.5 m

(C) 100 m (D) 60 m

20. Two men start together to walk a certain distance, one at 4 km/h and another at 3 km/h. The former arrives half an hour before the later. Find the distance.

(A) 6 km (B) 9 km

(C) 8 km (D) 7 km

21. A car covers a certain distance going at a speed of 60 km/h and returns to the starting point at a speed of 40 km/h. The average speed for the whole journey is :

(A) 48 km/h (B) 50 km/h

(C) 45 km/h (D) 40 km/h

22. The speed of a stream is 3 km/h and the speed of a man in still water is 5 km/h. The time taken by the man to swim 26 km downstream is :

(A) $5\frac{1}{5}$ h (B) $8\frac{2}{3}$ h

(C) $3\frac{1}{4}$ h (D) 13 h

23. The speed of a train going from Nagpur to Allahabad is 100 km/h. While its speed is 150 km/h, when coming back from Allahabad to Nagpur. Then, the average speed during the whole journey is :

(A) 135 km/h (B) 120 km/h

(C) 125 km/h (D) 140 km/h

24. A person can row $7\frac{1}{2}$ km/h in still water and he finds that it takes him twice as long to row up as to row down the river. The speed of the stream is :

(A) 2 km/h (B) 3 km/h

(C) $2\frac{1}{2}$ km/h (D) $3\frac{1}{2}$ km/h

25. A ship is moving at a speed of 30 km/h. To know the depth of the ocean beneath it. It sends a radiowave which travels at a speed 200 m/s. The ship receives the signal after it has moved 500 m. The depth of the ocean is :

(A) $\frac{\sqrt{143}}{2}$ km (B) 12 km

(C) $\sqrt{6}$ km (D) 8 km

26. With average speed of 40 km/h, a train reaches its destination in time. If it goes with an average speed of 35 km/h, it is late by 15 min. The total journey is :

(A) 30 km (B) 40 km

(C) 70 km (D) 80 km

27. A train running at 36 km/h crosses a pole in 25 s. Length of the train is :

(A) 250 m (B) 300 m

(C) 225 m (D) 275 m

28. A bus can complete a journey in 6 h, if it travels a 60 km/h. At what speed (km/h) the bus must travel in order to complete the journey in 9 h ?

(A) 60 (B) 40

(C) 30 (D) 35

29. A man can row 6 km/h in still water. If the speed of the current is 2 km/h, it takes 3 h more in upstream than in the downstream for the same distance. The distance is :

(A) 30 km (B) 24 km

(C) 20 km (D) 32 km

30. A student goes to school at the rate of $2\frac{1}{2}$ km / h and reaches 6 min. late. If he travels at the speed of 3 km/h, he is 10 min early. The distance (in km) between the school and his house is :

(A) 5 (B) 4

(C) 3 (D) 1

31. Walking $\frac{6}{7}$ th of his usual speed, a man is 12 min too late. The usual time taken by him to cover that distance is :

(A) 1 h (B) 1 h 12 min

(C) 1 h 15 min (D) 1 h 20 min

SOLUTIONS

1. (B) Speed of first train $= \frac{150}{15}$

$= 10$ m/s.

Second case, $\frac{150+150}{x+10} = 12$

Where, x = speed of other train

$\Rightarrow \quad 300 = 12x + 120$

$\Rightarrow \quad x = \frac{180}{12} = 15$ m/s

$\therefore$ Required speed $= 15 \times \frac{18}{5}$

$= 54$ km/h

2. (D) Let, x and $(80 - x)$ km are the distance covered by travelling by taxi and car respectively.

According to the question,

$$1.5x + 0.50(80 - x) = 50$$

$$1.5x + 0.50 \times 80 - 0.50x = 50$$

$$x + 40 = 50$$

$$x = 10 \text{ km}$$

3. (D) Let, the speed of boat in still water be x km and speed of current be y km. According to the condition,

$$x + y = \frac{9}{3} = 3$$

$$x + y = 3 \quad \text{...(i)}$$

and $\quad x - y = 6 \quad \text{...(ii)}$

By solving Eqs. (i) and (ii), we get

$2x = 9, x = 4.5$ km/h

So,the speed of boat in still water $= 4.5$ km/h

4. (A) Let, x be the speed of boat in still water and y be the speed of current.

According to question,

$$\frac{30}{x-y} + \frac{44}{x+y} = 10 \quad \text{...(i)}] \times 4$$

$$\frac{40}{x-y} + \frac{55}{x+y} = 13 \quad \text{...(ii)}] \times 3$$

On subtracting

$$\frac{176-165}{x+y} = 40 - 39$$

$$\Rightarrow \quad x + y = 11 \quad \text{...(iii)}$$

Put the value of $x + y$ in Eq. (i), we have

$$x - y = 5 \quad \text{...(iv)}$$

From Eq. (iii) – Eq. (iv), we have

$$2y = 6$$

or $\quad y = 3$

$\therefore$ Speed of current $= 3$ km/h

5. (D) The relative speed of the train

$$= (36 + 45) \times \frac{5}{18}$$

$= 22.5$ m/s time

So, the length of fast train

= speed × time

$= 22.5 \times 8$

$= 180$ m

6. (B) According to the question,

Length of the train = 110m

$$\text{Speed} = \frac{60 \times 5}{18} \text{ m/s} = 16\frac{2}{3} \text{ m/s}$$

$$\text{Speed of man} = 6 \times \frac{5}{18} \text{ m/s}$$

$$= 1\frac{2}{3} \text{ m/s}$$

$$\text{Relative Speed} = 16\frac{2}{3} + 1\frac{2}{3}$$

$$= 18\frac{1}{3}$$

$$\text{Required time} = \frac{100}{18\frac{1}{3}}$$

$$= \frac{110 \times 3}{55} = 6 \text{ s}$$

7. (D) Speed of boat= $x = 9$ km/h

Speed of stream of river

$= y = 3$ km/h

Speed of boat in downstream

$= 9 + 3 = 12$ km/h

Speed of boat in upstream

$= 9 - 3 = 6$ km/h

Time taken by boat in upstream and downstream

$= t_1 + t_2 = 3$h

Let the distance covered by boat = D

$$\because \quad t_1 + t_2 = D\left(\frac{1}{12} + \frac{1}{6}\right)$$

$$\Rightarrow \quad 3 = D\left(\frac{1+2}{12}\right)$$

$$\Rightarrow \quad 3 = \frac{D \times 3}{12}$$

$$\therefore \quad D = 12 \text{ km}$$

8. (D) Speed of steamer = 4.5 km/h

Speed of stream = 1.5 km/h

Speed of steamer in upstream

$= 4.5 - 1.5$

$= 3 \text{ km/h} = V_1$

Speed of steamer in downstream

$= 4.5 + 1.5$

$= 6 \text{ km/h} = V_2$

$$\therefore \quad \text{Average speed} = \frac{2V_1V_2}{V_1+V_2}$$

$$= \frac{2\times3\times6}{3+6}$$

$$= \frac{36}{9}$$

$= 4 \text{ km/h}$

9. (C) $\text{Speed of the train} = \frac{10}{12} \times 60 \text{ km/h}$

$= 50 \text{ km/h}$

New speed of the train

$= (50 - 5) \text{ km/h}$

$= 45 \text{ km/h}$

$\because \quad \text{Distance} = 10 \text{ km}$

$$\therefore \quad \text{Required time} = \frac{10}{45} \times 60 \text{ min}$$

$$= \frac{40}{3} \text{ min}$$

$$= 13\frac{1}{3} \text{ min}$$

$= 13 \text{ min } 20 \text{ s}$

10. (B) Speed of the train = 132 km/h

$$= 132 \times \frac{5}{18} \text{ m/s}$$

$$= \frac{110}{3} \text{ m/s}$$

$$\therefore \quad \text{Required time} = \frac{110+165}{110/3}$$

$$= \frac{275}{110/3}$$

$$= \frac{275\times3}{110} = 7.5 \text{ s}$$

11. (C) Speed upstream $= (9 - 1.5)$ km/h

$= 7.5$ km/h

Speed downstream $= (9 + 1.5)$ km/h

$= 10.5$ km/h

$$\therefore \quad \text{Required time} = \frac{105}{7.5} + \frac{105}{10.5}$$

$= 14 + 10 = 24$ h

12. (C) Suppose speed of the current

$= x$ km/h

$$\text{Then,} \quad \left(\frac{28}{3} + x\right) = 3\times\left(\frac{28}{3} - x\right)$$

$$\Rightarrow \quad \left(\frac{28+3x}{3}\right) = 3\times\left(\frac{28-3x}{3}\right)$$

$$\Rightarrow \quad \frac{28+3x}{3} = (28 - 3x)$$

$$\Rightarrow \quad 28 + 3x = 84 - 9x$$

$$\Rightarrow \quad 12x = 56$$

$$\Rightarrow \quad x = \frac{56}{12}$$

$$\Rightarrow \quad x = \frac{14}{3} = 4\frac{2}{3} \text{ km/h}$$

13. (C) Suppose speed of the current

$= x$ km/h

$$\text{Then,} \quad \frac{26}{10+x} = \frac{14}{10-x}$$

($\because$ Speed of the boat = 10 km/h)

$$\Rightarrow \quad 260 - 26x = 140 + 14x$$

$$\Rightarrow \quad 40x = 120$$

$$\Rightarrow \quad x = \frac{120}{40} = 3 \text{ km/h}$$

14. (C) Suppose distance between A and B

$= x$ km

$$\text{Then,} \quad \frac{x}{25} + \frac{x}{4} = 5\frac{48}{60}$$

$$\Rightarrow \quad \frac{4x+25x}{100} = \frac{29}{5}$$

$$\Rightarrow \quad \frac{29x}{100} = \frac{29}{5}$$

$$\Rightarrow \quad x = \frac{100}{5} = 20 \text{ km}$$

15. (C) Suppose length of train = x m

$$\text{Then,} \quad \frac{x+200}{10} = \frac{x}{5}$$

$$\Rightarrow \quad 2x = x + 200$$

$$\Rightarrow \quad x = 200 \text{ m}$$

$\therefore$ Speed of the train

$$= \frac{200}{5} \text{ m/s}$$

$= 40$ m/s

$$= 40 \times \frac{18}{5} \text{ km/h}$$

$= 144$ km/h

16. (C) Let, length of each train be x m.

$$\text{Then,} \left(65+85\right)\times\frac{5}{18} = \frac{x+x}{6}$$

$$\frac{150\times5\times6}{18} = 2x$$

$$\Rightarrow \quad 2x = 250$$

$$\Rightarrow \quad x = 125 \text{ m}$$

17. (C) Distance covered by the train in 4.5 m

$= 80 \times 45$ km $= 360$ km

Now, to cover this distance is 4 h, the speed of the train

$$= \frac{360}{4} = 90 \text{ km/h}$$

18. (A) Let the distance covered from one side $= x$ km

$$\text{The time taken with 60 km/h} = \frac{x}{60}$$

$$\text{The time taken with 40 km/h} = \frac{x}{40}$$

$$\text{Now, Average speed} = \frac{\text{Total distance}}{\text{Total time taken}}$$

$$= \frac{x+x}{\frac{x}{60}+\frac{x}{40}}$$

$$= \frac{2x}{\frac{40x+60x}{2400x}}$$

$$= \frac{2x\times2400}{100x}$$

$= 48$ km/h

19. (C) Speed of bus in m/s

$$= 72\times\frac{5}{18} = 20 \text{ m/s}$$

$\therefore$ Distance travelled in 5 s

$= 20 \times 5$ (speed $\times$ time)

$= 100$ m

20. (A) Let the distance $= x$ km

By given condition,

$$\frac{x}{3} - \frac{x}{4} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{4x-3x}{12} = \frac{1}{2}$$

$$\Rightarrow \quad x = \frac{12}{2}$$

$$\therefore \quad x = 6 \text{ km}$$

21. (A) Let the distance covered from one side $= x$ km

$$\text{The time taken with 60 km/h} = \frac{x}{60}$$

$$\text{The time taken with 40 km/h} = \frac{x}{40}$$

Now, Average speed

$$= \frac{\text{Total Distance}}{\text{Total Time taken}}$$

$$\frac{x+x}{\frac{x}{60}+\frac{x}{40}} = \frac{2x}{\frac{40x+60x}{2400x}}$$

$$= \frac{2x \times 2400}{100x}$$

$$= 48 \text{ km/h}.$$

22. (C) Speed of stream = 3 km/h

Speed of man in still water = 5 km/h

Now, Speed of man downstream

$$= (3 + 5) = 8 \text{ km/h}$$

∴ Time taken to travel 26 km

$$\text{downstream} = \frac{26}{8}$$

$$= \frac{13}{4} = 3\frac{1}{4} \text{ h}$$

23. (B) Let the distance from Nagpur to Allahabad = x km

Now, time taken to travel from Nagpur to Allahabad

$$= \frac{x}{100} \text{ h}$$

Also, time taken to travel from Allahabad to Nagpur

$$= \frac{x}{150} \text{ h}$$

∴ Average speed

$$= \frac{\text{Total Distance travelled}}{\text{Total Time taken}}$$

$$= \frac{x+x}{\frac{x}{100}+\frac{x}{150}}$$

$$= \frac{2x}{\frac{3x+2x}{300}} = \frac{2x \times 300}{5x}$$

$$= 120 \text{ km/h}$$

24. (C) Let speed of stream be x km/h and covered distance be y km.

Then, according to the question,

$$\frac{y}{(7.5-x)} = 2 \times \left(\frac{y}{(7.5+x)}\right)$$

$$\Rightarrow 15 - 2x = 7.5 + x$$

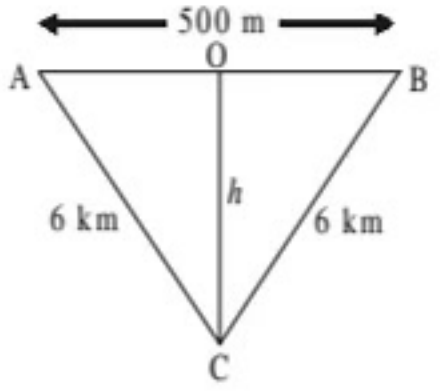
$$3x = 7.5$$

$$x = \frac{7.5}{3} = \frac{5}{2} = 2\frac{1}{2} \text{ km/h}$$

25. (A) Let, the depth of ocean = h km

∵ Speed of ship = 30 km/h

$$= \frac{30 \times 5}{18} = \frac{25}{3} \text{ m/s}$$



∴ Time taken to cover 500 m at the speed of $\frac{25}{3}$ m/s

$$= \frac{500}{\frac{25}{3}} = 60 \text{ s}$$

∴ Distance covered by radiowave in 60 s

$$= 200 \times 60$$

$$= 12000 \text{ m or } 12 \text{ km}$$

In Δ OCB, $OC^2 = CB^2 - OB^2$

$$\Rightarrow OC = \sqrt{(6)^2 - \left(\frac{1}{4}\right)^2}$$

$$\therefore OC = \frac{5\sqrt{23}}{4}$$

$$= 5.9 \text{ (approx)}$$

which is equal to $\frac{\sqrt{143}}{2}$ km

26. (C) Let, the total journey = x km.

According to question,

$$\frac{x}{35} - \frac{x}{40} = \frac{15}{60}$$

$$\Rightarrow \frac{x}{280} = \frac{1}{4}$$

$$\therefore x = \frac{280}{4}$$

$$= 70 \text{ km}$$

27. (A) Speed of train in metre/second

$$= 36 \times \frac{5}{18} = 10 \text{ m/s}$$

Time taken to cross a pole = 25 s

Now, length of train

= Speed of train × time taken to cross the pole

$$= 10 \times 25 = 250 \text{ m}$$

28. (B) Speed of bus = 60 km/h

Time taken = 6h

∴ Distance covered= Speed of Bus × Time taken

$$= 60 \times 6 = 360 \text{ km}$$

Now, Speed of bus to cover the same distance in 9 h

$$\text{Speed of bus} = \frac{\text{Distance covered}}{\text{Time taken}}$$

$$= \frac{360}{9} = 40 \text{ km/h}$$

29. (B) A man speed in downstream

$$= (6 + 2) \text{ km/h} = 8 \text{ km/h}$$

A man speed in upstream

$$= (6 - 2) \text{ km/h} = 4 \text{ km/h}$$

Let required distance = x km.

According to the given condition,

$$\frac{x}{4} - \frac{x}{8} = 3$$

$$\Rightarrow \frac{6x - 3x}{24} = 3$$

$$\Rightarrow \frac{3x}{24} = 3$$

$$\Rightarrow x = 24 \text{ km}$$

30. (B) Let the distance between school and his house = x km

According to the given condition,

$$\frac{x}{\frac{5}{2}} - \frac{x}{3} = \frac{6+10}{60}$$

$$\Rightarrow \frac{2x}{5} - \frac{x}{3} = \frac{16}{60}$$

$$\Rightarrow \frac{6x - 5x}{15} = \frac{4}{15}$$

$$\Rightarrow \frac{x}{15} = \frac{4}{15}$$

$$\Rightarrow x = 4 \text{ km}$$

31. (B) Since, speed and time are inversely proportional to each other.

According to the given condition,

$$\text{usual time} - \frac{6}{7} \times \text{usual time} = 12$$

$$\therefore \frac{1}{7} \times \text{usual time} = 12$$

$$\Rightarrow \text{usual time} = 72 \text{ min}$$

$$= 1 \text{ h } 12 \text{ min}$$

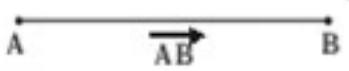
Chapter 15

Geometry

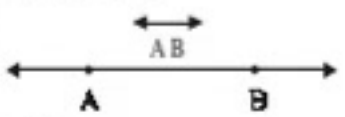
1. Geometrical Terms

- **Point**—A point is an exact position or location on a plane surface.
- **Flat shapes**— In geometry, a plane is a flat, two-dimensional surface. Plane means flat or a flat surface. They are flat, having only two dimensions- length and width. Example— book, money, glass, kite, postcard, cricket ground, etc.
- **Curved shapes**— Objects having the shapes of sphere or unplained surface of a cylinder have curved surfaces. The surface of a football, cricket ball, round bottle, orange, grapes, mango etc. are curved surfaces.
- **2-D shapes**— Figures with two dimensions are called 2-D shapes. Example, triangle, square, rectangle, etc.
- **Solid or 3-D shapes**— Figures with two dimensions are called 3-D shapes. Example, Cube, Cuboid, Cylinder, Cone, Sphere, Pyramid etc. These shapes consist of faces, edges, and vertices.
- **Line Segment**—A line segment is a part of a line with a certain distance. In the figure given below, A and B are the two points of the line segment. The length of the line segment is shown by the symbol $\overline{AB}$.

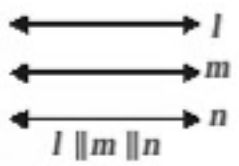
- **Ray**—If a line segment extends in one direction, the obtained figure is called a ray. It is shown by the symbol $\overrightarrow{AB}$. A is called initial point. It has no definite length.



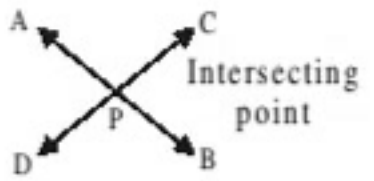
- **Line**—If a line segment extends in both directions, the obtained figure is called a line. It is shown by the symbol $\overleftrightarrow{AB}$. Line has no end points as well as no fixed length. A line consists of infinite points.



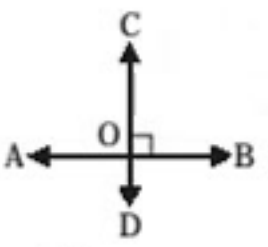
- **Parallel Lines**—If two or more lines are always on equi-distance. They never intersect each other. It is shown by the symbol '||'.



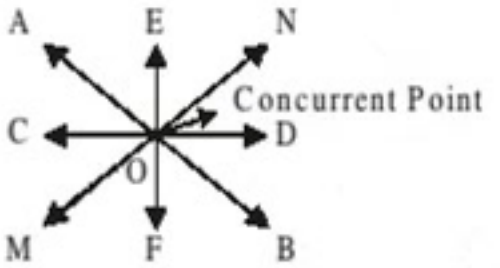
- **Intersecting Lines**—If two lines having a common point, then they are known as intersenting lines. In the given figure, 'P' is the intersecting point.



- **Perpendicular Lines**—If two lines intersect each other at 90°are called perpendicular lines. It is shown by the symbol ⊥. In the given figure, $\overrightarrow{AB}$ and $\overrightarrow{CD}$ intersect each other at O point, perpendicularly. So, $\overrightarrow{AB} \perp \overrightarrow{CD}$.

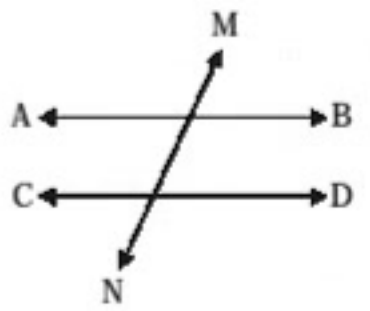


- **Concurrent Lines**—If three or more lines pass through a point, then they are called concurrent lines. The intersecting point is called concurrent point.

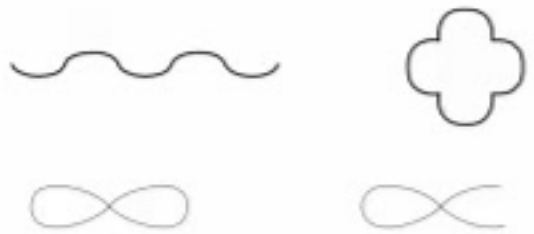


In the given figure, $\overleftrightarrow{AB}$ $\overleftrightarrow{CD}$ $\overleftrightarrow{EF}$ and $\overleftrightarrow{GH}$ pass through a point O. So, these are concurrent lines.

- **Transversal Line**—If two or more parallel lines are intersected by another line, then this line is called transversal line. In the given figure, line $\overleftrightarrow{GH}$ is a transversal line.



- **Curve**—A curved line or just curve is a line which is not straight. Ideally, a curve needs to be continuous and smooth.
- **Plane Curve**—If all the points of a curve are on a plane surface, then it is called Plane curve, otherwise non-plane curve.
- **Simple Curve**—A curve without intersecting its path is called simple curve, otherwise non-simple curve.



- **Open Curve–** A curve with two end points does not form a closed circuit, called open curve. Otherwise, it is called closed curve.

(Open Curve) (Closed Curve)

2. Angle

An angle is a figure consisting of two rays with end points. It is shown by the symbol ∠ .

Sexagesimal System–

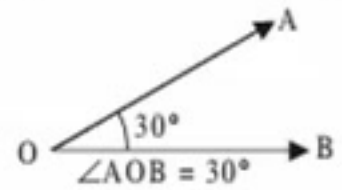
1 right angle = 90° = 90 degree

1° = 60′ = 60 minutes

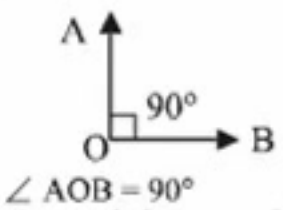
1′ = 60″ = 60 seconds

2.1 Types of Angles

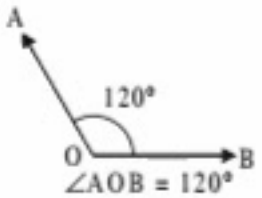
- **Acute Angle–** An angle between 0° and 90° is called acute angle. Example–10°, 25°, 30°, 50°,....,etc.
or $0 < \theta < 90°$ [where θ is an angle]



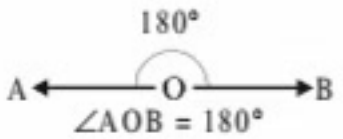
- **Right Angle–** Angle of 90° is called right angle. It means, θ = 90°.



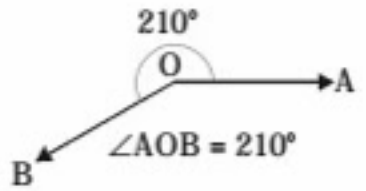
- **Obtuse Angle–** An angle between 90° and 180° is called obtuse angle. Example– 110°, 125°, 130°, 175°,.....,etc.
or $90 < \theta < 180°$ [where θ is an angle]



- **Straight Angle–** Angle of 90° is called straight angle. It means, θ = 180°



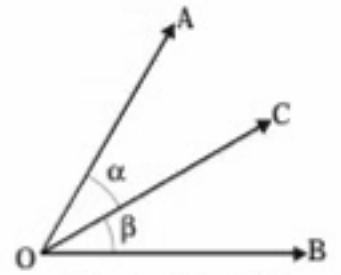
- **Reflex Angle–**An angle between 180° and 360° is called reflex angle. Example– 190°, 225°, 330°, 275°,.....,etc.
or $180 < \theta < 360°$ [where θ is an angle]



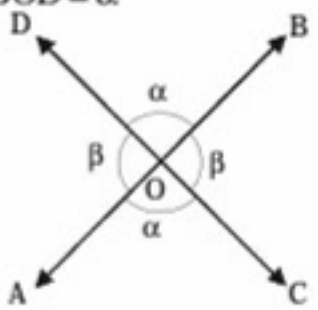
- **Complete Angle–**Angle of 360° is called complete angle. It means, θ = 360°. It represents a circle.

∠AOB = 360°

- **Adjacent Angles–**Two angles are called adjacent angles when they have a common vertex and a common side (or, arm). In the given figure, ∠AOC and ∠COB have a common vertex O. Its common arm is $\overrightarrow{OC}$. So, ∠AOC and ∠COB are adjacent angles.



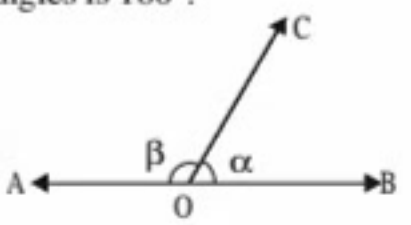
- **Vertically Opposite Angle–**If two lines AB and CD intersects at a point O, then the pair of ∠AOC and ∠BOD or the pair of ∠AOD and ∠BOC are called vertically opposite angles. Here,
∠AOD = ∠BOC = β
∠AOC = ∠BOD = α



Note

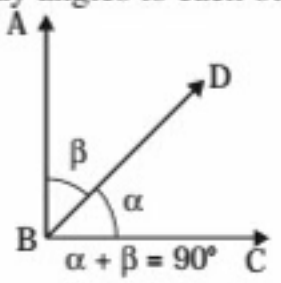
There is always the same vertex of vertically opposite angles.

- **Angles of Linear Pair–** Two angles are called to form a linear pair of angles, if they are adjacent angles. The sum of such angles is 180°.

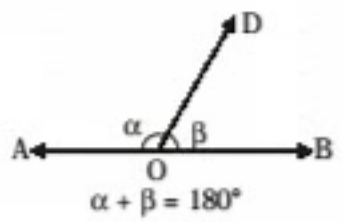


Here α + β = 180°

- **Complementary Angles–**Two angles are called complementary angles, if their sum is 90°. Example, if α + β = 90°, then angles α and β are said to be complementray angles to each other.



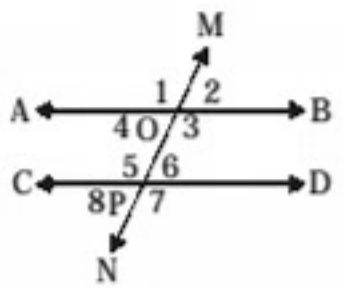
- **Supplementary Angle**—Two angles are called supplementary angles, if their sum is 180°. Example, if $\alpha + \beta = 180°$, then angles α and β are said to be supplementray angles to each other.



$\alpha + \beta = 180°$

2.2 Angles made by Traversal line

Accoding to the figure, traversal line $\overrightarrow{\text{ड घ}}$ intersects the parallel lines $\overrightarrow{\text{। ठ}}$ and $\overrightarrow{\text{ढ क}}$ at the points O and P. We have,

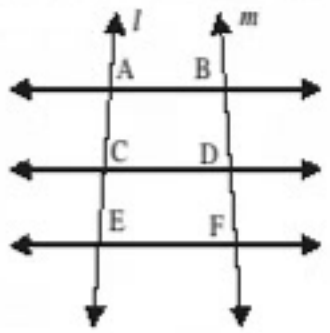


- **Corresponding angles—**
  In the given figure:
  $\angle 2 = \angle 6$
  $\angle 3 = \angle 7$
  $\angle 1 = \angle 5$
  $\angle 4 = \angle 8$
  are called corresponding angles.
- **Alternate angles—**
  In the given figure :
  $\angle 4 = \angle 6$
  $\angle 3 = \angle 5$
  $\angle 2 = \angle 8$
  $\angle 1 = \angle 7$
  are called alternate angles.
- **Interior angles—**
  In the given figure: $\angle 4$ and $\angle 5$
  $\angle 3$ and $\angle 6$
  are called interior angles.
- **Exterior angles—**
  In the given figure : $\angle 1$ and $\angle 2$
  $\angle 7$ and $\angle 8$
  are called exterior angles.

Important points :

- Each pair of corresponding angles are the same.
- Each pair of alternate angles are the same.
- The sum of interior angles on the same side of traversal is 180°.
  $\angle 3 + \angle 6 = 180°$ and $\angle 4 + \angle 5 = 180°$
- If two lines are the parallel to the third line, then all the three lines will be parallel to each other.
- Several points on a line are called colinear points.
- Only one line can pass through the two points.
- From a point, uncountable lines can be drawn.

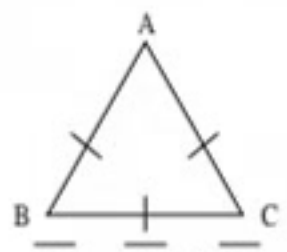
- All the perpendiculars drawn at a line are parallel to each other.
- **Intersection of three lines by two traversal lines–**
  As per given figure, the lines AB, CD and EF are parallel to each other. The traversal lines l and m intersect them, then



1. $\frac{\text{। ड}}{\text{ड ढ}} = \frac{\text{ठ क}}{\text{क ढ}}$
2. If AC = CE, then BD = DF.

3. Triangle

A closed figure consisting of three vertices, three sides and three angles are called triangle. It is shown by the symbol Δ.



In the above figure, we have

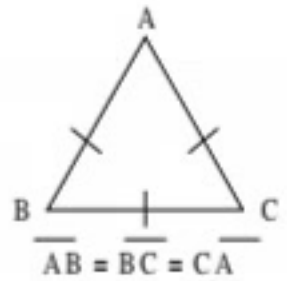
(i) Vertices ⟶ A, B, C,

(ii) Sides ⟶ $\overline{\text{। ठ}}$ $\overline{\text{ठ ढ}}$ $\overline{\text{। ढ}}$

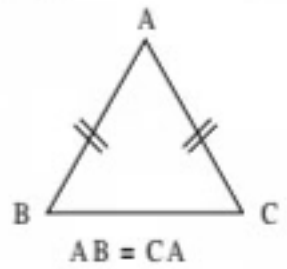
(iii) Angles ⟶ $\angle A$, $\angle B$, $\angle C$

3.1 Types of Triangle on the Basis of Sides

- **Equilateral Triangle**—All three sides of an equilateral triangle are equal.



- **Isosceles Triangle**—Any two sides of an isosceles triangle are equal.

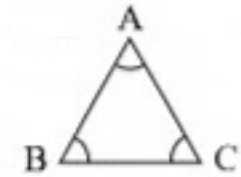


- **Scalene Triangle**—All three sides of an equilateral triangle are different.

$AB \neq BC \neq CA$

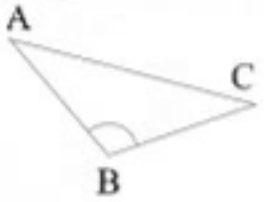
3.2 Types of Triangle on the Basis of Angles

- **Acute-angled Triangle**—Each angle of the acute-angled triangle is less than 90°.



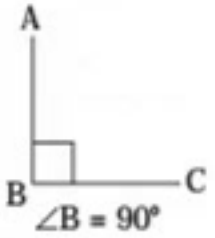
$\angle A < 90°, \angle B < 90°$ and $\angle C < 90°$

- **Obtuse-angled Triangle**—Any one angle of the obtuse-angled triangle is greater than 90°.



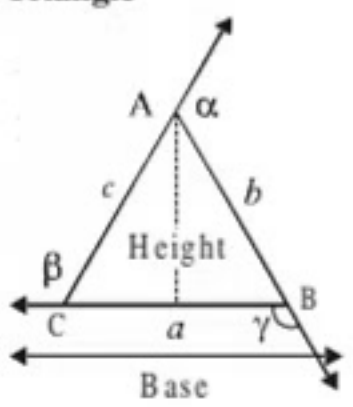
and $\angle B > 90°$

- **Right-angled Triangle**—Any one angle of the right-angled triangle is of 90°.



$\angle B = 90°$

3.3 Properties of a Triangle



- The sum of interior angles of a triangle is 180°.

$$\angle A + \angle B + \angle C = 180°$$

- The sum of exterior angles of a triangle is 360°.

$$\alpha + \beta + \gamma = 360°$$

- In a triangle, an exterior angle equals the sum of the two interior opposite angles.

$$\alpha = \angle B + \angle C$$
$$\beta = \angle A + \angle C$$
$$\gamma = \angle A + \angle B$$

- The sum of any two sides of a triangle is always greater than its third side.

$$a + b > c$$
$$b + c > a$$
$$c + a > b$$

- The difference of any two sides of a triangle is always less than its third side.

$$|a - b| < c$$
$$|b - c| < a$$
$$|c - a| < b$$

- In a triangle, the arm opposite to the largest angle is the largest arm and the arm opposite to the smallest angle is the smallest arm.
- Perimeter of triangle $\Delta ABC = (a + b + c)$.
- Area of triangle $\Delta ABC = \sqrt{(-)(-)(-)}$

where, $s = \frac{+ + }{2}$

- Area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$
- Area of equilateral triangle $= \frac{\sqrt{3}}{4} (\text{side})^2$
- **Pythagoras Theorem**–In a right-angled triangle, the square of the hypotenuse equals to the sum of the square of its sides. Let ΔABC is a right angle triangle, where AC, AB and BC be the hypotenuse, base and height respectively. We have,

$$AC^2 = AB^2 + BC^2$$

4. Quadrialateral

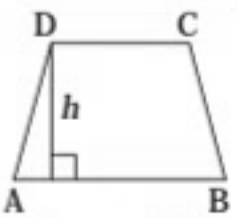
In a plane, a closed figure with four sides are called quadrilateral. it is shown by the symbol .

Characteristics of Quadrilateral–

- Vertices ⟶ Four (A, B, C and D)
- Sides ⟶ Four $\left(\overline{AB}, \overline{BC}, \overline{CD} \text{ and } \overline{DA}\right)$
- Angle ⟶ Four ($\angle A$, $\angle B$, $\angle C$ and $\angle D$)
- Opposite Angles ⟶ Two ($\angle A$, $\angle C$ and $\angle B$, $\angle D$)
- Diagonals ⟶ Two (and)
- Area of quadrilateral $= \frac{1}{2} \times \times (_1 + _2)$,

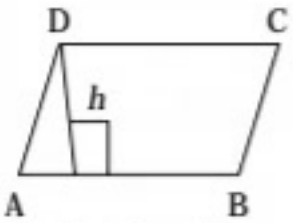
4.1 Types of Quadrilateral

- **Trapezium :** A trapezium is a quadrilateral wherein one pair of the opposite sides are parallel while the other isn't. In the given figure, ABCD is a trapezium with the height *h*.



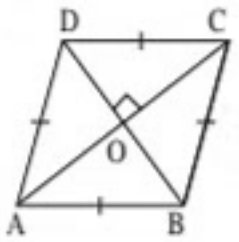
- **Parallelogram :** The parallelogram is a four-sided plane rectilinear figure with opposite sides parallel. A parallelogram is a simple object in 2D space having two parallel sides. The opposite sides and the angles are identical in parallelogram.

$$AB = CD \text{ and } AD = BC$$
$$\angle A = \angle C \text{ and } \angle B = \angle D$$



- **Rhombus :** A rhombus is a quadrilateral with four congruent sides. Congruents figures are identical in size, shape and measure. The diagonals of a rhombus intersect each other at right angles.

$\overline{AB} = \overline{BC} = \overline{CD} = \overline{DA}$ and $AC \perp BD$



- **Rectangle :** A quadrilateral whose opposite sides are equal and each interior angle is 90^o.

(i) $\angle A = \angle B = \angle C = \angle D = 90°$

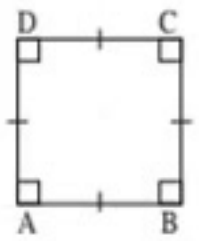
(ii) $\overline{AB} = \overline{CD}$ and $\overline{AD} = \overline{BC}$

- **Square :** A rectangle with four equal sides is called square. The diagonals of a square are alsp equal.

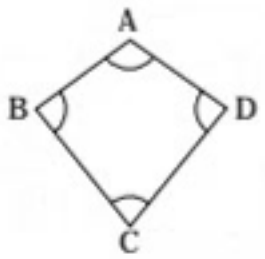
(i) $\overline{\text{ल}} = \overline{\text{ख}} = \overline{\text{ख}} = \overline{\text{क}}$

(ii) $\angle A = \angle B = \angle C = \angle D = 90°$

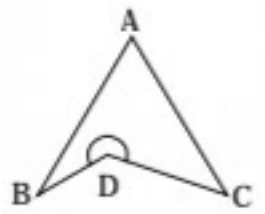
(iii) $AC \perp BD$



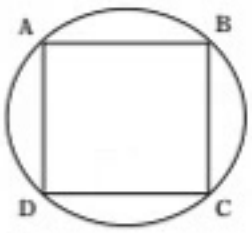
- **Convex :** A quadrilateral whose each interior angle is less than 180^o is called convex quadrilateral.



- **Concave :** A quadrilateral whose one interior angle is greater than 180^o is called concave quadrilateral. In the figure shown below, $\angle BDC > 180°$.



- **Cyclic Quadrilateral–**A quadrilateral whose all vertices are on the circumference of a circle is called cyclic quadrilateral.

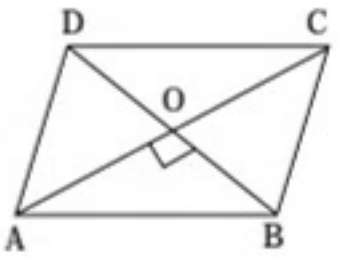


(i) $\angle A + \angle C = 180°$

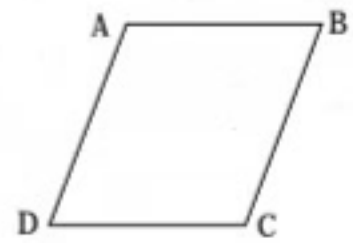
(ii) $\angle B + \angle D = 180°$

Important Points :

- The sum of all the interior angles of a quadrilateral is 360°.
- In a parallelogram, opposite sides and opposite angles are equal.
- In a parallelogram, the diagonals bisect each other.
- The diagonals of a rectangle are equal and bisect each other.
- The diagonals of a rhombus intersect each other at right angles.
- The diagonals of a square are equal and intersect each other at right angles.
- The sum of all the exterior angles of a quadrilateral is 360°.
- In parallelogram, the bisector of two adjacent angles intersect at right angles. Here, $\angle AOB = 90°$



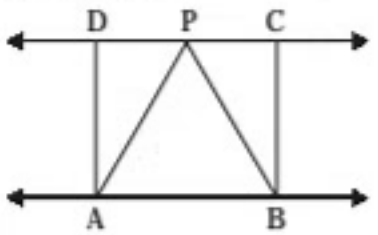
- A parallelogram formed inside a circle is a rectangle.
- A parallelogram outside a circle is a rhombus.
- A rectangle is formed inside the circle, then the diagonal of the rectangle is diameter of the circle.
- All rectangles are parallelograms but vice-versa is not true.
- Sum of interior angles formed between the parallel lines of a rhombus is 180^o.



$\angle A + \angle B = \angle B + \angle C = \angle C + \angle D$
$= \angle D + \angle A = 180°$

- All rhonbus figures are parallelograms but vice-versa is not true.
- The area of a rectangle and a parallelogram formed between the same base and a pair of the same parallel lines is the same.

- The area of a rectangle formed between the same base and a pair of the same parallel lines is 2 times the area of triangle formed on the same base.



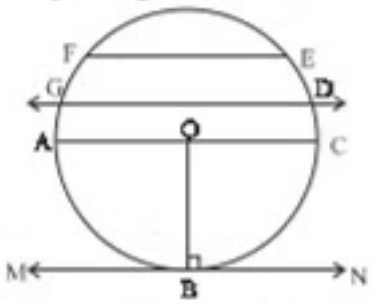
$ar(\square ABCD) = 2 \times ar(\Delta PAB)$

Perimeter of Different Shapes–

- Perimeter of qudrilateral = sum of four sides
- Perimeter of rectangle = 2 (Length x Breadth)
- Perimeter of square = Perimeter of rhombus = 4 × side
- Perimeter of parallelogram ABCD = 2(AB + BC)
- Perimeter of trapezium ABCD = (AB + BC + CD + DA)

5. Circle

A circle is the locus of all points equidistant from a central point. A distance around the circle is called circumference of the circle. A circle is a round shaped figure that has no corners or edges.



5.1 Terminology related to Circle–

- **Arc**–A curved line that is part of the circumference of a circle. In the above figure, $\overset{\frown}{AB}, \overset{\frown}{BC}, \overset{\frown}{CD}$ etc. are the arcs.
- **Center**–Center is a point that is equidistant from all the points lie on the circle. The point O is center of the circle.
- **Chord**–A line segment within a circle that touches 2 points on the circle. FE and AC are the two chords. A chord divides the circle into two circle sectors.
- **Diameter**–The longest distance from one end of a circle to the other. It is the longest chord of the circle. It is twice the radius of the circle.

  Diameter = 2 × Radius
- **Radius Sector**–An area covered by an arc between any two radii is called radius sector. In the figure, OBC is a radius sector.
- **Semi-circle**–A semi-circle is half of a circle, formed by cutting a whole circle along a line segment passing through the center of the circle. This line segment is called the diameter of the circle. Area of semi-circle is half of the area of the circle. In the figure, ABC is a semi-circle.
- **Secant Line**–A line intersecting the circumference of the circle at any two points is called secant line of the circle. In the figure, GD is a secant line.
- **Tangent Line**–A line touches the circumference of the circle at a point. this line is called tangent line. Distance between the center and the tangent line of the circle is called perpendicular. In the figure, MBN is a tangent line.

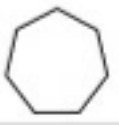
5.2 Properties of Circle

- Angle in a semi-circle is a right angle, i.e. 90°. In other words, Angle subtended by a diameter/semi-circle on any point of circle is 90°.
- The perpendicular bisector of a chord passes through the center of the circle. In other words, The perpendicular from the center to the chord bisects the chord.
- The opposite angles of a cyclic quadrilateral add to 180°.
- Tangents to the circle from a point have the same length.
- Equal chords are equidistance from the center and vice versa.
- Equal arcs subtend equal angles and vice versa.
- Any three non-colinear points lie on a unique circle.
- Angle at center is twice angle at circumference of the circle.
- An inscribed angle is an angle where its vertex is a point on the circumference of the circle and its sides are chords of the circle that passes through the vertex. So, All inscribed angles to a common chord are equal to each other.
- An angle of large circle sector is an acute angle and an angle of smaller circle sector is an obtuse angle.
- The alternate segment theorem (also known as the tangent-chord theorem) states that in any circle, the angle between a chord and a tangent through one of the end points of the chord is equal to the angle in the alternate segment.

6. Polygon

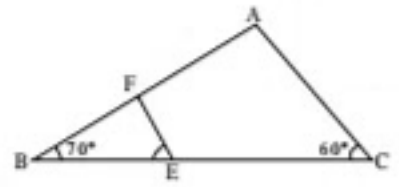
A polygon is a plane figure that is described by a finite number of straight line segments connected to form a closed polygonal chain or polygonal circuit. A polygon with equal sides is called regular polygon.

Classification of Polygons

No. of sides (n)	Figure name	Structure
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	
7	Septagon	
8	Octagon	

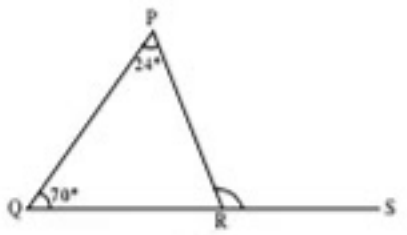
Important Questions

1. In the figure, $\angle B = 70°$, $\angle C = 60°$. E is the mid-point of BC and F is the mid-point of AB. Then find the value of $\angle FEB$. **(NCERT)**



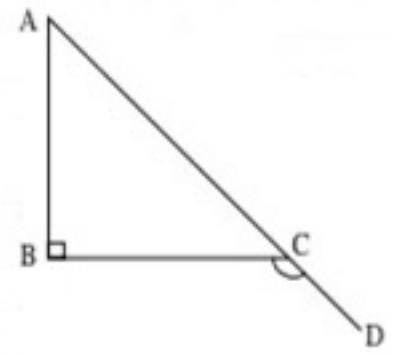
(A) 40° (B) 60°
(C) 70° (D) 50°

2. In the figure, $\angle PQR = 70°$, $\angle QPR = 24°$ then find $\angle PRS$.



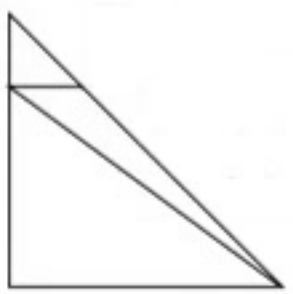
(A) 110° (B) 94°
(C) 55° (D) 111°

3. In figure, If AB = BC and $\angle ABC = 90°$, find $\angle BCD$.



(A) 135° (B) 150°
(C) 120° (D) 210°

4. How many triangles are there ? **(NCERT)**

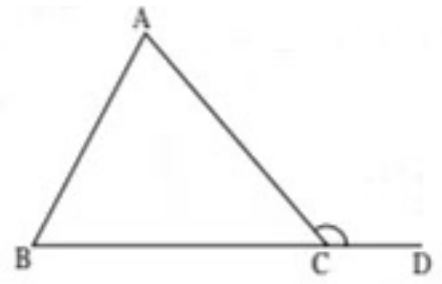


(A) 5 (B) 4
(C) 3 (D) 6

5. Two sides of a isosceles triangle are always—
(A) Decrease
(B) Increase
(C) Equal
(D) Either decrease or increase

6. The sides of a triangle are 8 cm, 17 cm and 15 cm. It is called— **(NCERT)**
(A) Equilateral triangle
(B) Isosceles triangle
(C) Obtuse angled triangle
(D) Right angled triangle

7. In figure, of AB = BC and $\angle BAC = 80°$, find $\angle ACD$.



(A) 130° (B) 150°
(C) 120° (D) 210°

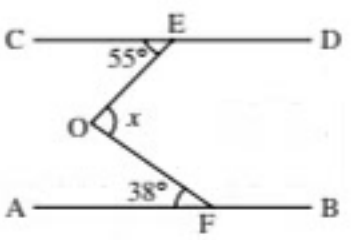
8. If a side of an equilateral triangle increases, then find the exterior angle. **(NCERT)**
(A) 60° (B) 90°
(C) 120° (D) 180°

9. Find the expression for the number of diagonals drawn from the vertices of a polygon of 'n' sides.
(A) $2n + 1$ (B) $n - 2$
(C) $5n + 2$ (D) $n - 3$

10. Find the sum of interior angles of a square.
(A) 180° (B) 360°
(C) 270° (D) None of these

11. Which of the following is true ?
(A) A square is a special type of a rectangle
(B) A rectangle is a special type of a square.
(C) Each ray is a segment.
(D) Straight line angle is of 90°

12. In figure, AB||CD, Find $\angle x$.



(A) 87° (B) 267°
(C) 93° (D) None of these

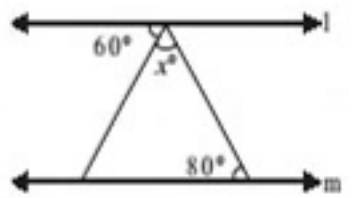
13. What will be the complementary angle of 55° ?
(A) 180° (B) 90°
(C) 125° (D) 35°

14. In figure, two parallel lines are cut by an oblique line. Find the alternate angles. **(NCERT)**

(A) (1, 8) and (2, 7)
(B) (1, 2) and (7, 8)
(C) (2, 6) and (1, 5)
(D) (1, 5) and (7, 3)

15. It two complementary angles are in the ratio of 2 : 3, then find the angles.
(A) 25°, 65° (B) 108°, 72°
(C) 36°, 54° (D) 40°, 60°

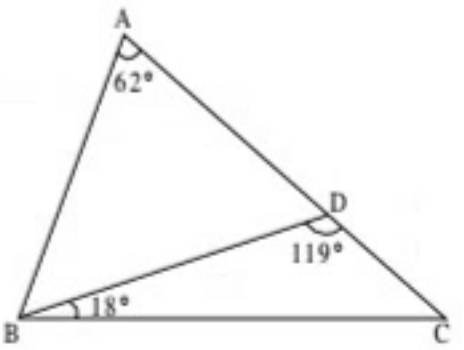
16. In figure, if $l \parallel m$, then find x. **(NCERT)**



(A) 60° (B) 80°
(C) 40° (D) 140°

17. In a right angled triangle, the product of two sides equals to the half of square of hypotenuse. Find the acute angle.
(A) 15° (B) 30°
(C) 45° (D) 60°

18. In figure of triangle ABC, find $\angle ABD$.



(A) 57 (B) 61
(C) 72 (D) 80

19. A polygon has 30 sides and 12 faces. Find the number of vertices of the polygan.
(A) 20 (B) 24
(C) 12 (D) 15

20. How many symmetrical lines are there in a parallelogram ?

(A) 2 (B) 4

(C) 3 (D) None of these

21. ABCD is a rhombus. If $\angle ACB = 30°$, find $\angle ADB$. **(NCERT)**

(A) 30° (B) 120°

(C) 60° (D) 45°

22. Vertices of a quadrilateral ABCD are on the circumference of a circle. If AB is the diameter of the circle and $\angle ADC = 130°$, find $\angle BAC$.

(A) 50° (B) 40°

(C) 30° (D) 20°

23. The angles of a quadrilateral are in the ratio of 2 : 3 : 5 : 8. Find the sum of supplementary angle of its largest angle and complementary angle of its smallest angle.

(A) 70° (B) 80°

(C) 50° (D) 60°

24. Each interior angle of a regular polygon is 156°. Find the number of sides of a polygon. **(NCERT)**

(A) 15 (B) 12

(C) 10 (D) 8

25. If sides of a regular polygon are 'n', then find its symmetrical lines.

(A) $\frac{n}{2}$ (B) n^2

(C) $2n$ (D) n

26. The sum of all interior angles of a polygon is 144°. Find the number of sides of the polygon.

(A) 9 (B) 10

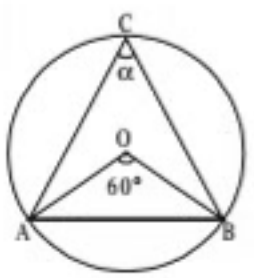
(C) 12 (D) 8

27. A straight line intersecting on two points of a circle is called—

(A) Tangent (B) Secant

(C) Chord (D) Segment

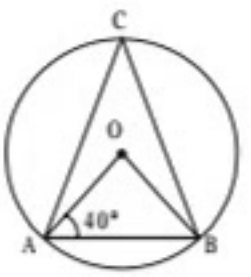
28. Find the value of angle α in the following figure. (0 is the centre).



(A) 60° (B) 20°

(C) 30° (D) 40°

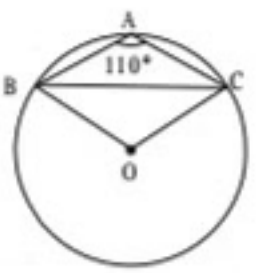
29. In the given figure, If $\angle OAB = 40°$, find $\angle ACB$. **(NCERT)**



(A) 50° (B) 40°

(C) 60° (D) 70°

30. In the given figure, O is the centre. Find $\angle OCB$ **(NCERT)**

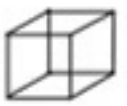


(A) 10° (B) 20°

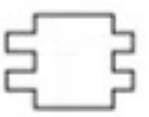
(C) 30° (D) 40°

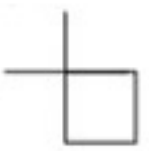
31. Which of the following figure is not a two dimensional figure ?

(A)  (B) 

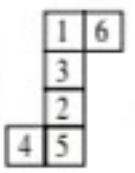
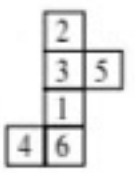
(C)  (D) 

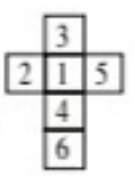
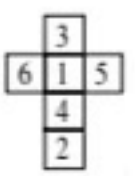
32. Which of the following is define as open curve ?

(A)  (B) 

(C)  (D) 

33. The sum of numbers shown on the opposite faces in a dice is 7. Which of the following options would be similar ?

(A)  (B) 

(C)  (D) 

34. Find the sum of 4 and two-third right angles.

(A) 420° (B) 400°

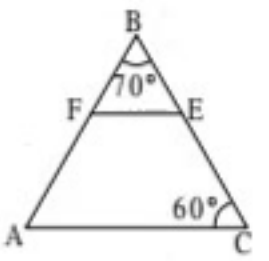
(C) 300° (D) 310°

SOLUTIONS

1. (B) ∵ E and F are the mid-points of BC and AB.

$\therefore$ EF ∥ AC

$\Rightarrow$ $\angle FEB = \angle ACB = 60°$



2. (B) As per figure,

$\angle PRS = \angle PQR + \angle QPR$

$= 70° + 24° = 94°$

3. (A) ∵ In Δ ABC,

$\angle B = 90°$ and $AB = BC$

$\therefore \quad \angle A = \angle C = \frac{90}{2} = 45°$

∵ $\angle BCD + \angle BCA = 180°$

$\Rightarrow \quad \angle BCD = 180° - \angle BCA$

$= 180° - 45° = 135°$

4. (A) Numbers of triangles = 5

5. (C) Any two sides of an isosceles triangle are always equal.

6. (D) ∵ $17^2 = 289$

$15^2 + 8^2 = 225 + 64 = 289$

$\therefore \quad 17^2 = 15^2 + 8^2$

$\Rightarrow$ It is a right angled triangle.

7. (A) ∵ AB = AC

$\Rightarrow \quad \angle B = \angle C$

$\therefore \quad \angle BAC + \angle B + \angle C = 180°$

$\Rightarrow \quad 80 + \angle B + \angle B = 180°$

$\Rightarrow \quad 80 + 2\angle B = 180°$

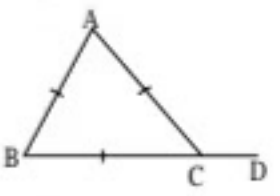
$\Rightarrow \quad 2\angle B = 180° - 80° = 100°$

$\Rightarrow \quad \angle B = 50°$

$\therefore \quad \angle ACD = \angle BAC + \angle B$

$= 80° + 50° = 130°$

8. (C) Let, BC is exended upto the point D which makes an exterior angle $\angle ACD$ with the side AC. In ΔABC,



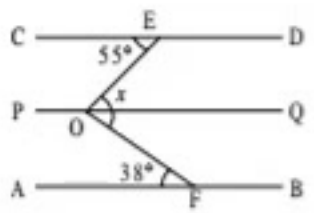
$$\angle A = \angle B = \angle C = \frac{180}{3} = 60°$$

$\therefore$ exterior $\angle ACD = 180 - \angle C$

$= 180 - 60 = 120°$

9. (D) $n - 3$ **10.** (B) 360° **11.** (A)

12. (C) Draw a line PQ parallel to the lines AB and CD through the point O. We have,



$\angle x° = \angle EOF = 55° + 38° = 93°$

13. (D) $90 - 55° = 35°$

14. (A) (1,8) and (2,7)

15. (C) Sum of complementary angles = 90°

and ratio between angles = 2 : 3

$\therefore$ 1st angle $= \frac{2}{5} \times 90° = 36°$

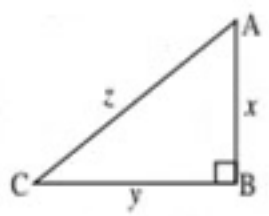
2nd angle $= 90 - 36 = 54°$

16. (C) According to the question,

$x + 60 + 80 = 180$ (Linear pair)

$\Rightarrow \quad x = 180 - 140 = 40°$

17. (C) Let AB = x, BC = y and AC = z



According to the question,

$$xy = \frac{z^2}{2}$$

$\Rightarrow \quad z^2 = 2xy \quad$ ...(i)

Pythagorous theorem,

$z^2 = x^2 + y^2 \quad$ ...(ii)

From eq. (i) and (ii),

$x^2 + y^2 = 2xy$

$\Rightarrow \quad x^2 + y^2 - 2xy = 0$

$\Rightarrow \quad (x - y)^2 = 0$ or $x - y = 0$

$\Rightarrow \quad x = y$

$\Rightarrow \quad \angle A = \angle C = \frac{90°}{2} = 45°$

18. (A) In ΔBCD,

$\angle BCD = 180° - (18° + 119°)$

$= 180° - 137° = 43°$

$\therefore$ In ΔABD,

$\angle BDA = 180° - 119° = 61°$

$\therefore \angle ABD = 180° - 61° - 62° = 57°$

19. (A) Let V be the number of vertices, F be the number of faces and E be the number of sides.

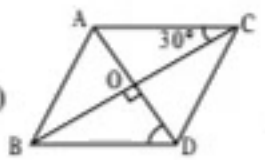
$\therefore V + F - E = 2$ (As per rule)

$\Rightarrow \quad V = 2 - F + E$

$V = 2 - 12 + 30 = 20$

20. (A) 2

21. (C)



$\because$ Diagonals of a rhombus intersect each other at a perpendicular.

$\therefore \quad \angle BOD = 90°$

$\because \quad AC \parallel BD$

$\Rightarrow \quad \angle OBD = \angle ACB = 30°$

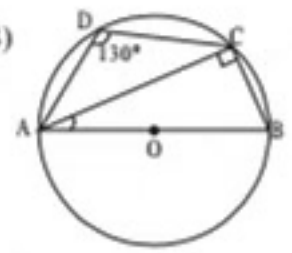
In ΔBOD,

$\angle BOD + \angle OBD + \angle BDO = 180°$

$\angle BDO = 180 - (90 + 30)$

$\therefore \quad \angle ADB = \angle BDO = 60°$

22. (B)



$\because \angle ABCD$ is a cyclic quadrilateral.

$\therefore \quad \angle ABC = 180 - \angle ADC$

$\angle ABC = 180 - 130 = 50°$

$\because$ AB is the diameter of circle.

$\therefore \angle BAC + \angle ACB + \angle ABC = 180°$

$\angle BAC = 180 - (\angle ACB + \angle ABC)$

$\angle BAC = 180 - (90 + 50)$

$\angle BAC = 40°$

23. (A) Ratio = 2 : 3 : 5 : 8

Smallest angle $= \frac{2}{18} \times 360° = 40°$

Largest angle = $8 \times 20° = 160°$

According to the question,

Sum = $(90 - 40) + (180 - 160)$

$= 50 + 20$

$= 70°$

24. (A) Sum of the interior angles of a regular polygon = $(2n - 4) \times 90°$

According to the question,

$(2n - 4) \times 90° = 156° \times n$

$\Rightarrow \quad 180n - 360 = 156n$

$\Rightarrow \quad 24n = 360$

$\Rightarrow \quad n = \frac{360}{24} = 15$

25. (D) n

26. (B) Sum of the interior angles of a polygon $= (n - 2) \times 180°$

According to the question,

$(n - 2) \times 180° = 144° \times n$

$\Rightarrow \quad 180n - 360 = 144n$

$\Rightarrow \quad 36n = 360$

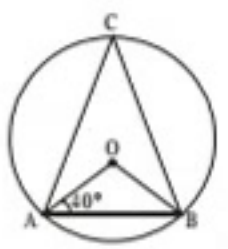
$\Rightarrow \quad n = 10$

27. (B) Secant line

28. (C) $\because$ An angle formed on the center is twice the angle formed on the remaining circumference.

$\therefore \quad \alpha = \frac{1}{2} \times 60° = 30°$

29.(A)



$\because \quad OA = OB$ (radius)

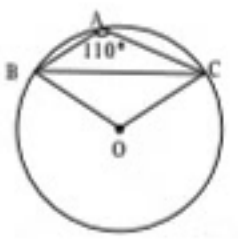
$\Rightarrow \quad \angle OAB = \angle OBA = 40°$

$\therefore \quad \angle AOB = 180° - 80° = 100°$

So, $\angle ACB = \frac{1}{2} \angle AOB$

$= \frac{1}{2} \times 100° = 50°$

30. (B)



$\because \quad OB = OC$ (radius)

$\therefore \angle OCB = \angle OBC$

Exterior $\angle BOC = 2 \times \angle BAC$

$= 2 \times 110 = 220°$

$\therefore$ Interior $\angle BOC = 360° - 220° = 140°$

Now in ΔBOC,

$\angle OCB + \angle OBC + \angle BOC = 180$

$\angle OCB + \angle OCB + 140 = 180$

$2 \angle OCB = 40°$

$\Rightarrow \quad \angle OCB = 20°$

31. (C) **32.** (C) **33.** (C)

34. (A) $\left(4 + \frac{2}{3}\right) \times 90° = \frac{14}{3} \times 90° = 420°$

Chapter 16

Area and Perimeter

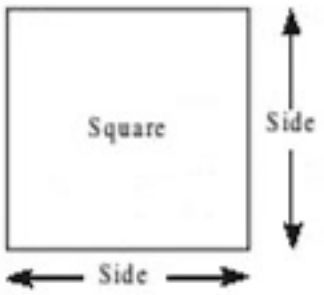
1. Area and Perimeter

Area means "the size of a surface". The amount of space inside the boundary of a flat (2-D) object such as a triangle or circle, or surface of a solid (3-D) object.

1.1 Square objects

A quadrilateral object whose all sides are equal and parallel is called square objects. Each angle of such figure is 90°. The sum of the lengths of all edges is known as perimeter.

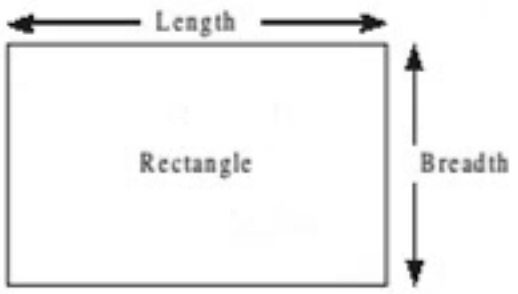
Different shapes having different areas and perimeter.



Related formulas

- Perimeter = 4 × side
- Area = side × side
- Diagonal = $\sqrt{2}$ × side
- Side = $\frac{\text{Perimeter}}{4} = \sqrt{\text{Area}} = \frac{\text{Diagonal}}{\sqrt{2}}$

1.2 Rectangular objects—A quadrilateral object whose opposite sides are parallel and same is called rectangular objects.



Related formulas

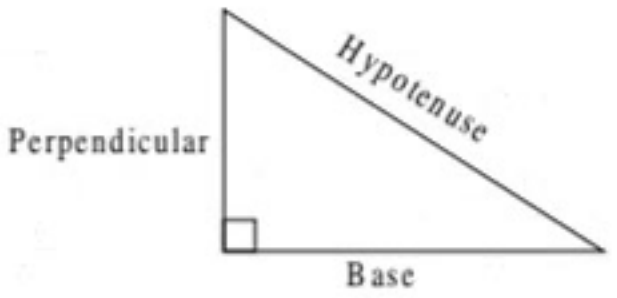
- Perimeter = 2(Length + Breadth)
- Area = Length × Breadth
- Diagonal = $\sqrt{(\text{Length})^2 + (\text{Breadth})^2}$
- Length = $\sqrt{(\text{Diagonal})^2 - (\text{Breadth})^2}$

$$= \frac{\text{Perimeter}}{2} - \text{Breadth} = \frac{\text{Area}}{\text{Breadth}}$$

- Breadth = $\sqrt{(\text{Diagonal})^2 - (\text{Length})^2}$

$$= \frac{\text{Perimeter}}{2} - \text{Length} = \frac{\text{Area}}{\text{Length}}$$

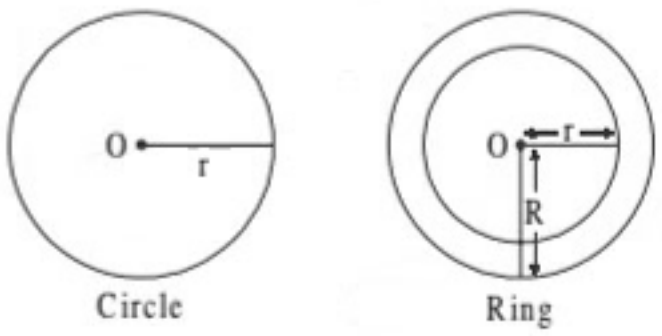
1.3 Triangle—A closed figure with three straight lines is called a triangle.



Related formulas

- Perimeter = Sum of all sides
- Area = $\frac{1}{2}$ × Base × Height
- Hypotenuse = $\sqrt{(\text{Perpendicular})^2 + (\text{Base})^2}$
- Perpendicular = $\sqrt{(\text{Hypotenuse})^2 - (\text{Base})^2}$
- Base = $\sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2}$

1.4 Circle—A round shaped figure that has no corners and no edges.



Related formulas

- Circumference = $2\pi r = \pi d$,
- Area of a circle = πr^2
- Area of semicircle = $\frac{1}{2}\pi r^2$
- Perimeter of semicircle = $(\pi r + 2r)$
- Area of a ring = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$

Important Questions

1. Length and breadth of a rectangular plot are 25 m and 30 m respectively. Find the area of plot. **(NCERT)**

(A) 250 sq m (B) 300 sq m
(C) 750 sq m (D) 550 sq m

2. Length and breadth of a rectangular towel are 125 cm and 60 cm respectively. Find the perimeter of the towel ?

(A) 750 cm (B) 370 cm
(C) 185 cm (D) 150 cm

3. 260 m thorned wire is required for fencing around the square plot. Find the side of the field.

(A) 65 m (B) 26 m
(C) 52 m (D) 50 m

4. The length and breadth of floor of a room are 8 m and 7 m respectively. There is a carpet that covers the floor completely. Find the area of the carpet.

(A) 65 m^2 (B) 56 m^2
(C) 30 m^2 (D) 38 m^2

5. Dev had to walk 40 m to take two rounds of a square plot. Find the side of the square plot.

(A) 2 m (B) 3 m
(C) 5 m (D) 10 m

6. Find the perimeter of stool whose side is 60 cm. **(NCERT)**

(A) 80 cm (B) 100 cm
(C) 120 cm (D) 240 cm

7. Gopi's field shape is square whose side is 75 m. Narayan's field shape is rectangular whose length is 85 m. Whose area will be larger while both have same perimeter ?

(A) Gopi, 100 m^2 (B) Narayan, 100 m^2
(C) Gopi, 90 m^2 (D) Narayan, 90 m^2

8. Radha takes two rounds daily around the side-by-side of the square park of side 60 m. Find out how much distance she cover daily ?

(A) 120 m (B) 240 m
(C) 480 m (D) 960 m

9. Suresh has 70 cm long ribbon. He wants to use 26 cm ribbon for the rectangular photo frame. Find the width of the ribbon ?

(A) 10 cm (B) 11 cm
(C) 12 cm (D) 13 cm

10. Ranu wants to lay the carpnt in his meeting hall. The length of the hall is 50 m. If the breadth of the hall is half of its length, then find the area of the carpet. **(NCERT)**

(A) 1150 m^2 (B) 1225 m^2
(C) 1250 m^2 (D) 2500 m^2

11. Find the maximum distance between two points on the perimeter of a rectangular field whose length and breadth are 80 m and 60 m.

(A) 100 m (B) 150 m
(C) 160 m (D) 480 m

12. There is a rectangular field in the stadium, whose one side is 15 m and one of its diagonals is 17 m. What will be the area of the field ?

(A) 150 m^2 (B) 170 m^2
(C) 120 m^2 (D) 205 m^2

13. Find the cost of carpeting a room 14 m long and 9 m broad with a carpet 75 cm wide at the rate of ₹ 8.50 per square meter.

(A) ₹ 1926 (B) ₹ 1428
(C) ₹ 1532 (D) ₹ 1450

14. A rectangular sheet has an area of 100 square meters and a perimeter of 50 meters. Find its diagonal.

(A) 17.5 m (B) 19.6 m
(C) 20.6 m (D) 23.5 m

15. In measuring the sides of a rectangular plot, one side is taken 5% in excess and the other 6% is deficit. Find the error percent in area calculated.

(A) 0.5% (B) 1.0%
(C) 1.3% (D) 2.3%

16. A rectangular garsy pot 99 m by 55 m has a gravel path 2.5 m wide all around it on the inside. Find the cost of gravelling the path at 45 paise per sq. m.

(A) ₹ 105.35 (B) ₹ 150.65
(C) ₹ 222.25 (D) ₹ 326.25

17. The diagonal of a rectangular field 15 m and its area is 108 m^2. What will be the total expenditure in fencing the field at the rate of 5 per metre ?

(A) 170 (B) 190
(C) 210 (D) 230

18. The perimeters of two squares are 40 cm and 32 cm. Find the perimeter of a third square whose area is equal to the difference of the areas of the two squares.

(A) 24 (B) 26
(C) 25 (D) 28

19. The length of a rectangle R is 10% more than the side of a square S. The width of the rectangle R is 10% less than the side of the square S. What is the ratio of the area of R to that of 5 ?

(A) 9 : 11 (B) 11 : 9
(C) 100 : 99 (D) 99 : 100

20. Find the largest size of a bamboo that can be placed in a square of area 100 m^2.

(A) 14.14 m (B) 1.414 m
(C) 12.34 m (D) 1.234 m

21. A rectangular courtyard, 3.78 in long and 5.25 m broad, is to be paved exactly with square tiles, all of the same size. Find the least number of square tiles covered.

(A) 450 (B) 540
(C) 405 (D) 504

22. Find the area of a square in sq. meters, one of whose diagonals is 3.8 m long.

(A) 7.22 (B) 6.27
(C) 5.72 (D) 5.50

23. The diagonals of two squares are in the ratio of 2 : 5. Find the ratio of their areas.

(A) 4 : 25 (B) 25 : 4
(C) 1 : 1 (D) 5 : 2

24. If each side of a square is increased by 25%, Find the percentage change in its area.

(A) 56% (B) 25%
(C) 25.56% (D) 56.25%

25. If the diagonal of a square is decreased by 15%, find the percentage decrease in its area.

(A) 27.75% (B) 25.57%
(C) 27% (D) 25%

26. A square park is surrounded by a path of uniform width 2 meters all around it. The area of the path is 288 sq. metres. Find the perimeter of the park.

(A) 136 m (B) 142 m
(C) 156 m (D) 462 m

27. Area of a play ground of school is 38400 m^2. If the ratio between length and breadth is 3 : 2, then find the perimeter of the playground.

(A) 500 m (B) 800 m
(C) 1200 m (D) 1000 m

28. Perimeter of a rectangle and a square is same. The length and breadth of the rectangle are 25 cm and 15 cm. Find the side of the square.

(A) 10 cm (B) 20 cm
(C) 30 cm (D) 40 cm

29. Side of a square carrom board is 40 cm. Find the value of the area covered by it.

(A) 4200 cm^2 (B) 3600 cm^2

(C) 2500 cm^2 (D) 1600 cm^2

30. Area of a playwood is 44100 cm^2. If the breadth of the playwood is 210 cm, then find its length. **(NCERT)**

(A) 200 cm (B) 180 cm

(C) 210 cm (D) 150 cm

31. If each side of a square remain $\frac{1}{3}$ rd of itself, then find the changes in the area of square.

(A) $\frac{1}{7}$ (B) $\frac{1}{3}$

(C) $\frac{1}{8}$ (D) $\frac{1}{9}$

32. Side of a square is $\sqrt{2}$ cm. Find its diagonal. **(NCERT)**

(A) 2 (B) $3\sqrt{6}$

(C) $6\sqrt{2}$ (D) $6\sqrt{3}$

33. A diagonal of a rectangle is 3 times the shorter side, find ratio between the sides of the rectangle.

(A) $2\sqrt{2}:1$ (B) $2\sqrt{3}:1$

(C) $2\sqrt{2}:2$ (D) $3\sqrt{2}:1$

34. Ratio of sides of a rectangular garden is 5 : 4. Its area is 500 m^2. Find the perimeter of the rectangle.

(A) 50 m (B) 80 m

(C) 25 m (D) 90 m

35. Area of a rectangular plot is 150 sq. m and its perimeter is 50 m. The length and width of the plot will be—

(A) 15 m, 10 m (B) 50 m, 3 m

(C) 30 m, 5 m (D) 7 m , 2 m

36. Perimeters of two squares are 24 cm and 32 cm. Find the perimeter of third square whose area is equal to the sum of areas of these squares. **(NCERT)**

(A) 24 cm (B) 40 cm

(C) 42 cm (D) 20 cm

37. Find the ratio between area of a square and area of another square made on diagonal of the first square.

(A) $1:\sqrt{2}$ (B) 1 : 2

(C) 2 : 1 (D) $\sqrt{2}:1$

38. Two perpendicular cross roads of 10 m width run through the middle of a square field of side 110 m. Find the area of the remaining field except cross roads.

(A) 36100 m^2 (B) 10000 m^2

(C) 63100 m^2 (D) 16300 m^2

39. Side of a square is increased by 10%. What percentage area will be increased ?

(A) 27 (B) 30

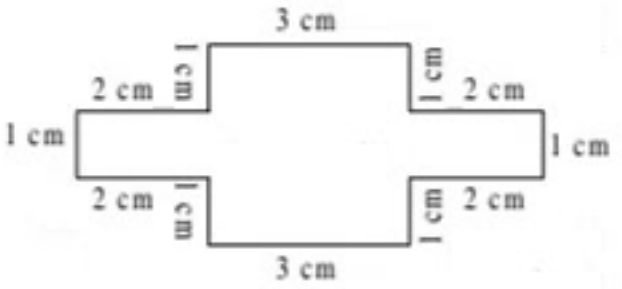
(C) 19 (D) 21

40. A rectangle's dimension are 20 m and 15 m. If the length is increased by 20% and the breadth is increased by 30%, find the percentage increase in its area ?

(A) 54% (B) 56%

(C) 50% (D) 52%

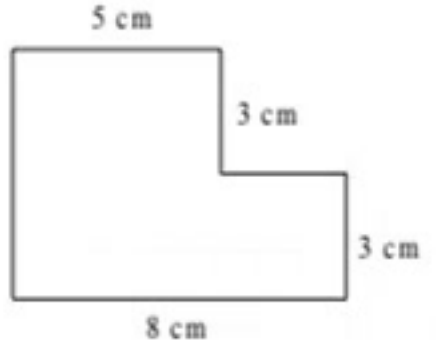
41. Find the perimeter of the following figure.



(A) 14 cm (B) 16 cm

(C) 18 cm (D) 20 cm

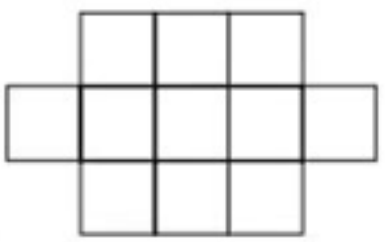
42. Find the area of the following figure.



(A) 25 cm^2 (B) 30 cm^2

(C) 36 cm^2 (D) 39 cm^2

43. Find the perimeter of the following figure. (size of each block-1 cm × 1 cm)



(A) 12 cm (B) 14 cm

(C) 15 cm (D) 16 cm

SOLUTIONS

1. (C) Area of plot = length × breadth
= 25 × 30
= 750 sq. m

2. (B) Perimeter of the towel
= 2 (length + breadth)
= 2 (125 + 60)
= 2 × 185
= 370 cm

3. (A) Side of square plot = $\frac{\text{Perimeter}}{4}$

$= \frac{260}{4} = 65$ m

4. (B) Area of carpet = Area of room's floor
$= 8 \times 7$
$= 56\ m^2$

5. (C) Distance covered in two rounds = 40 m

$\therefore$ Distance covered in on rounds = 20 m

$\because$ Perimeter of the plot = Distance covered in one round

$4 \times$ side = 20

side = $\frac{20}{4} = 5$ m

6. (D) Perimeter of the stool = 4 × side
= 4 × 60
= 240 cm

7. (A) Area of square field = side × side
= 75 × 75
= 5625 m^2

$\because$ Perimeter of rectangular field = Perimeter of square field

$2 (85 + b) = 4 \times 75$

$85 + b = 150$

$b = 150 - 85$

$b = 65$ m

$\therefore$ Area of rectangular field
= 85 × 65
= 5525 m^2

Therefore, Difference = 5625 – 5525
= 100 m^2

$\Rightarrow$ Required answer = Gopi, 100 m^2

8. (C) 1 round of square park = perimeter of the park

$\therefore$ 2 rounds of the park = 2 × perimeter
= 2 × 4 × side
= 8 × 60
= 480 m

9. (D) Perimeter of frame = Length of ribbon

2 (26 + breadth) = 78

breadth = 39 – 26 = 13 cm

10. (C) Breadth = $\frac{\text{Length}}{2} = \frac{50}{2}$

= 25 m

$\therefore$ Area of carpet = 25 × 50
= 1250 m^2

11. (A) Required distance = diagonal of rectangular field

$$= \sqrt{l^2 + b^2}$$

$$= \sqrt{(80)^2 + (60)^2}$$

$$= \sqrt{6400 + 3600}$$

$$= \sqrt{10000} = 100 \text{ m}$$

12. (C) Let, the other side be x m.

We have,

$(\text{length})^2 + (\text{breadth})^2 = (\text{diagonal})^2$

$$(15)^2 + x^2 = (17)^2$$

$$x^2 = 289 - 225$$

$$x = \sqrt{64} = 8 \text{ m}$$

$\therefore$ Area $= l \times b$

$$= 15 \times 8 = 120 \text{ m}^2$$

13. (B) Area of carpet = Area of room

$$= 14 \times 9$$

$$= 126 \text{ m}^2$$

$$\text{Length of the carpet} = \frac{\text{Area}}{\text{Width}}$$

$$= 126 \times \frac{4}{3}$$

$$= 168 \text{ m}$$

$\therefore$ Cost of carpeting = ₹ (168×8.50)

= ₹ 1428

14. (C) Perimeter of sheet $= 2(l + b)$

$\therefore$ $2(l + b) = 50$

$\Rightarrow$ $l + b = 25$

or $b = 25 - l$ ...(1)

Now, Area $= l \times b$

$\therefore$ $l \times b = 100$

$l(25 - l) = 100$ [from eq. (1)]

$\Rightarrow$ $25 = l - l^2 = 100$

$\Rightarrow$ $l^2 - 25l + 100 = 0$

$\Rightarrow$ $(l - 20)(l - 5) = 0$

$\therefore$ $l = 20$ m

and breadth, $b = 25 - 20 = 5$ m

Hence, diagonal $= \sqrt{l^2 + b^2}$

$$= \sqrt{(20)^2 + (5)^2}$$

$$= \sqrt{425}$$

$$= 20.6 \text{ m}$$

15. (C) Let the original length and breadth be l and b respectively.

After changes,

New length after 5% excess

$$= l + \frac{5l}{100}$$

$$= \frac{105}{100} l$$

New breadth after 6% deficit

$$= b - \frac{6b}{100} = \frac{94}{100} b$$

$\therefore$ Calculated area $= \frac{105\,l}{100} \times \frac{94}{100} b$

$$= \frac{987}{1000} l\,b$$

Error in area $= l\,b - \frac{987}{1000} l\,b$

$$= \frac{13}{1000} l\,b$$

$\therefore$ Error % $= \frac{13}{1000}\, l\,b \times \frac{1}{l\,b} \times 100$

$= 1.3\%$

16. (D) Area of the plot $= 95 \times 55$

$= 5225 \text{ m}^2$

Area of the plot excluding the path

$= (95 - 5) \times (55 - 5)$

$= 90 \times 50$

$= 4500 \text{ m}^2$

$\therefore$ Area of the path $= 5225 - 4500$

$= 725 \text{ m}^2$

Cost of gravelling the path

$$= 725 \times \frac{45}{100}$$

= ₹ 326.25

17. (C) Diagonal = 15

$\therefore$ $\sqrt{l^2 + b^2} = 15$

$\Rightarrow$ $l^2 + b^2 = (15)^2 = 225$

and area = 108

$l \times b = 108$

$\because$ $(l + b)^2 = l^2 + b^2 + 2\,l\,b$

$(l + b)^2 = 225 + 2 \times 108$

$(l + b) = \sqrt{225 + 216}$

$(l + b) = \sqrt{441} = 21$

Now, perimeter $= 2(l + b)$

$= 2 \times 21 = 42$ m

Hence, cost of fencing $= 42 \times 5$

= ₹ 210

18. (A) Perimeter of 1st square = 40 cm

$\therefore$ side of 1st square $= \frac{40}{4}$

$= 10$ cm

and, perimeter of 2nd square = 32 cm

$\therefore$ side of 2nd square $= \frac{32}{4}$

$= 8$ cm

According to question,

Area of 3rd square = Difference between area of 1st & 2nd squares.

$(\text{side})^2 = (10)^2 - (8)^2$

$(\text{side})^2 = 100 - 64 = 36$

$\therefore$ side $= \sqrt{36} = 6$ cm

$\therefore$ Perimeter of 3rd square $= 4 \times 6$

$= 24$ cm

19. (D) Required ratio

$$= \left[\frac{110}{100} \times \text{side} \times \frac{90}{100} \times \text{side}\right]$$

: (side × side)

$$= \frac{99}{100} : 1$$

or 99 : 100

20. (A) Area of square = 100 m^2

$\therefore$ side $= \sqrt{100} = 10$ m

Now, diagonal = Largest size of a bamboo placed in the square

$= \text{side}\ \sqrt{2}$

$= 10\sqrt{2}$

$= 10 \times 1.414$

$= 14.14$ m

21. (A) Area $= 3.78 \text{ m} \times 5.25 \text{ m}$

$= (378 \times 525) \text{ cm}^2$

Largest square tile = H.C.F (378, 525)

$\Rightarrow$ side = 21

$\therefore$ Area of a tile $= (\text{side})^2$

$= (21)^2$

$= 441 \text{ cm}^2$

Hence, No. of tiles $= \frac{378 \times 525}{441}$

$= 450$

22. (A) Using formula,

$$\text{Area of square} = \frac{1}{2} \times (\text{diagonal})^2$$

$$= \frac{1}{2} \times (3.8)^2$$

$$= \frac{14.44}{2} = 7.22 \text{ m}^2$$

23. (A) $\because$ $(\text{Diagonal})^2 = 2 \times \text{Area of square}$

Here, $\frac{d_1}{d_2} = \frac{2}{5}$

On squaring both sides,

$$\frac{d_1^2}{d_2^2} = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

$$\frac{2 \times A_1}{2 \times A_2} = \frac{4}{25}$$

or $A_1 : A_2 = 4 : 25$

24. (D) Required changes $= \left(2a + \frac{a^2}{100}\right)\%$

$= \left(2\times 25 + \frac{25\times 25}{100}\right)$

$= 50 + 6.25$

$= 56.25\%$

25. (A) $\because$ $\text{Area} = \frac{1}{2}(\text{diagonal})^2$

$\Rightarrow$ $A = \frac{1}{2} D^2$

$\because$ New digonal, $D_1 = (100 - 15)\%$ of D

$= \frac{85\,D}{100}$

$\therefore$ New Area, $A_1 = \frac{1}{2} \times \left(\frac{85\,D}{100}\right)^2$

$= \frac{7225}{20000} D^2$

$= \frac{289}{800} D^2$

So, Decrease in area $= \frac{D^2}{2} - \frac{289}{800} D^2$

$= \frac{111}{800} D^2$

Hence, required %

$= \frac{111\,D^2/800}{D^2/2} \times 100$

$= \frac{111\times 2\times 100}{800}$

$= 27.75\%$

26. (A) Let side of park = x meter

$\therefore$ Perimeter of the park = $4x$ meters ...(1)

and Area of path = $[(x+4)^2 - x^2]$

$= x^2 + 16 + 8x - x^2$

$= 8x + 16$

According to question,

$8x + 16 = 288$

$8x = 272$ or $x = 34$

Hence, required perimeter = $4x$

$= 4 \times 34 = 136$ m

27. (B) Let, the length of playground = $3x$ and the breadth of playground = $2x$

According to question,

$\text{Area} = 38400$

$l \times b = 38400$

$3x \times 2x = 38400$

$x^2 = 6400$

$x = \sqrt{6400} = 80$

$\therefore$ Perimeter $= 2(3x + 2x)$

$= 10x = 10 \times 80$

$= 800$ m

28. (B) According to question,

Perimeter of square = Perimeter of rectangle

$4 \times \text{side} = 2(l + b)$

$2 \times \text{side} = (25 + 15)$

$\text{side} = \frac{40}{2} = 20$ cm

29. (D) Side of square corromboard = 40 cm

According to question,

Covered area by corromboard

= area of square

$= (\text{side})^2$

$= (40)^2 = 1600$ cm^2

30. (C) Area of plywood = 44100 cm^2

$l \times b = 44100$

$l \times 210 = 44100$

$l = \frac{44100}{210}$

$= 210$ cm

31. (D) Let, side of a square= x cm

$\therefore$ Area of the square= x^2 cm^2

Now, according to question

New side of the square $= \frac{x}{3}$ cm

$\therefore$ New area of the square $= \left(\frac{x}{3}\right)^2$

$= \frac{x^2}{9}$ cm^2

$\therefore$ New area $= \frac{1}{9} \times$ original area

32. (A) Side of square = $\sqrt{2}$ cm

$\because$ Diagonal of square = side $\times \sqrt{2}$

$= \sqrt{2} \times \sqrt{2}$

$= 2$ cm

33. (A) Let, the Length and breadth of a rectangle are x and y respectively. Where $(x > y)$

Given, diagonal = $3y$

$\sqrt{l^2 + b^2} = 3y$

$\sqrt{x^2 + y^2} = 3y$

$x^2 = 9y^2 - y^2 = 8y^2$

$x = 2\sqrt{2}\ y$

$\therefore$ $x : y = 2\sqrt{2} : 1$

34. (D) Let, length of reatangle = $5x$ meters

Breadth of rectangle = $4x$ meters

According to question,

$\text{Area} = l \times b$

$\therefore$ $5x \times 4x = 500$

$x^2 = 25$

or $x = 5$ m

$\therefore$ Perimeter $= 2(5x + 4x)$

$= 18x$

$= 90$ m

35. (A) Let, the Length of rectangular plot = x metres

and the breadth of rectangular plot = y metres

Given,

$\text{Area} = l \times b \Rightarrow xy = 150$ ...(1)

and Perimeter $= 2(l + b)$

$\Rightarrow$ $2(x + y) = 50$

or $x + y = 25$ ...(2)

$\because$ $(x-y)^2 = (x+y)^2 - 4xy$

$(x-y)^2 = (25)^2 - 4 \times 150$

$(x-y)^2 = 625 - 600$

$x - y = \sqrt{25} = 5$ ...(3)

On solving eq. (2) & (3)

$x = 15$, $y = 10$

36. (B) Given, $P_1 = 24$ cm and $P_2 = 32$ cm

$4 \times a_1 = 24$ cm and $4 \times a_2 = 32$ cm

$a_1 = 6$ cm and $a_2 = 8$ cm

According to question,

Area of 3rd square $= A_1 + A_2$

$(\text{side})^2 = a_1^2 + a_2^2$

$(a_3)^2 = (6)^2 + (8)^2$

$(a_3)^2 = 100$

or $a_3 = 10$ cm

$\therefore$ Perimeter of 3rd square $= 4 \times a_3$

$= 4 \times 10$

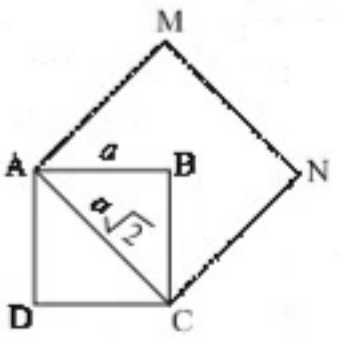
$= 40$ cm

37. (B) Let side of square ABCD = a unit

$\therefore$ Diagonal of square ABCD

$= a\sqrt{2}$ unit

Area of square ABCD = st ...(1)



Now, side of square AMNC = Diagonal of square ABCD

$A_1 = a\sqrt{2}$ unit

$\therefore$ Area of square AMNC

$$= \left(a\sqrt{2}\right)^2$$

$A_2 = 2a^2$ sq. unit

$A_1 : A_2 = a^2 : 2a^2$

$= 1 : 2$

38. (B) AB = BC = CD = AD

= 110 m

Area of II ABCD = 110 × 110

= 12100 m^2

Area of II LMNP

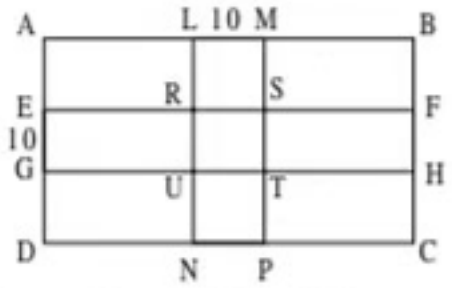
= Area of II EFGH

= 10 × 110

= 1100 m^2

Area of II RSTU

= 10 × 10 = 100 m^2



$\therefore$ Area of the remaining field except cross roads

= ar (ABCD) – ar (LMNP) – ar (EFGH) + ar (RSTU)

= 12100 – 1100 – 1100 + 100

= 12200 – 2200

= 10000 m^2

39. (D) % Increased $= \left(2a + \frac{a^2}{100}\right)\%$

$= \left(20 + \frac{100}{100}\right)\%$

= 21%

40. (B) Original area of rectangle = 20 × 15

= 300 m^2

According to question,

New length $= 20 \times \frac{120}{100} = 24$ m

New breadth $= 15 \times \frac{130}{100} = 19.5$ m

$\therefore$ New area = 24 × 19.5

= 468 m^2

$\therefore$ Required % $= \frac{468 - 300}{300} \times 100$

$= \frac{168}{3} = 56\%$

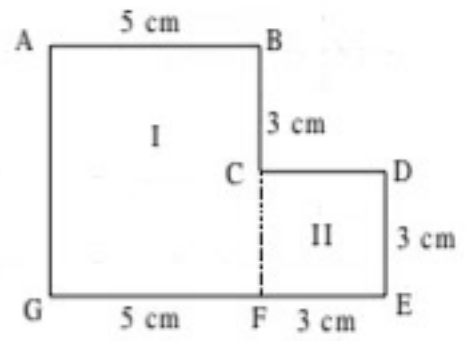
41. (D) Peimeter = (2 + 1 + 3 + 1 + 2 + 1 + 2 + 1 + 3 + 1 + 2 + 1) cm

= 20 cm

42. (D) Area of II ABCD = $l \times b$

I = 5 × 6

I = 30 cm^2



Area of II CDEF = $l \times b$

II = 3 × 3

II = 9 cm^2

$\therefore$ Total area = I + II

= 30 + 9 = 39 cm^2

43. (D) $\because$ size of each block = 1 cm × 1 cm

i.e. side of each block = 1 cm

$\therefore$ Perimeter of the figure = 1 × 16

= 16 cm

Chapter 17 Volume and Surface Area

1. **Volume**—An amount of space occupied by an object is called its volume.

   1 cu. meter $= 1\text{ m} \times 1\text{ m} \times 1\text{ m} = (\text{meter})^3$
   $= 10,00000$ cu. cm

   and 1 Liter = 1000 cu. cm

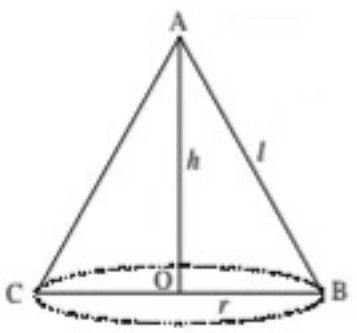
2. **Total Surface Area**—the sum of all surfaces of the three-dimensional object is called total surface area.

3. **Cuboid's Formulas**—
   - Volume = Length × Breadth × Height = lbh
   - Total Surface Area (TSA) = $2(lb + bh + lh)$
   - Diagonal $= \sqrt{(\text{Length})^2 + (\text{Breadth})^2 + (\text{Height})^2}$
     $= \sqrt{l^2 + b^2 + h^2}$

4. **Cube's Formulas**—
   If a is the side of cube, then
   - Volume $= (\text{side})^3 = a^3$
   - TSA $= 6(\text{side})^2 = 6a^2$
   - Diagonal $= \sqrt{3} \times \text{side} = \sqrt{3}\,a$

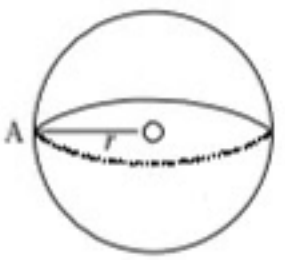
5. **Right Circular Cylinder's Formulas**—
   If r and h be the radius of the base and height respectively, then
   - Volume = Base area × height $= \pi r^2 h$
   - Curved surface area $= 2\pi r h$
   - Total surface area $= 2\pi r^2 + 2\pi r h$
     $= 2\pi r(r + h)$

6. **Cone**—



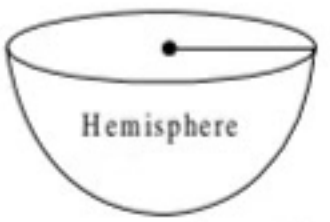
   In the figure,
   Radius of cone = OB = r
   Straight height = AO = h
   Slant height = AB = l
   - $l = \sqrt{h^2 + r^2}$;
   - $h = \sqrt{l^2 - r^2}$;
   - $r = \sqrt{l^2 - h^2}$;
   - Volume $= \frac{1}{3}\pi r^2 h$
   - Total surface area $= \pi r l + \pi r^2$
     $= \pi r(l + r)$

7. **Sphere**—



   In the figure, Radius of the sphere = AO = r
   - Volume $= \frac{4}{3}\pi r^3$
   - Total surface area $= 4\pi r^2$

8. **Hemisphere**—



   - Curved surface area $= 2\pi r^2$
   - Total surface area $= 3\pi r^2$
   - Volume $= \frac{2}{3}\pi r^3$

Important Questions

1. A cuboidal box is 13 cm lengthy, 11 cm wide and 9 cm high. Side of a cubical box is 12 cm. Anirudh wants to pack 3060 boxes of 1 cm side in these boxes. Find the number of unpacked boxes.

(A) 30 (B) 45
(C) 15 (D) 28

2. Ratio of volumes of two cubes is 27 : 64. Find the ratio between their total surface areas. **(NCERT)**

(A) 3 : 8
(B) 3 : 4
(C) 9 : 16
(D) 27 : 64

3. Ratio between the length, breadth and height of a cuboid is 6 : 5 : 4. If its total surface area is 5328 sq. cm, then find the dimensions of the cuboid.

(A) 20 cm, 22 cm, 23 cm

(B) 38 cm, 25 cm, 23 cm

(C) 36 cm, 30 cm, 24 cm

(D) 22 cm, 23 cm, 21 cm

4. Areas of three faces of a cuboid are x, y, and z resepctively. Volume of the cuboid will be–

(A) xyz (B) $x^2y^2z^2$

(C) $\sqrt{xyz}$ (D) $3\sqrt{xyz}$

5. A solid cuboid is formed by joining 24 cubes of 1 cm side. If perimeter of the base of the cuboid is 12 cm, find the height of the cuboid.

(A) 1 cm (B) 2 cm

(C) 3 cm (D) 4 cm

6. Dimesnions (in cm) of four cuboids are given below– **(NCERT)**

	Length	Width	Height
(i)	20	25	18
(ii)	23	20	20
(iii)	20	22	21
(iv)	24	20	19

Which of the following boxes will be used to pack wood cubes of 18440 cu. cm.

(A) (i) and (iii) (B) (ii) and (iii)

(C) (ii) and (iv) (D) (i) and (ii)

7. Juice packet of 1 liter is of cuboidal shape and its base is a square with dimension 8 × 8 cm. Find the height (in cm) of the juice packet.

(A) 20 (B) 22

(C) 16 (D) 18

8. The length, width, and height of a water tank are 11 m, 10 m, and 6 m respectively. The tank is filled with water up to 6m height. Find the empty portion of the water tank. **(NCERT)**

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$

(C) $\frac{1}{6}$ (D) $\frac{2}{3}$

9. Two cubes have their volumes in the ratio 1 : 64. Find the ratio of their surface areas.

(A) 1 : 4 (B) 4 : 1

(C) 16 : 1 (D) 1 : 16

10. If each edge of a cube is increased by 50%, find the percentage increase in its surface area.

(A) 50% (B) 100%

(C) 125% (D) 150%

11. There solid cubes of sides 8 cm, 6 cm and 1 cm are joined together to form a new cube. Find the side of new cube.

(A) 8 m (B) 9 m

(C) 10 m (D) 10 m

12. A cube of edge 20 cm is immersed fully in a rectangular water tank. If the dimensions of base of tank are 25 cm × 20 cm, then find the rise in water level of the water tank.

(A) 16 cm (B) 20 cm

(C) 25 cm (D) No change

13. A ractangular dice (6 cm × 12 cm × 18 cm) is cut up into an exact number of equal cubes. Find the least possible number of cubes.

(A) 30 (B) 18

(C) 12 (D) 6

14. The surface area of a cubical box is 2166 sq. cm. Find its volume.

(A) 5689 cm^3 (B) 8569 cm^3

(C) 6859 cm^3 (D) None of these

15. The diagonal of a cube is $5\sqrt{3}$ cm. Find its volume.

(A) 25 cm^3 (B) 125 cm^3

(C) 75 cm^3 (D) 225 cm^3

16. The dimensions of an open tank are 50 m, 40 m and 10 m. Its thickness is 1 m. If 1 cubic meter of material used in the tank weighs 40gm, find the weight of the tank.

(A) 1433.60 kg (B) 1200.25 kg

(C) 1096.48 kg (D) 980.50 kg

17. Find the surface area of a cuboid 16 m long, 12 m wide and 5 m high.

(A) 545 m^2 (B) 456 m^2

(C) 564 m^2 (D) 664 m^2

18. An auditorium is 12 meters long, 9 meters wide, and 8 meters high. Find the length of its diagonal.

(A) 15 m (B) 16 m

(C) 17 m (D) 18 m

19. Find the number of bricks, each measuring 24 cm × 12 cm × 8 cm, required to construct a wall 24 m long, 8 m high and 60 cm thick, it 10% of the wall is filled with mortar.

(A) 54000 (B) 45000

(C) 32000 (D) 24000

20. A rectangular sheet of paper 10 cm long and 8 cm wide has squares of side 2 cm cut from each of its corners. The sheet is then folded to form a tray of depth 2 cm. Find the volume of this tray.

(A) 48 cm^3 (B) 84 cm^3

(C) 26 cm^3 (D) 62 cm^3

21. Find the volume of cube whose edge is 3.5 m.

(A) 45.625 m^3 (B) 54.765 m^3

(C) 25.250 m^3 (D) 42.875 m^3

22. Find the length of a wooden plank of width 60 cm, thickness 12 cm and volume 12240 cm^3.

(A) 15 cm (B) 16 cm

(C) 17 cm (D) 18 cm

23. A cuboidal water tank is 5 m long, 3 m wide and 3.5 m deep. How many liters of water can it hold ?

(A) 47.5×10^3 L (B) 52.5×10^3 L

(C) 65.4×10^3 L (D) 35.7×10^3 L

24. What can be possible volume of a box to carry 150 books 25 cm long, 16 cm wide and 2.5 cm thick ?

(A) 150,000 cm^3 (B) 120000 cm^3

(C) 180,000 cm^3 (D) 140000 cm^3

25. Find the maximum length of a rod that can be kept in a rectangular box of dimensions 5 m × 4 m × $2\sqrt{10}$ m.

(A) 15 m (B) 14 m

(C) 11 m (D) 9 m

SOLUTIONS

1. (B) Number of cubes to be packed into the cuboidal box

$= \frac{13\times11\times9}{1\times1\times1} = 1287$

Number of packed cubes into the cuboidal box

$= 12 \times 12 \times 12$

$= 1728$

∴ Number of un-packed cubes into the cuboidal box = 3060 – (1728 + 1287) = 45

2. (C) Ratio between volume of cubes

$$\frac{a_1^3}{a_2^3} = \frac{27}{64} \Rightarrow \frac{a_1}{a_2} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{3}{4} \Rightarrow \frac{a_1^2}{a_2^2} = \frac{9}{16}$$

$\therefore$ Ratio in areas = 9 : 16

3. (C) Let, length = $6x$, Breadth = $5x$, and height = $4x$

$\because$ Total surface area = 5328 cm^2

$$2(lb + bh + hl) = 5328$$
$$\Rightarrow lb + bh + hl = 2664$$
$$\Rightarrow 30x^2 + 20x^2 + 24x^2 = 2664$$
$$\Rightarrow 74x^2 = 2664$$
$$\Rightarrow x^2 = 36$$
$$\Rightarrow x = 6$$

$\therefore$ Length= $6x = 6 \times 6 = 36$ cm

Breadth = $5x = 5 \times 6 = 30$ cm

Height = $4x = 4 \times 6 = 24$ cm

4. (C) Let, the length, width and height of a cuboid are a, b and c. So, area of the three faces will be,

$$x = ab;\ y = bc;\ z = ca$$
$$\Rightarrow x\,y\,z = ab \times bc \times ca = a^2b^2c^2$$
$$\Rightarrow \sqrt{xyz} = abc$$

$\Rightarrow$ Volume of cuboid

$$= abc = \sqrt{xyz}$$

5. (C) Let, l, b, and h be the length, width, and height of a cuboid.

$\because$ Volume of cuboid = 24 (volume of 24 cubes)

$$lbh = 24 \quad \text{... (i)}$$

$\because$ $2(l + b) = 12$

$$l + b = 6 \quad \text{... (ii)}$$

$\because$ Possible combination

$l = 4$ and $b = 2$

$\therefore$ $lbh = 24$

$$h = \frac{24}{l \times b} = \frac{24}{4 \times 2} = 3 \text{ cm}$$

6. (B) Different volumes of all the dimensions are as follows–

Volume for 1st dimensions

$= 20 \times 25 \times 18 = 9000$

Volume for 2nd dimensions

$= 23 \times 20 \times 20 = 9200$

Volume for 3rd dimensions

$= 20 \times 22 \times 21 = 9240$

Volume for 4th dimensions

$= 24 \times 20 \times 19 = 9120$

$\therefore$ Volume of (2nd + 3rd)

$= 9200 + 9240$

$= 18440$ cu. cm

7. (C) $\because$ 1 liter = 1000 cu. cm

$\therefore$ Volume of Juice packet

$= 1000$

$\Rightarrow 8 \times 8 \times h$ (let) $= 1000$

$$\Rightarrow h = \frac{1000}{64} = 15{\cdot}625$$

$\cong 16$ cm

8. (B) Volume of water tank

$= 11 \times 10 \times 9$

$= 990$ cube m.

Volume of empty portion

$= 11 \times 10 \times (9 - 6)$

$= 110 \times 3$

$= 330$ cu. cm

$\therefore$Required portion $= \frac{330}{990} = \frac{1}{3}$ part

9. (D) Let their sides be m and n. We have,

$$\frac{V_1}{V_2} = \frac{1}{64}$$

$$\frac{m^3}{n^3} = \frac{1}{64} \Rightarrow \frac{m}{n} = \sqrt[3]{\frac{1}{64}} = \frac{1}{4}$$

$\therefore$ Ratio of their surface areas

$$= \frac{6m^2}{6n^2}$$

$$= \frac{m^2}{n^2} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

10. (C) Increase percent in surface area

$$= \left(2a + \frac{a^2}{100}\right)\%$$

$$= 2 \times 50 + \frac{50 \times 50}{100}$$

$= 100 + 25 = 125\%$

11. (B) Volume of new cube

$= V_1 + V_2 + V_3$

$V = (8)^3 + (6)^3 + (1)^3$

$V = 512 + 216 + 1$

$V = 729\ m^3$

Now, $(\text{side})^3 = 729$

$\text{side} = \sqrt[3]{729} = 9$ m

12. (A) Increase in volume = Volume of cube

$= 20 \times 20 \times 20$

$= 8000\ cm^3$

$\therefore$ Rise in water level

$$= \frac{\text{Increase in volume}}{\text{Base area}}$$

$$= \frac{8000}{25 \times 20} = \frac{800}{50} = 16 \text{ cm}$$

13. (D) Volume of dice $= 6 \times 12 \times 18$

$= 1296\ cm^3$

and, side of the largest cube

= H.C.F (6, 12, 18)

$= 6$

$\therefore$ Volume of 1 cube $= (6)^3$

$= 216\ cm^3$

So, number of cubes $= \frac{1296}{216}$

$= 6$

14. (C) Let, the edge of the box is x cm.

We have,

Surface area $= 2166\ cm^2$

$6x^2 = 2166$

$x^2 = 361$

or $x = 19$ cm

$\therefore$ Volume of the box $= (x)^3$

$= (19)^3$

$= 6859\ cm^3$

15. (B) Diagonal of a cube = $a\sqrt{3}$ (where a-side of cube)

$\therefore$ $a\sqrt{3} = 5\sqrt{3}$

or $a = 5$ cm

So, Volume of the cube= a^3

$= (5)^3$

$= 125\ cm^3$

16. (A) volume of the material used in the tank

= External volume – Internal volume

$= 50 \times 40 \times 10 - 48 \times 38 \times 9$

$= 20000 - 16416$

$= 3584\ m^3$

Now, weight of the material

$$= \frac{3584 \times 40}{100} \text{ kg}$$

$= 1433.6$ kg

17. (D) Surface area of cuboid = 2 $(lb + bh + hl)$

$= 2(16 \times 12 + 12 \times 5 + 5 \times 16)$

$= (192 + 60 + 80)$

$= 2 \times 332 = 664\ m^2$

18. (C) Auditorium's diagonal

$$= \sqrt{l^2 + b^2 + h^2}$$

Here, $l = 12$ m, $b = 9$ m and $h = 8$ m

$\therefore$ Diagonal's length

$$= \sqrt{(12)^2 + (9)^2 + (8)^2}$$
$$= \sqrt{144 + 81 + 64}$$
$$= \sqrt{289} = 17 \text{ m}$$

19. (B) Volume of 1 brick $= l \times b \times h$

$= 24 \times 12 \times 8$

$= 2304\ cm^3$

Volume of the wall $= l \times b \times h$

$= 2400 \times 800 \times 60$

$= 1152 \times 10^5\ cm^3$

$\because$ Volume of all bricks

= 90% of volume's wall

$$= \frac{90}{100} \times 1152 \times 10^5$$

$= 103680 \times 10^3\ cm^3$

$$\therefore \text{No. of bricks} = \frac{103680 \times 10^3}{2304}$$

$= 45 \times 10^3$ or 45000

20. (A) According to question,

Length of the tray $= 10 - 2 \times 2$

$l = 10 - 4$

$= 6$ cm

Breadth of the tray $= 8 - 2 \times 2$

$b = 8 - 4$

$= 4$ cm

$\therefore$ Depth of the tray $(h) = 2$ cm

So, Volume of the tray $= l \times b \times h$

$= 6 \times 4 \times 2$

$= 48\ cm^3$

21. (D) Volume of cube $= (\text{edge})^3$

$= (3.5)^3$

$= 42.875\ m^3$

22. (C) Volume of wooden plank $= l \times b \times h$

$\therefore \quad l \times b \times h = 12240\ cm^3$ (given)

$l \times 60 \times 12 = 12240$

$$l = \frac{12240}{60 \times 12}$$

$l = 17$ cm

23. (B) Volume of tank $= l \times b \times h$

$= 5 \times 3 \times 3.5$

$= 52.5\ m^3$

$\because$ 1 meter $= 10^3$ liters

$\therefore \quad 52.5\ m^3 = 52.5 \times 10^3$ liters

24. (A) Volume of 1 book $= l \times b \times h$

$= 25 \times 16 \times 2.5$

$= 1000\ cm^3$

$\therefore$ Volume of box to carry such 150 books

= 150 × volume of 1 book

$= 150 \times 1000$

$= 150{,}000\ cm^3$

25. (D) Maximum length of rod = Diagonal of box

$$= \sqrt{l^2 + b^2 + h^2}$$

$$= \sqrt{(5)^2 + (4)^2 + \left(2\sqrt{10}\right)^2}$$

$$= \sqrt{25 + 16 + 40}$$

$= \sqrt{81} = 9$ m

Chapter 18

Statistics

1. Introduction

The number, multipled by an integer (not a fraction) is called multiples. Normally, the skip counting or "count by" numbers are most often called multiples.

As we see every day, a lot of information in the form of facts, numerical figures, tables, graphs etc. come across. These are provided by newspapers, televisions, magazines and other means of communication. These may relate to cricket batting or bowling averages, profits of a company, temperatures of cities, expenditures in various sectors of a five year plan, polling results, and so on. These facts or figures, which are numerical or otherwise, collected with a definite purpose are called *data*. Data is the plural form of the Latin word *datum*. Every part of our lives utilizes data in one form or the other. So, it becomes essential for us to know how to extract meaningful information from such data. This extraction of meaningful information is studied in a branch of mathematics called *Statistics*.

The word 'statistics' is used as a singular noun, meaning the subject which deals with the collection, presentation, analysis of data as well as drawing of meaningful conclusions from the data.

2. Collection of Data

Let us take an example and see how the data is to be collected.

Divide the students of your class into four groups. Allot each group the work of collecting one of the following kinds of data :

- Weights of 20 students of your class.
- Number of absentees in each day in your class for a month.
- Number of members in the families of your classmates.
- Heights of 15 students of your school.

Let us move to the results students have gathered. How did they collect their data in each group ?

Case 1: Did they collect the information from each and every student, house or person concerned for obtaining the information ?

Case 2: Did they get the information from some source like available school records ?

In the first case, when the information was collected by the investigator herself or himself with a definite objective in her or his mind, the data obtained is called *primary data*. In the second case, when the information was gathered from a source which already had the information stored, the data obtained is called *secondary data*.

3. Presentation of Data

Presentation of data plays a very important role after collecting the data. Let us take few examples and learn how to represent the collected data in different conditions.

Example 1: Consider the marks obtained by 10 students in a mathematics test as given below :

40 35 88 75 62 55 36 95 73 60

The data in this form is called *raw data*.

By looking at it in this form, can you find the highest and the lowest marks ?

Did it take you some time to search for the maximum and minimum scores? Wouldn't it be less time consuming if these scores were arranged in ascending or descending order? So let us arrange the marks in ascending order as

35 36 40 55 60 62 73 75 88 95

Now, we can clearly see that the lowest marks are 35 and the highest marks are 95. The difference of the highest and the lowest values in the data is called the *range* of the data. So, the range in this case is 95 – 35 = 60.

Example 2: Consider the marks obtained (out of 100 marks) by 50 students of Class X of a school :

10 70 15 2 23 59 56 27 89 75
99 22 20 22 50 89 56 70 56 89
75 65 85 22 3 12 41 87 82 72
50 22 87 50 89 28 89 50 40 36
40 30 28 87 81 90 22 15 30 35

The number of students who have obtained a certain number of marks is called the *frequency* of those marks. For instance, 5 students got 89 marks. So the frequency of 89 marks is 5. To make the data more easily understandable, we write it in a table, as given below :

Marks	Number of students (frequency)	Marks	Number of students (frequency)	Marks	Number of students (frequency)
2	1	30	2	72	1
3	1	35	1	75	2
10	1	36	1	81	1

Marks	Number of students (frequency)	Marks	Number of students (frequency)	Marks	Number of students (frequency)
12	1	40	2	82	1
15	2	41	1	85	1
20	1	50	4	87	3
22	5	56	3	89	5
23	1	59	1	90	1
27	1	65	1	99	1
28	2	70	2	**Total**	**Õf = 50**

The Table shown above is called *an ungrouped frequency distribution table,* or simply a *frequency distribution table*.

To present such a large amount of data so that a reader can make sense of it easily, we condense it into groups like 0-10, 10-20, …, 90-100 (since our data is from 2 to 99). These groupings are called 'classes' or 'class-intervals', and their size is called the *class-size* or *class width*, which is 10 in this case. In each of these classes, the least number is called the *lower class limit* and the greatest number is called the *upper class limit, e.g.*, in 10-20, 10 is the 'lower class limit' and 20 is the 'upper class limit'.

Using tally marks, the data above can be condensed in tabular form as follows :

Class Interval (Maximum100)	Tally Marks	Frequency (f)
0 – 10	II	2
10 – 20	IIII	4
20 – 30	⧹⧸ ⧹⧸	10
30 – 40	IIII	4
40 – 50	III	3
50 – 60	⧹⧸ III	8
60 – 70	I	1
70 – 80	⧹⧸	5
80 – 90	⧹⧸ ⧹⧸ I	11
90 – 100	II	2
Total		**50**

Presenting data in this form simplifies and condenses data and enables us to observe certain important features at a glance. This is called *a grouped frequency distribution table*.

By convention, we consider 10 in the class 10 – 20 and not in 0 – 10. Similarly, 40 is considered in 40 – 50 and not in 30 – 40.

4. Graphical Representation of Data

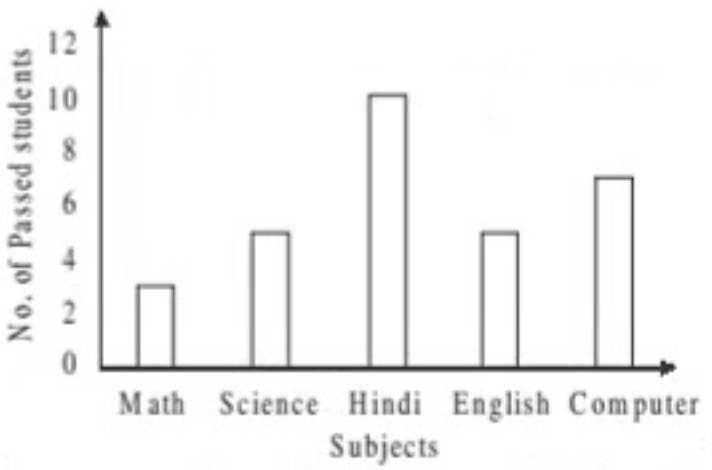
As we have already been discussed the representation of data by tables. Now, we will try to learn another representation of data, *i.e.*, the graphical representation. Usually comparisons among the individual items are best shown by means of graphs. The representation then becomes easier to understand than the actual data. Here, we shall study the following graphical representations :

I. **Bar Graphs—**A bar graph is a pictorial representation of data in which usually bars of uniform width are drawn with equal spacing between them on one axis (say, the *x*-axis), depicting the variable. The values of the variable are shown on the other axis (say, the *y*-axis) and the heights of the bars depend on the values of the variable.

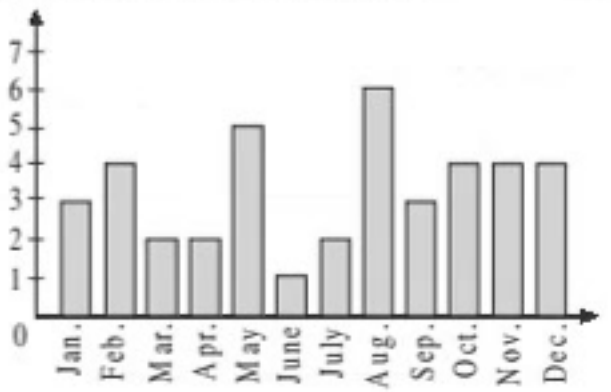
Example 1: Represent the following table by bar graph :

Subjects	No. of Passed Students
Math	3
Science	5
Hindi	10
English	5
Computer	7

Solution :



Example 2: In a company, 40 employees were asked about the months of their joining and the following graph was prepared for the data so obtained :



Observe the bar graph given above and answer the following questions :

(i) How many employees were joined in the month of May ?

(ii) In which month were the maximum number of employees joined ?

Solution : Note that the variable here is the 'month of joining', and the value of the variable is the 'Number of employees joined'.

(i) 5 employees were joined in the month of May.

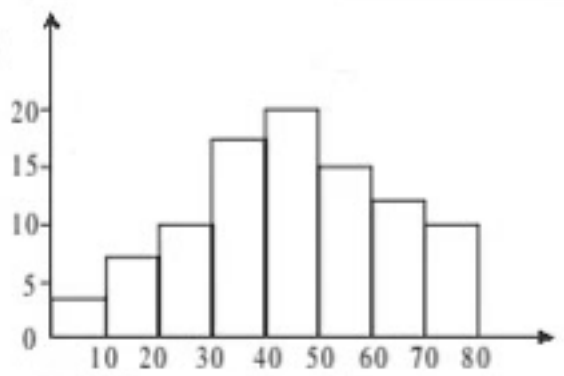
(ii) The Maximum number of employees were joined in the month of August.

II. Histogram—Such types of graphs are the form of representation like the bar graph, but it is used for continuous class intervals. This graph is made according to a frequency table. In such graphs the class intervals are the same and are marked on the X-axis. On each class interval, a rectangle is formed whose area represents the frequency of that class interval. The first class interval takes on the X-axis whose height is equal to the corresponding frequency. The second class interval on the X-axis adjacent to it is taken and the second rectangle is taken at a height equal to the corresponding frequency of the second class interval above it. In this way, we create the rectangles as much as the total number of class intervals. This type of graph is called histogram.

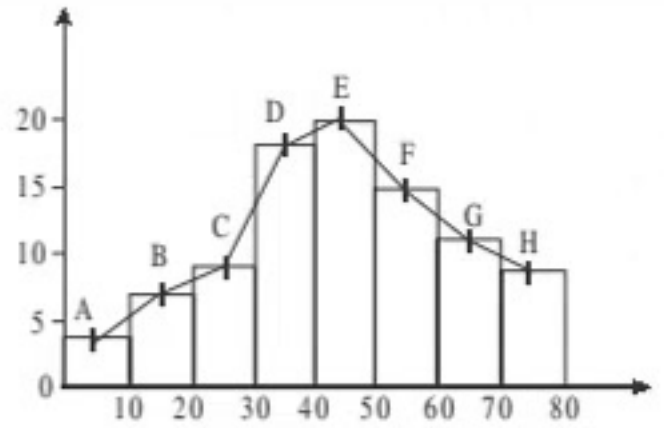
Example : Draw the histogram with the help of the following table :

Class Interval	Frequency
0 – 10	3
10 – 20	7
20 – 30	10
30 – 40	17
40 – 50	20
50 – 60	15
60 – 70	12
70 – 80	10

Solution :



III. Frequency Polygon—A polygon is a shape that has many sides. A frequency-polygon is a shape through which the values of different frequencies of different class intervals of a single frequency distribution are detected through multiple sides. Consider the histogram represented by above example. Let us join the mid-points of the upper sides of the adjacent rectangles of this histogram by means of line segments. Let us call these mid-points A, B, C, D, E, F, G and H. When joined by line segments, we obtain the figure ABCDEFGH (see figure) which is called the frequency polygon corresponding to the data.



Frequency polygons can also be drawn independently without drawing histograms. For this, we require the mid-points of the class-intervals used in the data. These mid-points of the class-intervals are called **class-marks**. To find the class-mark of a class interval, we find the sum of the upper limit and lower limit of a class and divide it by 2. We have,

$$\text{Class – marks} = \frac{\text{Upper Limit} + \text{Lower Limit}}{2}$$

IV. Pie Chart—When there is something given in the information or score percentage, then we make a Pie-chart to display those percentage marks. Apart from this, when the distribution of population or data is done on the basis of particular characteristics and it is possible to divide the population into several parts, then in such a situation, the use of the circle is very effective, attractive and widespread. This picture is circular so it is called a Pie-chart. A circle has a total of 360 degrees. If these 360 are considered equal to 100%, then the document can easily display percentage marks. The percentage is first converted to degrees when making a Pie-chart. Its formula is

$$\text{Degree} = \frac{360}{10} \times \text{Percentage}$$

Example : Navodaya class 9th following is the result of exam 2019-20. Display the results with a pie-chart.

Division	Percentage
First division	20
Second division	40
Third division	30
Fail	10

Solution : The percentage is first converted to degrees when making a pie-chart. Its formula is

$$\text{Degree} = \frac{360}{10} \times \text{Percentage}$$

$\because$ $100\% = 360^\circ$

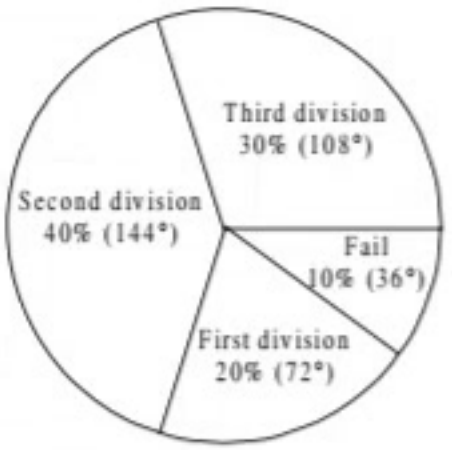
$$20\% = 360^\circ \times \frac{20}{100} = 72^\circ \quad \text{(First division)}$$

$$40\% = 360^\circ \times \frac{40}{100} = 144^\circ \quad \text{(Second division)}$$

$$30\% = 360^\circ \times \frac{30}{100} = 108^\circ \quad \text{(Third division)}$$

$$10\% = 360^\circ \times \frac{10}{10} = 36^\circ \quad \text{(Fail)}$$

By taking the results in degrees, we can make a pie-chart for further calculations.



5. Measures of Central Tendency

A number that represents all data will be neither a group with a minimum value nor a maximum value. Of course, that number will be the number between or around the group. This means that all the statistics of the group are distributed around the statistical mean. This trend of these figures is also called the central trend and this mean is the central measure or measure of central tendency. Therefore, the tendency of all the data of a group to be found near one of the figures of the group is called the central tendency. There are many types of measures of central tendency. Of these, we will study the following three—

I. Arithmetic Mean or Average

Average value in arithmetic is called arithmetic mean in statistics. Therefore, a value which is obtained by dividing the sum of the values of all terms by the number of terms.

Example : If the weight (in kg) of five persons are 50, 54, 53, 52 and 56, then find their arithmetic mean.

Solution : Average weight = Sum of all observations/Total number of observations

$$= (50 + 54 + 53 + 52 + 56) / 5$$
$$= 265 / 5$$
$$= 53 \text{ kg}$$

Hence, the arithmetic mean of all the weights = 53 kg

(i) Mean of Ungrouped Data

- **First method—**This method is also called direct method. If $x_1, x_2, x_3, \ldots, x_n$ terms are given, then the arithmetic mean is calculated by the following formula :

$$\text{Arithmetic mean, } M = \frac{\Sigma x}{n}$$

$$\text{Where, } \Sigma x = x_1 + x_2 + x_3 + \ldots + x_n$$

Example : In an examination, the obtained marks of students of 9th class students were following—

26, 20, 30, 36, 21, 38, 40, 22, 37

Find the arithmetic mean of their obtained marks.

Solution : We have,

$$\Sigma x = x_1 + x_2 + x_3 + \ldots + x_n$$
$$= 26 + 20 + 30 + 36 + 21 + 38 + 40 + 22 + 37$$
$$= 270$$

Total number of students, $n = 9$

$$\text{Arithmetic mean, } M = \frac{\Sigma x}{n} = \frac{270}{9} = 30 \text{ marks}$$

- **Shortcut method—**This method is also called indirect method. We use this method when the number of terms is larger with large numeral value.

$$\text{Arithmetic mean, } M = A + \frac{\Sigma d}{n}$$

Where, A = Assumed mean

Example : Five students have got 80, 82, 85, 90 and 103 marks respectively out of 200 marks. Find the arithmetic mean of these marks with the shortest method.

Solution : We have,

Obtained marks (x)	Assumed mean (A)	Deviation using Assumed mean (d)
80	85	80–85 = – 5
82		82–85 = – 3
85		85–85 = – 0
90		90–85 = 5
103		103–85 = 18
$n = 5$		$\Sigma d = 15$

$$M = A + \frac{\Sigma d}{n}$$
$$= 85 + \frac{15}{5} = 88$$

- **When Frequency is given—**If $x_1, x_2, x_3, \ldots, x_n$ terms with corresponding frequencies $f_1, f_2, \ldots, f_n$ are given, then the arithmetic mean is calculated by the following formula :

$$\text{Arithmetic mean } M = \frac{\Sigma fx}{n}$$

Where, $n = \Sigma f$

Example : After surveying 50 families, the following figures of living rooms with each family were obtained. Find the arithmetic mean.

No. of rooms (x)	No. of families (f)
1	12
2	24
3	8
4	6

Solution : We have

No. of rooms (x)	No. of families (f)	$f \times x$
1	12	$12 \times 1 = 12$
2	24	$24 \times 2 = 48$
3	8	$8 \times 3 = 24$
4	6	$6 \times 4 = 24$
	$n = \Sigma f = 50$	$\Sigma fx = 108$

$$\text{Now, mean} = M = \frac{\Sigma fx}{n} = \frac{108}{50} = 2.16$$

(ii) Mean of Grouped Data—Follow the following steps :

- Find the mean of all the class intervals.
- Multiplying the mean of each class interval by the corresponding frequency of that class, we find the sum of the product.

- The obtained sum is divided by the number of terms (the sum of the frequencies). This is the required mean.

If mean of the class interval = x, corresponding frequency = f, sum of frequencies = $n = \Sigma f$, then the arithmetic mean will be –

$$M = \frac{\Sigma fx}{n}$$

Example : The distribution of daily wages of workers in a private establishment is as follows—

Daily wages	No. of workers
3 – 5	7
5 – 7	10
7 – 9	23
9 – 11	51
11 – 13	6
13 – 15	3

Solution : We have,

Daily wages	Mean (x)	No. of workers (f)	$f \times x$
3 – 5	4	7	$7 \times 4 = 28$
5 – 7	6	10	$10 \times 6 = 60$
7 – 9	8	23	$23 \times 8 = 184$
9 – 11	10	51	$51 \times 10 = 510$
11 – 13	12	6	$6 \times 12 = 72$
13 – 15	14	3	$3 \times 14 = 42$
		$n = \Sigma f = 100$	$\Sigma fx = 896$

$$\text{Now, mean } M = \frac{\Sigma fx}{n} = \frac{896}{100} = 8.96$$

II. Median

If the data is arranged in ascending or descending form, then the data in the center is called the median. The **median** is that value of the given number of observations, which divides it into exactly two parts. So, when the data is arranged in ascending (or descending) order the median of ungrouped data is calculated as follows :

(i) Median of Ungrouped Data—First of all, write the data in ascending and descending order, then find the middle term of these data written in ascending or descending order.

- If number of observations is odd, then $\left(\frac{n+1}{2}\right)$th term will be middle term. Hence, the median will be the numeral value of the middle term.
- If the number of observations is even, then $\left(\frac{n}{2}\right)$th term and $\left(\frac{n}{2}+1\right)$th term be the two middle numbers. Hence, the median will be the half of the sum of values of these two middle numbers.

Median

$$= \frac{1}{2}\left[\frac{n}{2}\text{th term value} + \left(\frac{n}{2}+1\right)\text{th term value}\right]$$

Example : A student got the following marks in the nine question papers—

65, 36, 58, 62, 42, 40, 72, 82, 25

Find the median of the obtained marks.

Solution : Arrange the obtained marks in ascending order. We have,

25, 36, 40, 42, 58, 62, 65, 72, 82

Here, the total number of terms, $n = 9$, (an odd mumber)

$$\text{So, median} = \text{value of } \left(\frac{n+1}{2}\right)\text{th term}$$

= value of 5th term in the ascending order = 58 marks

III. Mode

In statistical data, if the frequency of a term is the highest, then the term is called the mode of the data.

Example : Let the weight of 15 students in a class are as follows :

51, 52, 53, 51, 51, 40, 39, 51, 45, 45, 50, 60, 51, 52, 51

These data can be presented in the form of a table as follows :

Weight (in kg)	39	40	45	50	51	52	53	60
Number of students	1	1	2	1	6	2	1	1

Here, the frequency of 51 is the highest. Hence, the mode of the observations is 51 kg.

IV. Relation among Mean, Median and Mode

a. $Mean - Mode = 3(Mean - Median)$

b. $3\ Median = 2Mean - Mode$

c. $Mode = 2Mean - 3Median$

Important Questions

1. Range of the data 25, 15, 23, 40, 27, 25, 23 and 42 is— **(NCERT)**

(A) 37 (B) 17

(C) 27 (D) 27.5

2. Which of the following is not a measure of central tendency?

(A) Mean (B) Median

(C) Mode (D) Range

3. For a given data, frequency 5 is shown by the following tally mark—

(A) ///// (B)

(C) (D)

4. If 9 is the mean of the observations $x, x+3, x+5, x+7, x+10$, then the mean of the last three observations—

(A) $10\frac{1}{3}$ (B) $10\frac{2}{3}$

(C) $11\frac{1}{3}$ (D) $11\frac{2}{3}$

5. Mean of the observations 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 will be— **(NCERT)**

(A) 10 (B) 11.5

(C) 10.5 (D) 11

6. Mean of the mean and mode of the observations 7, 6, 7, 9, 8, 8, 10, 8 is—

(A) 5·5 (B) 8

(C) 8·5 (D) 9

7. Standard deviation of set of the numbers 1, 4, 5, 7, 8, 10, 12, 13, 15, 17 is 4.85. If 10 is added to each number, then find the standard deviation of the new set.

(A) 48·50 (B) 4·85

(C) 0·485 (D) None of these

8. If the arithmetic of all the term of a table is 40 and their median is 50, then find its mode.

(A) 40 (B) 50

(C) 60 (D) 70

9. Sum of mean and median of the numbers 5·02, 5·18, 5·12, 5·007 and 5·018 is—

(A) 10·089 (B) 10·73

(C) 10·71 (D) 10·89

10. Mean of median, mode and range of the observations is—

(A) 10·3 (B) 10·5

(C) 8·6 (D) 8·8

11. The number of observations that fall under a class is called its— **(NCERT)**

(A) Mean

(B) Frequency

(C) Mode

(D) None of the above

12. If mean and median of a group of numbers are 8.9 and 9 respectively, then find their mode.

(A) 9·4 (B) 9·6

(C) 9·2 (D) 11·2

13. The method of displaying data through graphs where in the data is represented by rectangles whose base is x-axis, whose bases are of equal width and the space left between two rectangles is equal. **(NCERT)**

(A) Histogram (B) Frequency polygon

(C) Bar graph (D) None of these

14. The symbols 𝍸 and 𝍷 mean—

(A) 5, 5

(B) 5, Not defined

(C) Undefined, 5

(D) 3, 3

15. Mode of the data 25, 23, 22, 21, 24, 23, 25, 23, 22, 21, 23, 24, 23, 22, 23 is—

(A) 23 (B) 22

(C) 21 (D) 24

16. Mode of the data 29, 25, 38, 22, 38, 25, 38, 29 is—

(A) 29 (B) 38

(C) 25 (D) 22

17. Mean of the mode, median and range of the data 1, 2, 3, 3, 6, 4, 8, 14, 9, 4, 8, 4 is— **(NCERT)**

(A) 7 (B) 9

(C) 4 (D) 6

18. Sum of mean, mode and median of the observations 6, 3, 9, 5, 1, 2, 3, 6, 5, 1, 3 is—

(A) 10 (B) 11

(C) 12 (D) 13

19. Arithmetic mean of ungrouped data is—

(A) $\bar{x} = \frac{\Sigma x_i}{n}$ (B) $\bar{x} = \frac{n}{\Sigma x_i}$

(C) $\bar{x} = \frac{\Sigma x_i + n}{n}$ (D) None of these

20. Measures of central tendency means—

(A) Arithmetic mean

(B) Median

(C) Mode

(D) All the three

21. Arithmetic mean of grouped data is—

(A) $\bar{x} = \frac{\Sigma x_i}{n}$

(B) $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

(C) $\bar{x} = \frac{\Sigma f_i d_i}{\Sigma f_i}$

(D) $\bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$

22. If the number of obtained marks is odd, then median of the obtained marks is—

(A) $M = \frac{n+1}{2}^{th}$ term

(B) $M = \frac{\text{sum of the } \left[\frac{n}{2} + \left(\frac{n}{2}+1\right)\right]^{th} \text{ terms}}{2}$

(C) $M = \left(\frac{n}{2}\right)$th term

(D) None of the above

23. Which of the following is true?

(A) mode = 3 (median) + 2 (mean)

(B) mode = 2 (mean) – 3 (median)

(C) mode = 3 (median) – 2 (mean)

(D) None

24. Mean of the numbers 3, 5, 6, 8, 9, 7, 3, 4, 0 will be— **(NCERT)**

(A) 54 (B) 45

(C) 5 (D) 9

25. If mean of numbers 3, 8, 5, $2x + 1$, 5 is 8, then find x.

(A) 40 (B) 81

(C) 9 (D) 18

26. In class 11, the obtained marks in mathematics of the students are 45, 20, 41, 43, 25, 54, 47, 36, 40, 62, 55. Find its median.

(A) 41 (B) 47

(C) 45 (D) 43

27. Mean of 10 numbers is 12.5. Mean of first six numbers is 15 and mean of the last six numbers is 10, then find the sixth number.

(A) 10 (B) 15

(C) 12 (D) 13

28. M is the mean of a variable x. Find the mean of another variable $y = \left(\frac{x-4}{5}\right)$.

(A) $\frac{M-4}{5}$ (B) M

(C) M – 5 (D) 5M – 4

29. If M is the mean of y and $\frac{1}{y}$, then what will the mean of y^3 and $\frac{1}{y^3}$?

(A) $\frac{M(M^2-3)}{3}$ (B) M^3

(C) $M^3 - 3$ (D) $M(4M^2 - 3)$

30. Which relation among mode, median and mean is correct?

(A) Mean > median > mode

(B) Mean > mode > median

(C) Mean < mode < median

(D) Mode < mean < median

SOLUTIONS

1. (C) Arrange the data in ascending order,
15, 23, 23, 25, 25, 27, 40, 42
$\therefore$ range = 42 – 15 = 27

2. (D) Range

3. (B) 𝍸 = 5

4. (C) $\frac{x+x+3+x+5+x+7+x+10}{5} = 9$

$5x + 25 = 45$

$\Rightarrow \quad x = 4$

$\therefore$ Last 3 observations = $x + 5$, $x + 7$, $x + 10$

$= 9, 11, 14$

Required mean = $\frac{9+11+14}{3}$

$= \frac{34}{3} = 11\frac{1}{3}$

5. (D) Required mean = $\frac{\text{Sum of observations}}{\text{Number of observations}}$

$$= \frac{2+4+6+8+10+12+14+16+18+20}{10}$$

$$= \frac{110}{10} = 11$$

6. (B) Arranging the data,

6, 7, 7, 8, 8, 8, 9, 10

$\therefore$ mode = 8 and mean

$$= \frac{\frac{n}{2}\text{th term} + \left(\frac{n}{2}+1\right)\text{th term}}{2}$$

$$= \frac{1}{2}(4\text{th term} + 5\text{th term})$$

$$= \frac{1}{2}(8+8) = 8$$

$$\text{mean} = \frac{8+8}{2} = 8$$

7. (B) If 10 is added to each number the standard deviation will not change, because their variance remains the same, therefore, the new standard deviation = 4·85̇

8. (D) Mode = 3 × median − 2 × mean

$$= 3 \times 50 - 2 \times 40 = 70$$

9. (A) Mean

$$= \frac{5\text{■}2 + 5\text{■}8 + 5\text{■}2 + 5\text{■}07 + 5\text{■}18}{5}$$

$$= \frac{25{\cdot}345}{5} = 5{\cdot}069$$

$$\text{median} = \frac{5+1}{2} = 3\text{rd term} = 5{\cdot}02$$

So, required sum = 5·069 + 5·02

= 10·089

10. (C) Arranging the data in ascending order,

6, 6, 8, 8, 9, 9, 9, 14

$$\text{mean} = \frac{8+9}{2} = 8{\cdot}5$$

mode = 9

Range = 14 − 6 = 8

$\therefore$ Required mean $= \frac{8{\cdot}5+9+8}{3} = 8{\cdot}5$

$\approx$ 8·6 (appx.)

11. (B) Frequency

12. (C) $\because$ mode = 3 × median − 2 × mean

= 3 × 9 − 2 × 8·9

= 27 − 17·8

= 9·2

13. (A) Histogram

14. (B) 5, not define

15. (A) Arranging the data sequentially,

21, 21, 22, 22, 22, 23, 23, 23, 23, 23, 24, 24, 25, 25

$\therefore$ mode = 23

16. (B) Arranging the data sequentially,

22, 25, 25, 29, 29, 38, 38, 38

$\therefore$ mode = 38

17. (A) Arranging the data sequentially,

1, 2, 3, 3, 4, 4, 4, 6, 8, 8, 9, 14

$\therefore$ mode = 4

$$\text{mean} = \frac{1}{2}(6\text{th term} + 7\text{th term})$$

$$= \frac{1}{2}(4+4) = 4$$

range = 14 − 1 = 13

$\therefore$ Required mean $= \frac{4+4+13}{3} = \frac{21}{3}$

= 7

18. (A) Arranging the data sequentially,

1, 1, 2, 3, 3, 3, 5, 5, 6, 6, 9

$$\therefore \text{ median} = \frac{n+1}{2}\text{ th term}$$

= 6th term = 3

$$\text{mean} = \frac{1+1+2+3+3+3+5+5+6+6+9}{11}$$

$$= \frac{44}{11} = 4$$

mode = 3

$\therefore$ Sum = 3 + 4 + 3 = 10

19. (A) माध्य $= \frac{\Sigma x_i}{n}$

20. (D) All the three

21. (D) माध्य $= a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$

22. (A) $M = \left(\frac{n+1}{2}\right)$th term

23. (C) mode = 3(median) − 2(mean)

24. (C) माध्य

$$= \frac{3+5+6+8+9+7+3+4+0}{9}$$

$$= \frac{45}{9} = 5$$

25. (C) माध्य $= \frac{3+8+5+2x+1+5}{5}$

$$8 = \frac{22+2x}{5}$$

$$40 - 22 = 2x$$

$$2x = 18$$

$$x = 9$$

26. (D) Arranging the data sequentially,

20, 25, 36, 40, 41, 43, 45, 47, 54, 55, 62

Here, n = 11 (odd)

So, median $M = \left(\frac{n+1}{2}\right)$th term

$$= \left(\frac{11+1}{2}\right)\text{th term}$$

= 6th term

median M = 43

27. (B) Sixth number

= 6 × 15 + 5 × 10 − 10 × 12.5

= 90 + 50 − 125

= 15

28. (A) Mean of variable x = M

$$y = \text{mean of } \frac{x-4}{5}$$

$$= \frac{M-4}{5}$$

29. (D) $\frac{y+\frac{1}{y}}{2} = M$

$$y + \frac{1}{y} = 2M \qquad \ldots(1)$$

$$y^3 + \frac{1}{y^3}$$

$$= \left(y+\frac{1}{y}\right)\left(y^2 - y \times \frac{1}{y} + \frac{1}{y^2}\right)$$

$$= 2M\left[\left(y+\frac{1}{y}\right)^2 - 2y \times \frac{1}{y} - 1\right]$$

$$= 2M[(2M)^2 - 3]$$

$$\text{mean} = \frac{y^3 + \frac{1}{y^3}}{2}$$

$$= \frac{2M(4M^2-3)}{2}$$

$$= M(4M^2 - 3)$$

30. (A)

Chapter 19

Data Representation

1. Introduction

During your daily classes, you must have observed that your teacher recording the attendance in your class every day, or recording marks obtained by you after every test or examination. Similarly, you must have also seen a cricket score board. Twoscore boards have been illustrated here :

Name of the Bowlers	Overs	Maiden overs	Runs given	Wickets taken
P	20	2	50	4
Q	20	1	40	2
R	20	4	30	3
S	20	2	20	1

Name of the Batsmen	Runs	Balls faced	Time (in min.)
A	45	60	75
B	50	50	80
C	30	50	60
D	22	30	40

These above tables provide 'Data'. A data is a collection of numbers gathered to give some information.

2. Recording Data

Let us take an example of a class which is preparing to go for a picnic. The teacher asked the students to give their choice of fruits out of banana, apple, orange or guava. Kanchan is asked to prepare the list. She prepared a list of all the children and wrote the choice of fruit against each name. This list would help the teacher to distribute fruits according to the choice.

Anil	Banana	Yash	Apple
Ansh	Guava	Rajat	Banana
Aryan	Apple	Drashti	Guava
Reha	Apple	Reva	Banana
Somya	Orange	Vishu	Guava
Anirudh	Apple	Atharv	Banana
Yogesh	Orange	Pooja	Banana
Dev	Apple	Ravi	Banana
Rahul	Banana	Vinay	Apple

If the teacher wants to know the number of bananas required for the class, she has to read the names in the list one by one and count the total number of bananas required. To know the number of apples, guavas and oranges separately she has to repeat the same process for each of these fruits. How tedious and time consuming it is! It might become more tedious if the list has, say, 100 students.

3. Organisation of Data

To resolve the above problem, Ansh prepares the following table:

Name of Fruits	Number of Students
Bananas	|||||||
Apples	||||||
Guavas	|||
Oranges	||

'Rajat, after seeing the table suggested a better method to count the students. He asked Ansh to organise the marks (|) in a group of five. Teacher suggested that the fifth mark in a group of five marks should be used as a cross, as shown by . These are tally marks. Thus,shows the count to be five plus two (*i.e.* seven). With this, the table looks like :

Name of Fruits	Digits	Number of Students
Bananas	7	
Apples	6	
Guavas	3	|||
Oranges	2	||

Example : Esha is asked to collect data for number of flowersto students in her Class VI. Her finding are recorded in the manner shown below :
5 4 7 5 6 7 6 5 6 6 5 4 5 6 8 7 4 6 5 6 4 6 5 7 6 7 5 7 6 4 8 7
Find out the number of flowers has by the maximum number of students?

Solution : We have,

Numbers of Flowers	Number of Students	Tally marks
4	5	
5	8	|||
6	10	
7	7	||
8	2	||

Hence, it is clear from the table that 10 students have 6 flowers.

4. Pictograph

A pictograph represents data through pictures of objects. It helps answer the questions on the data at a glance. Pictographs are often used by dairies and magazines to attract readers attention. Collect one or two such published pictographs and display them in your class. Try to understand what they say. It requires some practice to understand the information given by a pictograph.

Example : A survey was carried out on 30 students of class VI in a school. Data about the different modes of transport used by them to travel to school was displayed as pictograph. What can you conclude from the pictograph ?

Modes of Travelling	Number of Students ⊙ – 1 Student
Private car	⊙ ⊙ ⊙ ⊙
Public bus	⊙ ⊙ ⊙ ⊙ ⊙
School bus	⊙ ⊙ ⊙ ⊙ ⊙ ⊙ ⊙ ⊙ ⊙ ⊙
Cycle	⊙ ⊙ ⊙ ⊙
Walking	⊙ ⊙ ⊙ ⊙ ⊙ ⊙ ⊙

Solution : From the pictograph we find that :

(a) The number of students coming by private car is 4.

(b) Maximum number of students use the school bus. This is the most popular way.

(c) Cycle is used by only four students.

5. Tabulation

The data obtained by a survey is represented with the help of table. Questions have to be answered based on the information received from the table. The questions can be understood with the help of the examples.

Example : Direction (Q. No. 1 to 4)

The following table shows distance covered by two persons A and B in a single direction.

Hours	Covered Distance (In kms)	
	A	B
First	20	25
Second	30	40
Third	20	35
Fourth	15	25
Fifth	25	35
Sixth	15	10
Seventh	25	25
Eighth	35	15
Ninth	20	25
Tenth	30	45

1. In first 4 hours, the average speed of B (km/h) is %

(A) 21.25 (B) 22

(C) 31.25 (D) 32

2. Ratio between A's speed for first five hours and last five hours is :

(A) 25 : 22 (B) 22 : 25

(C) 15 : 22 (D) 20 : 21

3. Find the distance between A and B at the end of eighth hour.

(A) 30 km (B) 25 km

(C) 15 km (D) 12 km

4. After how many hours will the distance between A and B be maximum ?

(A) 2 hr (B) 3 hr

(C) 4 hr (D) 5 hr

Solution :

1. (C) Average speed of B in first 4 hours

$$= \frac{25 + 40 + 35 + 25}{4}$$

$$= \frac{125}{4} = 31.25 \text{ km/h}$$

2. (B) Average speed of A in first 5 hours

$= 20 + 30 + 20 + 15 + 25$

$= 110$ km/h

Average speed of A in last 5 hours

$= 30 + 20 + 35 + 25 + 15$

$= 125$ km/h

Ratio $= 110 : 125$

$= 22 : 25$

3. (B) Distance covered by A in 8 hrs :

$= 20 + 30 + 20 + 15 + 25 + 15 + 25 + 35$

$= 185$ km/h

Distance covered by B in 8 hrs :

$= 25 + 40 + 35 + 25 + 35 + 10 + 25 + 15$

$= 210$ km/h

Difference $= 210 - 185 = 25$ km/h

4. (D) Distance between them in fifth hour :

$= 160 - 110 = 50$ km/h (maximum)

Example : Direction (Q. No. 5 to 9)

Read carefully content of the given table and answer the questions :

Annual production of scooters in different factories

(In thousands)

Fact.	1985	1986	1987	1988	1989
P	20	15	24	13	17
Q	16	23	41	20	15
R	14	21	30	16	12
S	25	17	15	12	22
T	40	32	39	41	35
Sum	115	108	149	102	101

5. In which year the production of scooters in all factories was equal to the average annual production held in the time period of 1985-89 ?

(A) 1985 (B) 1986

(C) 1987 (D) 1988

6. In comparison of year 1988, which factory/ies show 25% depreciation in the production of scooters in the year 1989?

(A) P (B) S

(C) Q and R (D) P and T

7. In 1985, ratio between the production of scooters in factory P and T :

(A) 2 : 3 (B) 1 : 2

(C) 3 : 2 (D) 2 : 1

8. In which year the total production of scooters was maximum?

(A) 1989 (B) 1986

(C) 1987 (D) 1985

9. In which year the total production of scooters of all factories was 20% of the production produced in the time period of 1985-89 ?

(A) 1988 (B) 1985

(C) 1986 (D) 1989

Solution :

5. (A) Average production from 1985 to 1989 :

$$= \frac{115+108+149+102+101}{5}$$

$$= \frac{575}{5} = 115$$

It equals to the average production of year 1985.

6. (C) Q and R

7. (B) 20 : 40 = 1 : 2

8. (C) 1987

9. (B) 20% of the total production from 1985–1989

$$= 575 \times \frac{20}{100} = 115$$

It equals to the production of year 1985.

6. Line Graph

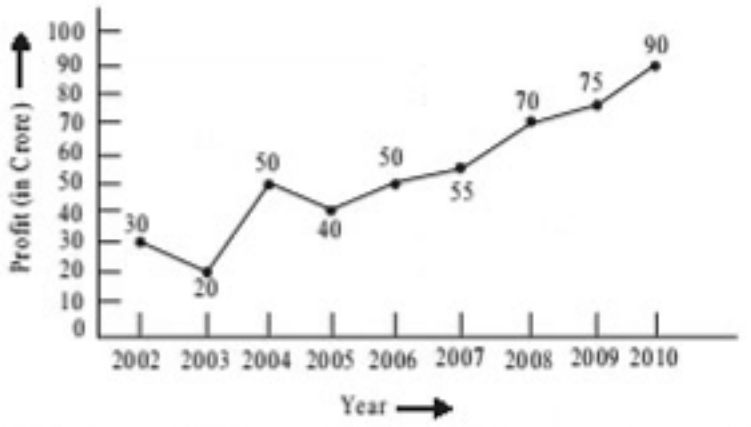
In line graph, any problem is defined or represented by line graph. Answers are to be given by deep study of the line graph. The questions related to the line graph can be understood by the following example :

Example : Direction (Q. No. 10 to 14)

Read carefully content of the given table and answer the questions :

Earned profit by a company in given years

(In Crores)



10. If the income in the year 2003 was 60 crores and expenditure of the year equals to the expenditure of the year 2005, then what was the income of the company in the year 2005?

(A) 75 (B) 80

(C) 65 (D) 70

11. If the expenditure of the year 2007 was 60 crores, then what was the income of the company in that year?

(A) 115 (B) 120

(C) 85 (D) 90

12. Find the year in which the percentage increase in profit was the highest compared to the previous year.

(A) 2006 (B) 2008

(C) 2002 (D) 2004

13. If the income for the year 2006 was 120 crores, then what percentage is the profit of the year? (in the nearest integers)

(A) 42% (B) 51%

(C) 41·66% (D) 43%

14. What percentage increase (in the nearest integers) occurs in the profit of the year 2009 compared to the previous year?

(A) 9% (B) 10%

(C) 7% (D) 8%

Solution :

10. (B) **For the year 2003 :**

Total income = 60 crores

Profit = 20 crores

Expenditures = 60 – 20 = 40 crores

For the year 2005 :

Expenditures = 40 crores

Profit = 40 crores

Income = 40 + 40 = 80 crores

11. (A) **In the year 2007 :**

Expenditures = ₹ 60 crores

Profit = ₹ 55 crores

Income = 60 + 55 = 115 crores

12. (D) 2004

13. (A) Profit % $= \frac{50}{120} \times 100$

$= \frac{500}{12} = 41\frac{2}{3}\%$

= 42% (approx.)

14. (C) Profit % $= \frac{75 - 70}{70} \times 100$

= 7.14 %

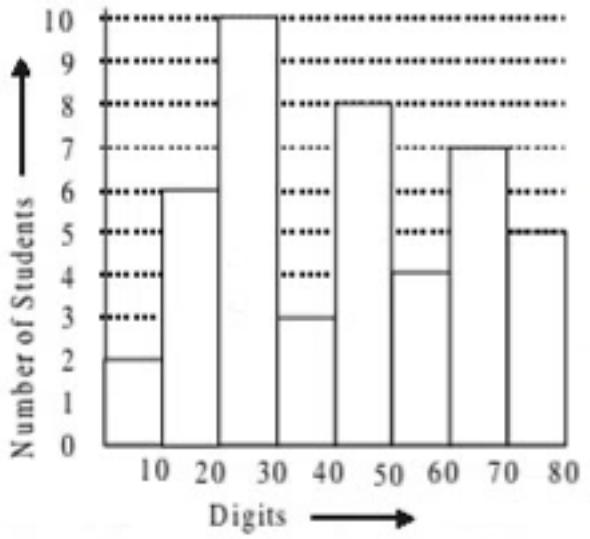
= 7 % (approx.)

7. Histogram

Answers are to be given by deep study of the histogram. The questions related to the histogram can be understood by the following example :

Example: Direction (Q. No. 15 to 19)

In histogram, marks of 45 students of a class are shown. Answer the question after deep study of the following histogram :



15. How many such students who got 50 or more than marks?

(A) 9 (B) 10

(C) 11 (D) 16

16. If 30 marks are set to pass, then what will be the number of failed students?

(A) 2 (B) 6

(C) 18 (D) 20

17. If 30 marks are the minimum marks to pass, then what will the percentage of passed students?

(A) 75% (B) 60%

(C) 50% (D) 40%

18. How many students are there who got less than 10 marks?

(A) 2 (B) 10

(C) 1 (D) 4

19. How many students are there who got more than 30 but less than 40 marks?

(A) 3 (B) 4

(C) 5 (D) 6

Solution :

15. (D) Required number of students

= 4 + 7 + 5 = 16

16. (C) Number of failed students

= 6 + 2 + 10 = 18

17. (B) Percentage of passed students

$= \frac{3+8+4+7+5}{45} \times 100 = 60\%$

18. (A) Required answer = 2

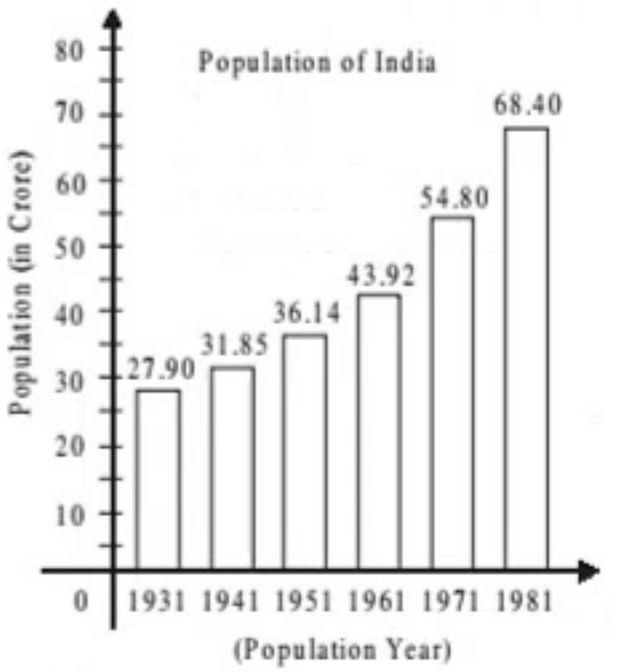
19. (A) Required answer = 3

8. Bar Graphs

Answers are to be given by deep study of the histogram. The questions related to the histogram can be understood by the following example :

Example: Direction (Q. No. 20 to 23)

Here, the bar graph shows the population of India (in crores) in the different years. Study the bar graph carefully and answer the questions :



20. The percentage increase in population in the Census year 1981 compared to the Census year :

(A) 24.8 (B) 20

(C) 16.7 (D) 22.9

21. Find the census year in which the percentage increase in population is the highest compared to the previous census year.

(A) 1951 (B) 1961

(C) 1971 (D) 1981

22. Find the census year in which the percentage increase in population is the lowest compared to the previous census year.

(A) 1961 (B) 1951

(C) 1971 (D) 1941

23. Annual increase in the population from year 1931 to 1981 is :

(A) 8100000 (B) 7600000

(C) 8900000 (D) 6700000

Solution :

20. (A) Required increase :

$$= \left(\frac{68.40 - 54.80}{54.80}\right) \times 100$$

$= 24.8\ \%$

21. (D) 1981

22. (D) 1941

23. (A) Annual growth $= \left(\frac{68.40 - 27.90}{1981 - 1931}\right)$

$$= \frac{40.5}{50} \times 10000000$$

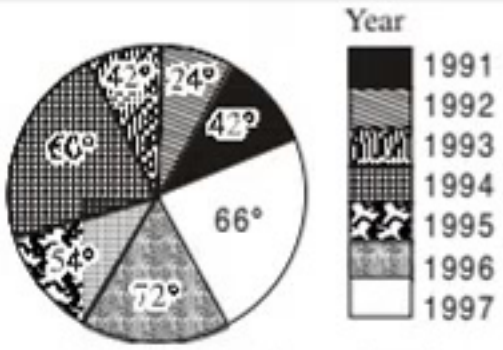
$= 8100000$ crores

9. Pie Chart

Displaying any problem through circular angles is called a pie chart. The questions related to the pie chart can be understood by the following example :

Example: Direction (Q. No. 24 to 27)

The pie chart shows earned profit in seven consecutive years by a company. Study the pie chart carefully and answer the questions:



24. If $x\%$ of sum of the earned profits equals to the earned profit in the year 1994, find x.

(A) $16\frac{2}{3}$ (B) $33\frac{1}{3}$

(C) $12\frac{1}{2}$ (D) $11\frac{2}{3}$

25. Ratio between expenditure and income in the years 1992, 1994, and 1996 are 6 : 5 : 8 and 2 : 3 : 4. Find expenditure ratio between the income of the year 1996 and the total expenditures of the years 1992 and 1994.

(A) 40 : 11 (B) 10 : 7

(C) 20 : 11 (D) 20 : 13

26. Profit in a year P is nearest to the average of earned profits in all the given years. Find the year P.

(A) 1991 (B) 1995

(C) 1993 (D) 1994

27. If income in the year 1997 was 5 times the expenditure of the same year, then find the ratio between earned profit in the year 1991 and the expenditure in the year 1997.

(A) 11 : 28 (B) 44 : 7

(C) 28 : 11 (D) 7 : 44

Solution :

24. (A) $360 \times \frac{x}{100} = 60$

$$\Rightarrow \quad x = \frac{60 \times 100}{360}$$

$$= \frac{50}{3} = 16\frac{2}{3}\%$$

25. (C) Let in the given years, respective incomes are $2x$, $3x$, $4x$ and the expenditures are $6y$, $5y$, $8y$.

$\therefore \quad 2x - 6y = 24$ ...(i)

and $3x - 5y = 60$ ...(ii)

From the eq.(i) $\times 3$ – (ii) $\times 2$,

$$6x - 18y - 6x + 10y = 72 - 120$$

$\Rightarrow \quad -8y = -48$

$\Rightarrow \quad y = 6$

From the eq.(i),

$$2x - 6 \times 6 = 24$$

$\Rightarrow \quad 2x = 36 + 24 = 60$

$\Rightarrow \quad x = 30$

$\therefore \quad x = 30$ and $y = 6$

Required ratio $= 4x : 11y$

$= 120 : 66$

$= 20 : 11$

26. (B) Average profit $= \frac{360}{7} = 51.4$

which is nearest to the profit in the year 1995.

27. (C) Ratio $= 42 : \frac{33}{2} = 28 : 11$

Example : Direction (Q. No. 28 to 30)

The pie chart shows the marks of different subjects of a students in a exam who got 720 marks. Study the pie chart carefully and answer the questions :

28. How many marks got in Science?

(A) 200 (B) 300

(C) 75 (D) 150

29. In which subject the student got $16\frac{2}{3}$: of his total marks?

(A) Bangla (B) History

(C) English (D) Math

30. In which subject the student get 180 marks?

(A) Math (B) Bangla

(C) English (D) Science

Solution :

28. (D) Obtained marks in Science $\%$

$$= \frac{75^\circ}{360^\circ} \times 720 = 150$$

29. (C) $100\% \longrightarrow 360^\circ$

$$1\% \longrightarrow \frac{360^\circ}{100}$$

$$\frac{50}{3}: \longrightarrow \left(\frac{360^\circ}{100} \times \frac{50}{3}\right)^\circ$$

$= 60^\circ$

= English

30. (A) 720 marks $\longrightarrow 360^\circ$

$$1 \text{ mark} \longrightarrow \left(\frac{360^\circ}{720}\right)^\circ$$

$$180 \text{ marks} \longrightarrow \frac{360^\circ}{720} \times 180$$

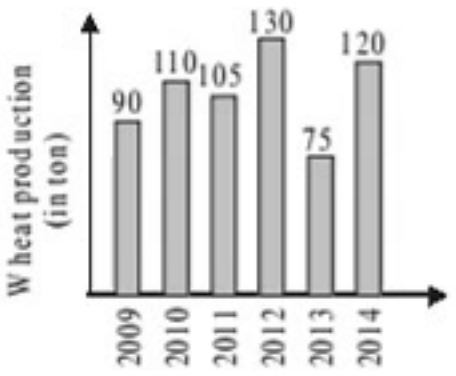
$= 90^\circ$

= Math

Important Questions

1. The given bar graph shows the wheat production (in tonnes) of a farm from the year 2009 to 2014. How many times the total production of 'odd', years is equal to the total production of 'even' years?

(NCERT)



(A) $\frac{2}{3}$ (B) 0.75

(C) 1.33 (D) None of these

Direction (Q. No. 2 to 4)

The given pie chart shows (in degree) the expenditure incurred on various sports during a particular year. Study the graph carefully and answer the questions.

2. What percentage of total expenditure was spent on tennis?

(A) $12\frac{1}{2}\%$ (B) $22\frac{1}{2}\%$

(C) 25% (D) 45%

3. If the total amount spent in that year is 2 crores, then the total amount spent on cricket and hockey was :

(A) ₹ 8,00,000 (B) ₹ 80,00,000

(C) ₹ 12000000 (D) ₹ 16000000

4. If the total amount spent during the year is 18000000, then the amount spent on basketball will be more than the amount spent on tennis, approximately—

(NCERT)

(A) ₹ 250000 (B) ₹ 360000

(C) ₹ 900000 (D) ₹ 410000

5. The following pie-chart shows a person's expenses. Its total income is 28000. If it saves 4900, then the value of x is–

(A) 136° (B) 126°

(C) 63° (D) 45°

6. According to the given graph, in which two consecutive months, the change in auto sales (auto sales) was highest?

(A) April - May

(B) May - June

(C) January - February

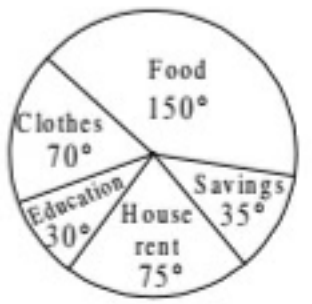
(D) February - March

7. If the figures to be displayed graphically on the Y-axis are 1.5, 2.1, 2.7 and 3.3 in lakhs, what would be the appropriate scale to represent a unit? (In thousand)

(A) ₹ 50 (B) ₹ 30

(C) ₹ 40 (D) ₹ 60

8. The expenditure incurred on various items in the following pie chart is in descending order.



(A) Education, Saving, Clothes, Rent, Food

(B) Food, rent, clothes, saving, education

(C) Food, rent, education, clothes, saving

(D) Rent, food, education, clothes, saving

9. The figures for the height of 35 leaves of a plant up to one mm are correctly in the following table. **(NCERT)**

Length (in mm)	Number of leaves
127–135	5
136–144	9
145–153	12
154–162	5
163–171	4

Is it ture that the length of the leaves mostly are of–

(A) 153 (B) 145

(C) 149 (D) None of these

10. The given pie-diagram shows the cost of building a house. If the total cost of construction of the house is 1500000, then wages will be spent.

(A) ₹ 90,000 (B) ₹ 2,50,000

(C) ₹ 3,60,000 (D) ₹ 3,75,000

Direction (Q. No. 11 to 14)

Study the table carefully and answer the questions.

Textile Company Pvt. Ltd. (In profit)

Year	Total sale	Total profit	Net profit
2010	351.6	155.5	54.2
2011	407.9	134.3	42.6
2012	380.1	149.9	38.9
2013	439.7	160.5	50.3
2014	485.9	203.3	65.8

11. In which year is the least difference between total sales and gross profit? **(NCERT)**

(A) 2010 (B) 2011

(C) 2012 (D) 2013

12. Total sales in the year 2013 is approximately what percent of the total sales in the year 2010?

(A) 75 (B) 85

(C) 110 (D) 125

13. In which years did the three items together increase in total sales, gross profit and net profit, as compared to the previous year?

(A) 2013 and 2014 both

(B) 2014 and 2012 both

(C) 2012 and 2013 both

(D) 2010 and 2011 both

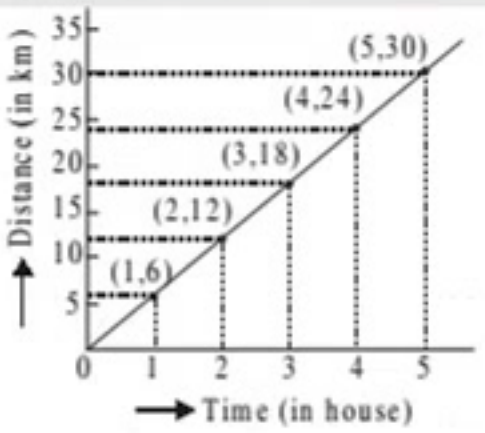
14. In which year was the percentage increase in gross profit higher than in the previous year?

(A) 2011 (B) 2012

(C) 2013 (D) 2014

Direction (Q. No. 15 to 18)

The graph given here shows the linear path followed by a bus moving at the same speed. Study the graph and answer.



15. Speed of the bus (in km/h) is :

(A) 12 (B) 6

(C) 18 (D) 24

16. Speed of the bus (in meter/min) is :

(A) 60 (B) 100

(C) 600 (D) 1000

17. The distance covered by the bus in 4.5 hours :

(A) 27 km (B) 30 km

(C) 36 km (D) 40 km

18. In what time does the bus cover the distance of 15 km? **(NCERT)**

(A) 3 hrs (B) 2 hrs

(C) 1.5 hrs (D) 2.5 hrs

SOLUTIONS

1. (B) Production of even years (2010, 2012, 2014)

$= 110 + 130 + 120 = 360$

Production of odd years (2009, 2011, 2013)

$= 90 + 105 + 75 = 270$

Required answer $= \frac{270}{360} = \frac{3}{4} = 0{\cdot}75$

2. (A) Expenses on Tennis $= \frac{45°}{360°} \times 100$

$= \frac{100}{8} = \frac{25}{2} = 12\frac{1}{2}\%$

3. (B) Expenses on Cricket and Hockey

$= \frac{81+63}{360°} \times 100$

$= \frac{144}{360} \times 100 = 40\%$

So, total expenses

$= \frac{40}{100} \times 2,00,00,000$

$= 80,00,000$

4. (C) Required answer

$= \frac{63-45}{360°} \times 18000000$

$= \frac{18}{360} \times 18000000$

$= 900000$

5. (C) $x° = \frac{4900 \times 360}{28000} = 63°$

6. (A) April - May

7. (B) ₹ 30 thousands

8. (B) Food, rent, clothes, saving, education

9. (C) 149 mm

10. (D) Wages % $= \frac{90°}{360°} \times 100\%$

$= 25\%$

$\therefore$ ₹ 25% of 15,00,000

$= \frac{25}{100} \times 1500000$

$=$ ₹ 3,75,000

11. (A) 2010

12. (D) Total sale in 2013 = 439·7

Total sale in 2010 = 351·6

$\therefore$ Required % $= \frac{439{\cdot}7}{351{\cdot}6} \times 100$

$= 125{\cdot}05\%$

$\cong 125\%$

13. (A) 2013 and 2014 both

14. (D) 2014

15. (B) 6 km/h

16. (B) Speed of bus $= \frac{6 \times 1000}{60}$

$= 100$ m/min

17. (A) Total distance

$= 24 + 3 = 27$ km

18. (D) 2·5 hours